UNIVERSITY OF SWAZILAND
FINAL EXAMINATION PAPER

PROGRAMME; BSc. AGRICULTURAL AND BIOSYSTEMS ENGINEERING YR 1

COURSE CODE: ABE104

TITLE OF PAPER: ENGINEERING MATHEMATICS

TIME ALLOWED: TWO (2) HOURS

SPECIAL MATERIAL REQUIRED: NONE

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY TWO OTHER QUESTIONS

SHOW ALL YOUR WORK. NO CREDIT SHALL BE AWARDED TO ANSWERS WIRHOUT CLEAR WORKING

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Question 1 (Compulsory)

a. Find the equation of the straight line through $(4, -7)$ and perpendicular to the line $4x - 3y = 9$. [4 marks]

b. Solve for x, given $4 \cdot e^{2x-5} = 789$ (correct to 3 s.f.). [4 marks]

c. Given the matrices $A = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$, and $B = \begin{pmatrix} -3 & 2 \\ 1 & -1 \\ 0 & 4 \end{pmatrix}$,

find the value of AB^T. [5 marks]

d. Given the vectors $\underline{A} = -2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$ and $\underline{B} = 6\hat{\imath} - 7\hat{\jmath}$, find

i. $\underline{A} \cdot \underline{B}$ [3 marks]

ii. $\underline{A} \times \underline{B}$ [5 marks]

e. Differentiate

i. $y = 5x^4 - \dfrac{8}{\sqrt{x}} + \dfrac{3}{x^4}$ [3 marks]

ii. $y = \sqrt{2x^2 + 1}$ [3 marks]

f. Evaluate $\displaystyle\int_1^9 \left(3x^2 - \frac{4}{\sqrt{x}} + \frac{3}{x}\right) \mathrm{d}x$ (correct to 1 d.p.) [5 marks]

g. Find the general solution of the ODE

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x\sqrt{y}.$$ [5 marks]

h. A password is to be created using two letters followed by 4 digits chosen from `0123456789`. How many different paswords are possible if repitition is not allowed? [3 marks]

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ANSWER ANY TWO (2) QUESTIONS FROM THE THE FOLLOWING

Question 2

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a. Solve for x, given

i. $\dfrac{20}{1+7e^{-x}} = 15$ [5 marks]

ii. $\log x - \log(5x-7) = 1$ [4 marks]

b. On 01 January 2016, a sum of E10,000 is invested at an interest rate of 9.4% compounded continuously. The total amount grows according to

$$A(t) = 10,000e^{0.094t}$$

where t is the number of years after 01 January 2016. Find

i. the total amount after 5 years [2 marks]

ii. the date when the total amount reaches E20,000 [5 marks]

c. A farm is in the shape of a triangle. The grid coordinates of the corners (in metres) are $A(40, 50)$, $B(2540, 4650)$ and $C(5420, 20)$. Find

i. the perimeter of the farm in kilometres (correct to 1 d.p.) [6 marks]

ii. the area of the farm in hactres (correct to 1 d.p.) [8 marks]

Question 3

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a. Differentiate

i. $y = (3x-1)e^{3x}$ [3 marks]

ii. $y = \dfrac{3x-2}{x}$ [3 marks]

iii. $y = \dfrac{x}{3x-2}$ [4 marks]

iv. $y = \ln\left(\dfrac{x}{1+x}\right)$ [4 marks]

b. Consider the function

$$y = x^3 - 3x^2 - 24x + 17.$$

Find the stationary points and classify them. [8 marks]

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c. A farmer has E54,000 to spend on constructing a rectangular holding for her livestock. If one side is to use heavy-duty fence costing E40/m while the other 3 sides use regular fence costing E10/m, find the dimensions of the *largest* holding that can be constructed with the available funds. [8 marks]

Question 4

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a. Integrate

$$\int 8x^2 e^{-2x} \mathrm{d}x.$$ [8 marks]

b. From the top of 250m tall tower, a bullet is fired vertically upwards with an initial velocity of 490m/s. Taking gravitational acceleration to be 9.8m/s^2, find

  i. the equation of the velocity $v(t)$ of the bullet [3 marks]

  ii. the equation of the height $v(t)$ of the bullet [3 marks]

  iii. the maximum height reached by the bullet [3 marks]

  iv. the time at which the bullet hits the ground [4 marks]

c. At time $t = 0$, a can of soft drink at temperature 45^0C is placed in a fridge. If the temperature inside the fridge is 5^0C, then temperature $T(t)$ of the soft drink declines according to the ODE

$$\frac{\mathrm{d}T}{\mathrm{d}t} = 0.04(5 - T),$$

where t is the number of minutes after placing the drink inside the fridge.

  i. Solve the ODE to obtain a formula for $T(t)$. [7 marks]

  ii. Hence determine the temperature of the soft drink 30 minutes. [2 marks]
