**UNIVERSITY OF SWAZILAND**
**MAIN EXAMINATION PAPER 2009/2010**

**TITLE OF PAPER:** BIOSTATISTICS

**COURSE CODE:** B305

**TIME ALLOWED:** THREE (3) HOURS

**INSTRUCTIONS:**

1. ANSWER ANY FOUR QUESTIONS.
2. EACH QUESTION CARRIES TWENTY FIVE (25) MARKS.
3. ILLUSTRATE YOUR ANSWERS WITH LARGE AND CLEARLY LABELED DIAGRAMS WHERE APPROPRIATE.
4. CLEARLY STATE YOUR NULL AND ALTERNATIVE HYPOTHESES AND YOUR CONCLUSIONS WHERE APPROPRIATE.

**SPECIAL REQUIREMENTS:**

1. CALCULATORS (CANDIDATES MUST BRING THEIR OWN).
2. GRAPH PAPER.
3. STATISTICAL TABLES (TO BE SUPPLIED BY THE LECTURER).
4. USEFUL EQUATIONS (TO BE SUPPLIED BY THE LECTURER).

**ANSWER ANY FOUR (4) OUT OF SIX (6) QUESTIONS**

## QUESTION 1

The following data were collected by an ecologist:

| Mass (g) | Altitude (m) |
|---|---|
| 53 | 200 |
| 57 | 400 |
| 64 | 600 |
| 71 | 800 |
| 80 | 1000 |
| 84 | 1200 |

a) Which one is the independent variable? [1 marks]

b) Calculate *a* and *b* for the regression of body length on altitude. [8 marks]

c) Is there a significant relationship between body length and altitude? Use an **appropriate** statistical test to support your answer. [6 marks]

d) Plot these data on an appropriate graph. Make sure that you include the best fit line (based on your calculations in (b) above). [10 marks]

**[TOTAL = 25 marks]**

## QUESTION 2

a) Show, by means of a sketch, what a normal distribution looks like. [2 marks]

b) A normally distributed population of plants has a mean flower diameter 103.1 mm and a standard deviation of 7.9 mm.

I. What proportion of this population is 100.0 mm larger? [4 marks]

II. If 1000 individuals were measured, how many are 100.0 mm or smaller? [1 mark]

III. What is the probability of selecting at random from this population a flower smaller than 95 mm? [3 marks]

**QUESTION 2 (continued)**

IV. What is the probability of choosing at random from this population a sample of 10 weights that has a mean greater than 105 mm? [5 marks]

c) Present the following data in a graph that shows the mean, 95% confidence intervals, range and number of observations for each season. [10 marks]

| Latitude | N | Mean time (min) | Standard error | Range |
|---|---|---|---|---|
| 06°S | 23 | 32.2 | 2.60 | 25.0-39.0 |
| 18°S | 29 | 38.4 | 1.77 | 31.0-43.5 |
| 33°S | 19 | 39.9 | 1.63 | 32.3-44.1 |

**[TOTAL = 25 marks]**

**QUESTION 3**

a) What are the assumptions of parametric tests? [6 marks]
b) The data given in the table below are not parametric. Why not? [2 marks]

| | **Mass (g)** | |
|---|---|---|
| | **Pop. A** | **Pop. B** |
| | 23 | 11 |
| | 27 | 12 |
| | 22 | 14 |
| | 18 | 9 |
| | 24 | 8 |
| Mean | 22.8 | 10.8 |
| Variance | 10.70 | 5.70 |

c) Using an appropriate transformation, transform these data. [6 marks]

d) Are the transformed data now parametric? If so, why? [2 marks]

e) Using an appropriate test, test whether the masses of the two populations are the same.

[9 marks]
**[TOTAL = 25 marks]**

## QUESTION 4

The following table shows the incubation period of three different bird species. The data meet the assumptions of parametric tests.

**Incubation period (days)**

| Species 1 | Species 2 | Species 3 |
|---|---|---|
| 13 | 13 | 16 |
| 12 | 12 | 14 |
| 13 | 14 | 15 |
| 12 | 13 | 15 |
| 12 | 14 | 16 |

a) Using an appropriate statistical test, establish whether the three different species have significantly different incubation periods.

[15 marks]

b) If you rejected the $H_0$ in (a) above, then do a multiple comparison to show which species differ (or are similar) in incubation period compared with other species.

[10 marks]

**[Total = 25 marks]**

## QUESTION 5

a) Consider a binomial distribution with P = 0.65. Calculate P(X=0) up to P(X=4). [8 marks]

b) Could the following data have come from the above distribution? Show your computations.

| X | Observed numbers |
|---|---|
| 0 | 2 |
| 1 | 12 |
| 2 | 30 |
| 3 | 41 |
| 4 | 18 |

[8 marks]

b) Six (6) out of 8 humans died after being exposed to a particular strain of the flu virus. Use the binomial test to test the null hypothesis that equal numbers of humans died and survived exposure to the virus.

[9 marks]
**[TOTAL = 25 marks]**

**QUESTION 6**

a) Define a normal distribution. [5 marks]

b) Present the following data in a histogram: [8 marks]

| Calories burnt | Number of records |
|---|---|
| <110 | 3 |
| 110 | 2 |
| 120 | 31 |
| 130 | 42 |
| 140 | 69 |
| 150 | 36 |
| 160 | 9 |
| >160 | 3 |

c) What is the difference between a histogram and a box plot? [3 marks]

d) Name and describe the different types of statistical data. [9 marks]
**[TOTAL = 25 marks]**

# B305 (BIOSTATISTICS) EXAMINATION

## USEFUL EQUATIONS

$S^2 = \sum (y_i - \bar{y})^2/(n-1)$

$SE = S/\sqrt{n}$

$Z = (y_i - \mu)/\sigma$

$Z = (\bar{y} - \mu)/SE$

C.I.: $\bar{y} \pm [t_{(\alpha,df)}]SE$

$t = (\bar{y} - \mu)/SE$

$t = (\bar{y}_1 - \bar{y}_2) / SE_{pooled}$

$SE_{pooled} = SD_{pooled}\sqrt{(1/n_1 + 1/n_2)}$

$SD_{pooled} = \sqrt{[(df_1 s_1^2 + df_2 s_2^2)/(df_1 + df_2)]}$

$\chi^2 = \Sigma (O - E)^2/E$

$\chi^2 = [n(|cell1*cell4-cell2*cell3|-n/2)^2]/[C_1*C_2*R_1*R_2]$

Total SS $= \Sigma\Sigma y^2 - (\Sigma\Sigma y)^2/N$

Groups SS $= \Sigma(\Sigma y)^2/n - (\Sigma\Sigma y)^2/N$

Error SS = Total SS – Groups SS

F = Groups MS/Error MS

$q = (\bar{y}_a - \bar{y}_b)/SE$

$y' = \log(y+1)$

$y' = \sqrt{(y+0.5)}$

$y' = \arcsin\sqrt{y}$

$\Phi = \sqrt{[(k-1)(\text{groups MS-error MS})]/(k)(\text{error MS})}$

$U = n_1 n_2 + n_1(n_1+1)/2 - R_1$

$H = 12/N(N+1)\Sigma[R^2/n] - 3(N+1)$

$r = \Sigma xy/\sqrt{(\Sigma x^2)(\Sigma y^2)}$

$b = \Sigma xy/ \Sigma x^2$

$\Sigma xy = \Sigma XY - (\Sigma X)(\Sigma Y)/n$

$\Sigma x^2 = \Sigma X^2 - (\Sigma X)^2/n$

$\Sigma y^2 = \Sigma Y^2 - (\Sigma Y)^2/n$

Total SS $= \Sigma y^2$

Regression SS $= (\Sigma xy)^2/ \Sigma x^2$

Residual SS = Total SS – Regression SS

$P(X) = n!/X!(n-X)![p^X(q^{n-X})]$

$p = [\Sigma(X)(Obs)/\Sigma(Obs)]/n$

$P(X) = \mu^X/e^\mu(X!)$

$\mu = [\Sigma(X*Obs)]/[\Sigma Obs]$

## TABLE B.1 CRITICAL VALUES OF THE CHI-SQUARE DISTRIBUTION

| ν | α = 0.999 | 0.995 | 0.99 | 0.975 | 0.95 | 0.90 | 0.75 | 0.50 | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.000 | 0.000 | 0.000 | 0.001 | 0.004 | 0.016 | 0.102 | 0.455 | 1.323 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 | 10.828 |
| 2 | 0.002 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 0.575 | 1.386 | 2.773 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 | 13.816 |
| 3 | 0.024 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 1.213 | 2.366 | 4.108 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 | 16.266 |
| 4 | 0.091 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 1.923 | 3.357 | 5.385 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 | 18.467 |
| 5 | 0.210 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 2.675 | 4.351 | 6.626 | 9.236 | 11.070 | 12.833 | 15.086 | 16.750 | 20.515 |
| 6 | 0.381 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 3.455 | 5.348 | 7.841 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 | 22.458 |
| 7 | 0.599 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 4.255 | 6.346 | 9.037 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 | 24.322 |
| 8 | 0.857 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 5.071 | 7.344 | 10.219 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 | 26.124 |
| 9 | 1.152 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 5.899 | 8.343 | 11.389 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 | 27.877 |
| 10 | 1.479 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 6.737 | 9.342 | 12.549 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 | 29.588 |
| 11 | 1.834 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 7.584 | 10.341 | 13.701 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 | 31.264 |
| 12 | 2.214 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 8.438 | 11.340 | 14.845 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 | 32.909 |
| 13 | 2.617 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 | 9.299 | 12.340 | 15.984 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 | 34.528 |
| 14 | 3.041 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 10.165 | 13.339 | 17.117 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 | 36.123 |
| 15 | 3.483 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 11.037 | 14.339 | 18.245 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 | 37.697 |
| 16 | 3.942 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 11.912 | 15.338 | 19.369 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 | 39.252 |
| 17 | 4.416 | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 | 12.792 | 16.338 | 20.489 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 | 40.790 |
| 18 | 4.905 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 13.675 | 17.338 | 21.605 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 | 42.312 |
| 19 | 5.407 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 | 14.562 | 18.338 | 22.718 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 | 43.820 |
| 20 | 5.921 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 15.452 | 19.337 | 23.828 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 | 45.315 |
| 21 | 6.447 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 | 16.344 | 20.337 | 24.935 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 | 46.797 |
| 22 | 6.983 | 8.643 | 9.542 | 10.982 | 12.338 | 14.041 | 17.240 | 21.337 | 26.039 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 | 48.268 |
| 23 | 7.529 | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 | 18.137 | 22.337 | 27.141 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 | 49.728 |
| 24 | 8.085 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 19.037 | 23.337 | 28.241 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 | 51.179 |
| 25 | 8.649 | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 | 19.939 | 24.337 | 29.339 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 | 52.620 |
| 26 | 9.222 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 20.843 | 25.336 | 30.435 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 | 54.052 |
| 27 | 9.803 | 11.808 | 12.879 | 14.573 | 16.151 | 18.114 | 21.749 | 26.336 | 31.528 | 36.741 | 40.113 | 43.195 | 46.963 | 49.645 | 55.476 |
| 28 | 10.391 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 22.657 | 27.336 | 32.620 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 | 56.892 |
| 29 | 10.986 | 13.121 | 14.256 | 16.047 | 17.708 | 19.768 | 23.567 | 28.336 | 33.711 | 39.087 | 42.557 | 45.722 | 49.588 | 52.336 | 58.301 |
| 30 | 11.588 | 13.787 | 14.953 | 16.791 | 18.493 | 20.599 | 24.478 | 29.336 | 34.800 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 | 59.703 |
| 31 | 12.196 | 14.458 | 15.655 | 17.539 | 19.281 | 21.434 | 25.390 | 30.336 | 35.887 | 41.422 | 44.985 | 48.232 | 52.191 | 55.003 | 61.098 |
| 32 | 12.811 | 15.134 | 16.362 | 18.291 | 20.072 | 22.271 | 26.304 | 31.336 | 36.973 | 42.585 | 46.194 | 49.480 | 53.486 | 56.328 | 62.487 |
| 33 | 13.431 | 15.815 | 17.074 | 19.047 | 20.867 | 23.110 | 27.219 | 32.336 | 38.058 | 43.745 | 47.400 | 50.725 | 54.776 | 57.648 | 63.870 |
| 34 | 14.057 | 16.501 | 17.789 | 19.806 | 21.664 | 23.952 | 28.136 | 33.336 | 39.141 | 44.903 | 48.602 | 51.966 | 56.061 | 58.964 | 65.247 |
| 35 | 14.688 | 17.192 | 18.509 | 20.569 | 22.465 | 24.797 | 29.054 | 34.336 | 40.223 | 46.059 | 49.802 | 53.203 | 57.342 | 60.275 | 66.619 |
| 36 | 15.324 | 17.887 | 19.233 | 21.336 | 23.269 | 25.643 | 29.973 | 35.336 | 41.304 | 47.212 | 50.998 | 54.437 | 58.619 | 61.581 | 67.985 |
| 37 | 15.965 | 18.586 | 19.960 | 22.106 | 24.075 | 26.492 | 30.893 | 36.336 | 42.383 | 48.363 | 52.192 | 55.668 | 59.893 | 62.883 | 69.346 |
| 38 | 16.611 | 19.289 | 20.691 | 22.878 | 24.884 | 27.343 | 31.815 | 37.335 | 43.462 | 49.513 | 53.384 | 56.896 | 61.162 | 64.181 | 70.703 |
| 39 | 17.262 | 19.996 | 21.426 | 23.654 | 25.695 | 28.196 | 32.737 | 38.335 | 44.539 | 50.660 | 54.572 | 58.120 | 62.428 | 65.476 | 72.055 |
| 40 | 17.916 | 20.707 | 22.164 | 24.433 | 26.509 | 29.051 | 33.660 | 39.335 | 45.616 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 | 73.402 |
| 41 | 18.576 | 21.421 | 22.906 | 25.215 | 27.326 | 29.907 | 34.585 | 40.335 | 46.692 | 52.949 | 56.942 | 60.561 | 64.950 | 68.053 | 74.745 |
| 42 | 19.239 | 22.138 | 23.650 | 25.999 | 28.144 | 30.765 | 35.510 | 41.335 | 47.766 | 54.090 | 58.124 | 61.777 | 66.206 | 69.336 | 76.084 |
| 43 | 19.906 | 22.859 | 24.398 | 26.785 | 28.965 | 31.625 | 36.436 | 42.335 | 48.840 | 55.230 | 59.304 | 62.990 | 67.459 | 70.616 | 77.419 |
| 44 | 20.576 | 23.584 | 25.148 | 27.575 | 29.787 | 32.487 | 37.363 | 43.335 | 49.913 | 56.369 | 60.481 | 64.201 | 68.710 | 71.893 | 78.750 |
| 45 | 21.251 | 24.311 | 25.901 | 28.366 | 30.612 | 33.350 | 38.291 | 44.335 | 50.985 | 57.505 | 61.656 | 65.410 | 69.957 | 73.166 | 80.077 |
| 46 | 21.929 | 25.041 | 26.657 | 29.160 | 31.439 | 34.215 | 39.220 | 45.335 | 52.056 | 58.641 | 62.830 | 66.617 | 71.201 | 74.437 | 81.400 |
| 47 | 22.610 | 25.775 | 27.416 | 29.956 | 32.268 | 35.081 | 40.149 | 46.335 | 53.127 | 59.774 | 64.001 | 67.821 | 72.443 | 75.704 | 82.720 |
| 48 | 23.295 | 26.511 | 28.177 | 30.755 | 33.098 | 35.949 | 41.079 | 47.335 | 54.196 | 60.907 | 65.171 | 69.023 | 73.683 | 76.969 | 84.037 |
| 49 | 23.983 | 27.249 | 28.941 | 31.555 | 33.930 | 36.818 | 42.010 | 48.335 | 55.265 | 62.038 | 66.339 | 70.222 | 74.919 | 78.231 | 85.351 |
| 50 | 24.674 | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 | 42.942 | 49.335 | 56.334 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 | 86.661 |

## TABLE B.4 CRITICAL VALUES OF THE *F* DISTRIBUTION

Numerator DF = 1

| α(2): | 0.50 | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.005 | 0.002 | 0.001 |
|---|---|---|---|---|---|---|---|---|---|
| α(1): | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0025 | 0.001 | 0.0005 |
| Denom. DF | | | | | | | | | |
| 1 | 5.83 | 39.9 | 161. | 648. | 4050. | 16200. | 64800. | 405000. | 1620000. |
| 2 | 2.57 | 8.53 | 18.5 | 38.5 | 98.5 | 199. | 399. | 999. | 2800. |
| 3 | 2.02 | 5.54 | 10.1 | 17.4 | 34.1 | 55.6 | 89.6 | 167. | 267. |
| 4 | 1.81 | 4.54 | 7.71 | 12.2 | 21.2 | 31.3 | 45.7 | 74.1 | 106. |
| 5 | 1.69 | 4.06 | 6.61 | 10.0 | 16.3 | 22.8 | 31.4 | 47.2 | 63.6 |
| 6 | 1.62 | 3.78 | 5.99 | 8.81 | 13.7 | 18.6 | 24.8 | 35.5 | 46.1 |
| 7 | 1.57 | 3.59 | 5.59 | 8.07 | 12.2 | 16.2 | 21.1 | 29.2 | 37.0 |
| 8 | 1.54 | 3.46 | 5.32 | 7.57 | 11.3 | 14.7 | 18.8 | 25.4 | 31.6 |
| 9 | 1.51 | 3.36 | 5.12 | 7.21 | 10.6 | 13.6 | 17.2 | 22.9 | 28.0 |
| 10 | 1.49 | 3.29 | 4.96 | 6.94 | 10.0 | 12.8 | 16.0 | 21.0 | 25.5 |
| 11 | 1.47 | 3.23 | 4.84 | 6.72 | 9.65 | 12.2 | 15.2 | 19.7 | 23.7 |
| 12 | 1.46 | 3.18 | 4.75 | 6.55 | 9.33 | 11.8 | 14.5 | 18.6 | 22.2 |
| 13 | 1.45 | 3.14 | 4.67 | 6.41 | 9.07 | 11.4 | 13.9 | 17.8 | 21.1 |
| 14 | 1.44 | 3.10 | 4.60 | 6.30 | 8.86 | 11.1 | 13.5 | 17.1 | 20.2 |
| 15 | 1.43 | 3.07 | 4.54 | 6.20 | 8.68 | 10.8 | 13.1 | 16.6 | 19.5 |
| 16 | 1.42 | 3.05 | 4.49 | 6.12 | 8.53 | 10.6 | 12.8 | 16.1 | 18.9 |
| 17 | 1.42 | 3.03 | 4.45 | 6.04 | 8.40 | 10.4 | 12.6 | 15.7 | 18.4 |
| 18 | 1.41 | 3.01 | 4.41 | 5.98 | 8.29 | 10.2 | 12.3 | 15.4 | 17.9 |
| 19 | 1.41 | 2.99 | 4.38 | 5.92 | 8.18 | 10.1 | 12.1 | 15.1 | 17.5 |
| 20 | 1.40 | 2.97 | 4.35 | 5.87 | 8.10 | 9.94 | 11.9 | 14.8 | 17.2 |
| 21 | 1.40 | 2.96 | 4.32 | 5.83 | 8.02 | 9.83 | 11.8 | 14.6 | 16.9 |
| 22 | 1.40 | 2.95 | 4.30 | 5.79 | 7.95 | 9.73 | 11.6 | 14.4 | 16.6 |
| 23 | 1.39 | 2.94 | 4.28 | 5.75 | 7.88 | 9.63 | 11.5 | 14.2 | 16.4 |
| 24 | 1.39 | 2.93 | 4.26 | 5.72 | 7.82 | 9.55 | 11.4 | 14.0 | 16.2 |
| 25 | 1.39 | 2.92 | 4.24 | 5.69 | 7.77 | 9.48 | 11.3 | 13.9 | 16.0 |
| 26 | 1.38 | 2.91 | 4.23 | 5.66 | 7.72 | 9.41 | 11.2 | 13.7 | 15.8 |
| 27 | 1.38 | 2.90 | 4.21 | 5.63 | 7.68 | 9.34 | 11.1 | 13.6 | 15.6 |
| 28 | 1.38 | 2.89 | 4.20 | 5.61 | 7.64 | 9.28 | 11.0 | 13.5 | 15.5 |
| 29 | 1.38 | 2.89 | 4.18 | 5.59 | 7.60 | 9.23 | 11.0 | 13.4 | 15.3 |
| 30 | 1.38 | 2.88 | 4.17 | 5.57 | 7.56 | 9.18 | 10.9 | 13.3 | 15.2 |
| 35 | 1.37 | 2.85 | 4.12 | 5.48 | 7.42 | 8.98 | 10.6 | 12.9 | 14.7 |
| 40 | 1.36 | 2.84 | 4.08 | 5.42 | 7.31 | 8.83 | 10.4 | 12.6 | 14.4 |
| 45 | 1.36 | 2.82 | 4.06 | 5.38 | 7.23 | 8.71 | 10.3 | 12.4 | 14.1 |
| 50 | 1.35 | 2.81 | 4.03 | 5.34 | 7.17 | 8.63 | 10.1 | 12.2 | 13.9 |
| 60 | 1.35 | 2.79 | 4.00 | 5.29 | 7.08 | 8.49 | 9.96 | 12.0 | 13.5 |
| 70 | 1.35 | 2.78 | 3.98 | 5.25 | 7.01 | 8.40 | 9.84 | 11.8 | 13.3 |
| 80 | 1.34 | 2.77 | 3.96 | 5.22 | 6.96 | 8.33 | 9.75 | 11.7 | 13.2 |
| 90 | 1.34 | 2.76 | 3.95 | 5.20 | 6.93 | 8.28 | 9.68 | 11.6 | 13.0 |
| 100 | 1.34 | 2.76 | 3.94 | 5.18 | 6.90 | 8.24 | 9.62 | 11.5 | 12.9 |
| 120 | 1.34 | 2.75 | 3.92 | 5.15 | 6.85 | 8.18 | 9.54 | 11.4 | 12.8 |
| 140 | 1.33 | 2.74 | 3.91 | 5.13 | 6.82 | 8.14 | 9.48 | 11.3 | 12.7 |
| 160 | 1.33 | 2.74 | 3.90 | 5.12 | 6.80 | 8.10 | 9.44 | 11.2 | 12.6 |
| 180 | 1.33 | 2.73 | 3.89 | 5.11 | 6.78 | 8.08 | 9.40 | 11.2 | 12.6 |
| 200 | 1.33 | 2.73 | 3.89 | 5.10 | 6.76 | 8.06 | 9.38 | 11.2 | 12.5 |
| 300 | 1.33 | 2.72 | 3.87 | 5.07 | 6.72 | 8.00 | 9.30 | 11.0 | 12.4 |
| 500 | 1.33 | 2.72 | 3.86 | 5.05 | 6.69 | 7.95 | 9.23 | 11.0 | 12.3 |
| ∞ | 1.32 | 2.71 | 3.84 | 5.02 | 6.64 | 7.88 | 9.14 | 10.8 | 12.1 |

TABLE B.4 (cont.) CRITICAL VALUES OF THE *F* DISTRIBUTION

Numerator DF = 2

| α(2): | 0.50 | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.005 | 0.002 | 0.001 |
|---|---|---|---|---|---|---|---|---|---|
| α(1): | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0025 | 0.001 | 0.0005 |
| Denom. DF | | | | | | | | | |
| 1 | 7.50 | 49.5 | 200. | 800. | 5000. | 20000. | 80000. | 500000. | 2000000. |
| 2 | 3.00 | 9.00 | 19.0 | 39.0 | 99.0 | 199. | 399. | 999. | 2000. |
| 3 | 2.28 | 5.46 | 9.55 | 16.0 | 30.8 | 49.8 | 79.9 | 148. | 237. |
| 4 | 2.00 | 4.32 | 6.94 | 10.6 | 18.0 | 26.3 | 38.0 | 61.2 | 87.4 |
| 5 | 1.85 | 3.78 | 5.79 | 8.43 | 13.3 | 18.3 | 25.0 | 37.1 | 49.8 |
| 6 | 1.76 | 3.46 | 5.14 | 7.26 | 10.9 | 14.5 | 19.1 | 27.0 | 34.8 |
| 7 | 1.70 | 3.26 | 4.74 | 6.54 | 9.55 | 12.4 | 15.9 | 21.7 | 27.2 |
| 8 | 1.66 | 3.11 | 4.46 | 6.06 | 8.65 | 11.0 | 13.9 | 18.5 | 22.7 |
| 9 | 1.62 | 3.01 | 4.26 | 5.71 | 8.02 | 10.1 | 12.5 | 16.4 | 19.9 |
| 10 | 1.60 | 2.92 | 4.10 | 5.46 | 7.56 | 9.43 | 11.6 | 14.9 | 17.9 |
| 11 | 1.58 | 2.86 | 3.98 | 5.26 | 7.21 | 8.91 | 10.8 | 13.8 | 16.4 |
| 12 | 1.56 | 2.81 | 3.89 | 5.10 | 6.93 | 8.51 | 10.3 | 13.0 | 15.3 |
| 13 | 1.55 | 2.76 | 3.81 | 4.97 | 6.70 | 8.19 | 9.84 | 12.3 | 14.4 |
| 14 | 1.53 | 2.73 | 3.74 | 4.86 | 6.51 | 7.92 | 9.47 | 11.8 | 13.7 |
| 15 | 1.52 | 2.70 | 3.68 | 4.77 | 6.36 | 7.70 | 9.17 | 11.3 | 13.2 |
| 16 | 1.51 | 2.67 | 3.63 | 4.69 | 6.23 | 7.51 | 8.92 | 11.0 | 12.7 |
| 17 | 1.51 | 2.64 | 3.59 | 4.62 | 6.11 | 7.35 | 8.70 | 10.7 | 12.3 |
| 18 | 1.50 | 2.62 | 3.55 | 4.56 | 6.01 | 7.21 | 8.51 | 10.4 | 11.9 |
| 19 | 1.49 | 2.61 | 3.52 | 4.51 | 5.93 | 7.09 | 8.35 | 10.2 | 11.6 |
| 20 | 1.49 | 2.59 | 3.49 | 4.46 | 5.85 | 6.99 | 8.21 | 9.95 | 11.4 |
| 21 | 1.48 | 2.57 | 3.47 | 4.42 | 5.78 | 6.89 | 8.08 | 9.77 | 11.2 |
| 22 | 1.48 | 2.56 | 3.44 | 4.38 | 5.72 | 6.81 | 7.96 | 9.61 | 11.0 |
| 23 | 1.47 | 2.55 | 3.42 | 4.35 | 5.66 | 6.73 | 7.86 | 9.47 | 10.8 |
| 24 | 1.47 | 2.54 | 3.40 | 4.32 | 5.61 | 6.66 | 7.77 | 9.34 | 10.6 |
| 25 | 1.47 | 2.53 | 3.39 | 4.29 | 5.57 | 6.60 | 7.69 | 9.22 | 10.5 |
| 26 | 1.46 | 2.52 | 3.37 | 4.27 | 5.53 | 6.54 | 7.61 | 9.12 | 10.3 |
| 27 | 1.46 | 2.51 | 3.35 | 4.24 | 5.49 | 6.49 | 7.54 | 9.02 | 10.2 |
| 28 | 1.46 | 2.50 | 3.34 | 4.22 | 5.45 | 6.44 | 7.48 | 8.93 | 10.1 |
| 29 | 1.45 | 2.50 | 3.33 | 4.20 | 5.42 | 6.40 | 7.42 | 8.85 | 9.99 |
| 30 | 1.45 | 2.49 | 3.32 | 4.18 | 5.39 | 6.35 | 7.36 | 8.77 | 9.90 |
| 35 | 1.44 | 2.46 | 3.27 | 4.11 | 5.27 | 6.19 | 7.15 | 8.47 | 9.52 |
| 40 | 1.44 | 2.44 | 3.23 | 4.05 | 5.18 | 6.07 | 6.99 | 8.25 | 9.25 |
| 45 | 1.43 | 2.42 | 3.20 | 4.01 | 5.11 | 5.97 | 6.86 | 8.09 | 9.04 |
| 50 | 1.43 | 2.41 | 3.18 | 3.97 | 5.06 | 5.90 | 6.77 | 7.96 | 8.88 |
| 60 | 1.42 | 2.39 | 3.15 | 3.93 | 4.98 | 5.79 | 6.63 | 7.77 | 8.65 |
| 70 | 1.41 | 2.38 | 3.13 | 3.89 | 4.92 | 5.72 | 6.53 | 7.64 | 8.49 |
| 80 | 1.41 | 2.37 | 3.11 | 3.86 | 4.88 | 5.67 | 6.46 | 7.54 | 8.37 |
| 90 | 1.41 | 2.36 | 3.10 | 3.84 | 4.85 | 5.62 | 6.41 | 7.47 | 8.28 |
| 100 | 1.41 | 2.36 | 3.09 | 3.83 | 4.82 | 5.59 | 6.37 | 7.41 | 8.21 |
| 120 | 1.40 | 2.35 | 3.07 | 3.80 | 4.79 | 5.54 | 6.30 | 7.32 | 8.10 |
| 140 | 1.40 | 2.34 | 3.06 | 3.79 | 4.76 | 5.50 | 6.26 | 7.26 | 8.03 |
| 160 | 1.40 | 2.34 | 3.05 | 3.78 | 4.74 | 5.48 | 6.22 | 7.21 | 7.97 |
| 180 | 1.40 | 2.33 | 3.05 | 3.77 | 4.73 | 5.46 | 6.20 | 7.18 | 7.93 |
| 200 | 1.40 | 2.33 | 3.04 | 3.76 | 4.71 | 5.44 | 6.17 | 7.15 | 7.90 |
| 300 | 1.39 | 2.32 | 3.03 | 3.73 | 4.68 | 5.39 | 6.11 | 7.07 | 7.80 |
| 500 | 1.39 | 2.31 | 3.01 | 3.72 | 4.65 | 5.35 | 6.06 | 7.00 | 7.72 |
| ∞ | 1.39 | 2.30 | 3.00 | 3.69 | 4.61 | 5.30 | 5.99 | 6.91 | 7.60 |

TABLE B.4 (cont.) CRITICAL VALUES OF THE *F* DISTRIBUTION

Numerator DF = 3

| α(2): | 0.50 | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.005 | 0.002 | 0.001 |
|---|---|---|---|---|---|---|---|---|---|
| α(1): | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0025 | 0.001 | 0.0005 |
| Denom. DF | | | | | | | | | |
| 1 | 8.20 | 53.6 | 216. | 864. | 5400. | 21600. | 86500. | 540000. | 2160000. |
| 2 | 3.15 | 9.16 | 19.2 | 39.2 | 99.2 | 199. | 399. | 999. | 2000. |
| 3 | 2.36 | 5.39 | 9.28 | 15.4 | 29.5 | 47.5 | 76.1 | 141. | 225. |
| 4 | 2.05 | 4.19 | 6.59 | 9.98 | 16.7 | 24.3 | 35.0 | 56.2 | 80.1 |
| 5 | 1.88 | 3.62 | 5.41 | 7.76 | 12.1 | 16.5 | 22.4 | 33.2 | 44.4 |
| 6 | 1.78 | 3.29 | 4.76 | 6.60 | 9.78 | 12.9 | 16.9 | 23.7 | 30.5 |
| 7 | 1.72 | 3.07 | 4.35 | 5.89 | 8.45 | 10.9 | 13.8 | 18.8 | 23.5 |
| 8 | 1.67 | 2.92 | 4.07 | 5.42 | 7.59 | 9.60 | 12.0 | 15.8 | 19.4 |
| 9 | 1.63 | 2.81 | 3.86 | 5.08 | 6.99 | 8.72 | 10.7 | 13.9 | 16.8 |
| 10 | 1.60 | 2.73 | 3.71 | 4.83 | 6.55 | 8.08 | 9.83 | 12.6 | 15.0 |
| 11 | 1.58 | 2.66 | 3.59 | 4.63 | 6.22 | 7.60 | 9.17 | 11.6 | 13.7 |
| 12 | 1.56 | 2.61 | 3.49 | 4.47 | 5.95 | 7.23 | 8.65 | 10.8 | 12.7 |
| 13 | 1.55 | 2.56 | 3.41 | 4.35 | 5.74 | 6.93 | 8.24 | 10.2 | 11.9 |
| 14 | 1.53 | 2.52 | 3.34 | 4.24 | 5.56 | 6.68 | 7.91 | 9.73 | 11.3 |
| 15 | 1.52 | 2.49 | 3.29 | 4.15 | 5.42 | 6.48 | 7.63 | 9.34 | 10.8 |
| 16 | 1.51 | 2.46 | 3.24 | 4.08 | 5.29 | 6.30 | 7.40 | 9.01 | 10.3 |
| 17 | 1.50 | 2.44 | 3.20 | 4.01 | 5.18 | 6.16 | 7.21 | 8.73 | 9.99 |
| 18 | 1.49 | 2.42 | 3.16 | 3.95 | 5.09 | 6.03 | 7.04 | 8.49 | 9.69 |
| 19 | 1.49 | 2.40 | 3.13 | 3.90 | 5.01 | 5.92 | 6.89 | 8.28 | 9.42 |
| 20 | 1.48 | 2.38 | 3.10 | 3.86 | 4.94 | 5.82 | 6.78 | 8.10 | 9.20 |
| 21 | 1.48 | 2.36 | 3.07 | 3.82 | 4.87 | 5.73 | 6.64 | 7.94 | 8.99 |
| 22 | 1.47 | 2.35 | 3.05 | 3.78 | 4.82 | 5.65 | 6.54 | 7.80 | 8.82 |
| 23 | 1.47 | 2.34 | 3.03 | 3.75 | 4.76 | 5.58 | 6.45 | 7.67 | 8.66 |
| 24 | 1.46 | 2.33 | 3.01 | 3.72 | 4.72 | 5.52 | 6.36 | 7.55 | 8.51 |
| 25 | 1.46 | 2.32 | 2.99 | 3.69 | 4.68 | 5.46 | 6.29 | 7.45 | 8.39 |
| 26 | 1.45 | 2.31 | 2.98 | 3.67 | 4.64 | 5.41 | 6.22 | 7.36 | 8.27 |
| 27 | 1.45 | 2.30 | 2.96 | 3.65 | 4.60 | 5.36 | 6.16 | 7.27 | 8.16 |
| 28 | 1.45 | 2.29 | 2.95 | 3.63 | 4.57 | 5.32 | 6.10 | 7.19 | 8.07 |
| 29 | 1.45 | 2.28 | 2.93 | 3.61 | 4.54 | 5.28 | 6.05 | 7.12 | 7.98 |
| 30 | 1.44 | 2.28 | 2.92 | 3.59 | 4.51 | 5.24 | 6.00 | 7.05 | 7.89 |
| 35 | 1.43 | 2.25 | 2.87 | 3.52 | 4.40 | 5.09 | 5.80 | 6.79 | 7.56 |
| 40 | 1.42 | 2.23 | 2.84 | 3.46 | 4.31 | 4.98 | 5.66 | 6.59 | 7.33 |
| 45 | 1.42 | 2.21 | 2.81 | 3.42 | 4.25 | 4.89 | 5.55 | 6.45 | 7.15 |
| 50 | 1.41 | 2.20 | 2.79 | 3.39 | 4.20 | 4.83 | 5.47 | 6.34 | 7.01 |
| 60 | 1.41 | 2.18 | 2.76 | 3.34 | 4.13 | 4.73 | 5.34 | 6.17 | 6.81 |
| 70 | 1.40 | 2.16 | 2.74 | 3.31 | 4.07 | 4.66 | 5.26 | 6.06 | 6.67 |
| 80 | 1.40 | 2.15 | 2.72 | 3.28 | 4.04 | 4.61 | 5.19 | 5.97 | 6.57 |
| 90 | 1.39 | 2.15 | 2.71 | 3.26 | 4.01 | 4.57 | 5.14 | 5.91 | 6.49 |
| 100 | 1.39 | 2.14 | 2.70 | 3.25 | 3.98 | 4.54 | 5.11 | 5.86 | 6.43 |
| 120 | 1.39 | 2.13 | 2.68 | 3.23 | 3.95 | 4.50 | 5.05 | 5.78 | 6.34 |
| 140 | 1.38 | 2.12 | 2.67 | 3.21 | 3.92 | 4.47 | 5.01 | 5.73 | 6.28 |
| 160 | 1.38 | 2.12 | 2.66 | 3.20 | 3.91 | 4.44 | 4.98 | 5.69 | 6.23 |
| 180 | 1.38 | 2.11 | 2.65 | 3.19 | 3.89 | 4.42 | 4.95 | 5.66 | 6.19 |
| 200 | 1.38 | 2.11 | 2.65 | 3.18 | 3.88 | 4.41 | 4.94 | 5.63 | 6.16 |
| 300 | 1.38 | 2.10 | 2.63 | 3.16 | 3.85 | 4.38 | 4.88 | 5.56 | 6.08 |
| 500 | 1.37 | 2.09 | 2.62 | 3.14 | 3.82 | 4.33 | 4.84 | 5.51 | 6.01 |
| ∞ | 1.37 | 2.08 | 2.61 | 3.12 | 3.78 | 4.28 | 4.77 | 5.42 | 5.91 |

TABLE B.4 (cont.) CRITICAL VALUES OF THE *F* DISTRIBUTION

Numerator DF = 4

| α(2): | 0.50 | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.005 | 0.002 | 0.001 |
|---|---|---|---|---|---|---|---|---|---|
| α(1): | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0025 | 0.001 | 0.0005 |
| Denom. DF | | | | | | | | | |
| 1 | 8.58 | 55.8 | 225. | 900. | 5620. | 22500. | 90000. | 562000. | 2250000. |
| 2 | 3.23 | 9.24 | 19.2 | 39.2 | 99.2 | 199. | 399. | 999. | 2000. |
| 3 | 2.39 | 5.34 | 9.12 | 15.1 | 28.7 | 46.2 | 73.9 | 137. | 218. |
| 4 | 2.06 | 4.11 | 6.39 | 9.60 | 16.0 | 23.2 | 33.3 | 53.4 | 76.1 |
| 5 | 1.89 | 3.52 | 5.19 | 7.39 | 11.4 | 15.6 | 21.0 | 31.1 | 41.5 |
| 6 | 1.79 | 3.18 | 4.53 | 6.23 | 9.15 | 12.0 | 15.7 | 21.9 | 28.1 |
| 7 | 1.72 | 2.96 | 4.12 | 5.52 | 7.85 | 10.1 | 12.7 | 17.2 | 21.4 |
| 8 | 1.66 | 2.81 | 3.84 | 5.05 | 7.01 | 8.81 | 10.9 | 14.4 | 17.6 |
| 9 | 1.63 | 2.69 | 3.63 | 4.72 | 6.42 | 7.96 | 9.74 | 12.6 | 15.1 |
| 10 | 1.59 | 2.61 | 3.48 | 4.47 | 5.99 | 7.34 | 8.89 | 11.3 | 13.4 |
| 11 | 1.57 | 2.54 | 3.36 | 4.28 | 5.67 | 6.88 | 8.25 | 10.3 | 12.2 |
| 12 | 1.55 | 2.48 | 3.26 | 4.12 | 5.41 | 6.52 | 7.76 | 9.63 | 11.2 |
| 13 | 1.53 | 2.43 | 3.18 | 4.00 | 5.21 | 6.23 | 7.37 | 9.07 | 10.5 |
| 14 | 1.52 | 2.39 | 3.11 | 3.89 | 5.04 | 6.00 | 7.06 | 8.62 | 9.95 |
| 15 | 1.51 | 2.36 | 3.06 | 3.80 | 4.89 | 5.80 | 6.80 | 8.25 | 9.48 |
| 16 | 1.50 | 2.33 | 3.01 | 3.73 | 4.77 | 5.64 | 6.58 | 7.94 | 9.08 |
| 17 | 1.49 | 2.31 | 2.96 | 3.66 | 4.67 | 5.50 | 6.39 | 7.68 | 8.75 |
| 18 | 1.48 | 2.29 | 2.93 | 3.61 | 4.58 | 5.37 | 6.23 | 7.46 | 8.47 |
| 19 | 1.47 | 2.27 | 2.90 | 3.56 | 4.50 | 5.27 | 6.09 | 7.27 | 8.23 |
| 20 | 1.47 | 2.25 | 2.87 | 3.51 | 4.43 | 5.17 | 5.97 | 7.10 | 8.02 |
| 21 | 1.46 | 2.23 | 2.84 | 3.48 | 4.37 | 5.09 | 5.86 | 6.95 | 7.83 |
| 22 | 1.45 | 2.22 | 2.82 | 3.44 | 4.31 | 5.02 | 5.76 | 6.81 | 7.67 |
| 23 | 1.45 | 2.21 | 2.80 | 3.41 | 4.26 | 4.95 | 5.67 | 6.70 | 7.52 |
| 24 | 1.44 | 2.19 | 2.78 | 3.38 | 4.22 | 4.89 | 5.60 | 6.59 | 7.39 |
| 25 | 1.44 | 2.18 | 2.76 | 3.35 | 4.18 | 4.84 | 5.53 | 6.49 | 7.27 |
| 26 | 1.44 | 2.17 | 2.74 | 3.33 | 4.14 | 4.79 | 5.46 | 6.41 | 7.16 |
| 27 | 1.43 | 2.17 | 2.73 | 3.31 | 4.11 | 4.74 | 5.40 | 6.33 | 7.06 |
| 28 | 1.43 | 2.16 | 2.71 | 3.29 | 4.07 | 4.70 | 5.35 | 6.25 | 6.97 |
| 29 | 1.43 | 2.15 | 2.70 | 3.27 | 4.04 | 4.66 | 5.30 | 6.19 | 6.89 |
| 30 | 1.42 | 2.14 | 2.69 | 3.25 | 4.02 | 4.62 | 5.25 | 6.12 | 6.82 |
| 35 | 1.41 | 2.11 | 2.64 | 3.18 | 3.91 | 4.48 | 5.07 | 5.88 | 6.51 |
| 40 | 1.40 | 2.09 | 2.61 | 3.13 | 3.83 | 4.37 | 4.93 | 5.70 | 6.30 |
| 45 | 1.40 | 2.07 | 2.58 | 3.09 | 3.77 | 4.29 | 4.83 | 5.56 | 6.13 |
| 50 | 1.39 | 2.06 | 2.56 | 3.05 | 3.72 | 4.23 | 4.75 | 5.46 | 6.01 |
| 60 | 1.38 | 2.04 | 2.53 | 3.01 | 3.65 | 4.14 | 4.64 | 5.31 | 5.82 |
| 70 | 1.38 | 2.03 | 2.50 | 2.97 | 3.60 | 4.08 | 4.56 | 5.20 | 5.70 |
| 80 | 1.38 | 2.02 | 2.49 | 2.95 | 3.56 | 4.03 | 4.50 | 5.12 | 5.60 |
| 90 | 1.37 | 2.01 | 2.47 | 2.93 | 3.53 | 3.99 | 4.45 | 5.06 | 5.53 |
| 100 | 1.37 | 2.00 | 2.46 | 2.92 | 3.51 | 3.96 | 4.42 | 5.02 | 5.48 |
| 120 | 1.37 | 1.99 | 2.45 | 2.89 | 3.48 | 3.92 | 4.36 | 4.95 | 5.39 |
| 140 | 1.36 | 1.99 | 2.44 | 2.88 | 3.46 | 3.89 | 4.32 | 4.90 | 5.33 |
| 160 | 1.36 | 1.98 | 2.43 | 2.87 | 3.44 | 3.87 | 4.30 | 4.86 | 5.29 |
| 180 | 1.36 | 1.98 | 2.42 | 2.86 | 3.43 | 3.85 | 4.27 | 4.83 | 5.26 |
| 200 | 1.36 | 1.97 | 2.42 | 2.85 | 3.41 | 3.84 | 4.26 | 4.81 | 5.23 |
| 300 | 1.35 | 1.96 | 2.40 | 2.83 | 3.38 | 3.80 | 4.21 | 4.75 | 5.15 |
| 500 | 1.35 | 1.96 | 2.39 | 2.81 | 3.36 | 3.76 | 4.17 | 4.69 | 5.09 |
| ∞ | 1.35 | 1.94 | 2.37 | 2.79 | 3.32 | 3.72 | 4.11 | 4.62 | 5.00 |

## TABLE B.12 CRITICAL VALUES OF THE KRUSKAL-WALLIS $H$ DISTRIBUTION

| $n_1$ | $n_2$ | $n_3$ | $\alpha$ = 0.10 | 0.05 | 0.02 | 0.01 | 0.005 | 0.002 | 0.001 |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 4.571 | | | | | | |
| 3 | 2 | 1 | 4.286 | | | | | | |
| 3 | 2 | 2 | 4.500 | 4.714 | | | | | |
| 3 | 3 | 1 | 4.571 | 5.143 | | | | | |
| 3 | 3 | 2 | 4.556 | 5.361 | 6.250 | | | | |
| 3 | 3 | 3 | 4.622 | 5.600 | 6.489 | (7.200) | 7.200 | | |
| 4 | 2 | 1 | 4.500 | | | | | | |
| 4 | 2 | 2 | 4.458 | 5.333 | 6.000 | | | | |
| 4 | 3 | 1 | 4.056 | 5.208 | | | | | |
| 4 | 3 | 2 | 4.511 | 5.444 | 6.144 | 6.444 | 7.000 | | |
| 4 | 3 | 3 | 4.709 | 5.791 | 6.564 | 6.745 | 7.318 | 8.018 | |
| 4 | 4 | 1 | 4.167 | 4.967 | (6.667) | 6.667 | | | |
| 4 | 4 | 2 | 4.555 | 5.455 | 6.600 | 7.036 | 7.282 | 7.855 | |
| 4 | 4 | 3 | 4.545 | 5.598 | 6.712 | 7.144 | 7.598 | 8.227 | 8.909 |
| 4 | 4 | 4 | 4.654 | 5.692 | 6.962 | 7.654 | 8.000 | 8.654 | 9.269 |
| 5 | 2 | 1 | 4.200 | 5.000 | | | | | |
| 5 | 2 | 2 | 4.373 | 5.160 | 6.000 | 6.533 | | | |
| 5 | 3 | 1 | 4.018 | 4.960 | 6.044 | | | | |
| 5 | 3 | 2 | 4.651 | 5.251 | 6.124 | 6.909 | 7.182 | | |
| 5 | 3 | 3 | 4.533 | 5.648 | 6.533 | 7.079 | 7.636 | 8.048 | 8.727 |
| 5 | 4 | 1 | 3.987 | 4.985 | 6.431 | 6.955 | 7.364 | | |
| 5 | 4 | 2 | 4.541 | 5.273 | 6.505 | 7.205 | 7.573 | 8.114 | 8.591 |
| 5 | 4 | 3 | 4.549 | 5.656 | 6.676 | 7.445 | 7.927 | 8.481 | 8.795 |
| 5 | 4 | 4 | 4.619 | 5.657 | 6.953 | 7.760 | 8.189 | 8.868 | 9.168 |
| 5 | 5 | 1 | 4.109 | 5.127 | 6.145 | 7.309 | 8.182 | | |
| 5 | 5 | 2 | 4.623 | 5.338 | 6.446 | 7.338 | 8.131 | 6.446 | 7.338 |
| 5 | 5 | 3 | 4.545 | 5.705 | 6.866 | 7.578 | 8.316 | 8.809 | 9.521 |
| 5 | 5 | 4 | 4.523 | 5.666 | 7.000 | 7.823 | 8.523 | 9.163 | 9.606 |
| 5 | 5 | 5 | 4.343 | 5.780 | 7.220 | 8.000 | 8.780 | 9.620 | 9.920 |
| 6 | 1 | 1 | ----- | | | | | | |
| 6 | 2 | 1 | 4.200 | 4.822 | | | | | |
| 6 | 2 | 2 | 4.545 | 5.345 | 6.182 | 6.982 | | | |
| 6 | 3 | 1 | 3.909 | 4.855 | 6.236 | | | | |
| 6 | 3 | 2 | 4.682 | 5.348 | 6.227 | 6.970 | 7.515 | 8.182 | |
| 6 | 3 | 3 | 4.538 | 5.615 | 6.590 | 7.410 | 7.872 | 8.628 | 9.346 |
| 6 | 4 | 1 | 4.038 | 4.947 | 6.174 | 7.106 | 7.614 | | |
| 6 | 4 | 2 | 4.494 | 5.340 | 6.571 | 7.340 | 7.846 | 8.494 | 8.827 |
| 6 | 4 | 3 | 4.604 | 5.610 | 6.725 | 7.500 | 8.033 | 8.918 | 9.170 |
| 6 | 4 | 4 | 4.595 | 5.681 | 6.900 | 7.795 | 8.381 | 9.167 | 9.861 |
| 6 | 5 | 1 | 4.128 | 4.990 | 6.138 | 7.182 | 8.077 | 8.515 | |
| 6 | 5 | 2 | 4.596 | 5.338 | 6.585 | 7.376 | 8.196 | 8.967 | 9.189 |
| 6 | 5 | 3 | 4.535 | 5.602 | 6.829 | 7.590 | 8.314 | 9.150 | 9.669 |
| 6 | 5 | 4 | 4.522 | 5.661 | 7.018 | 7.936 | 8.643 | 9.458 | 9.960 |
| 6 | 5 | 5 | 4.547 | 5.729 | 7.110 | 8.028 | 8.859 | 9.771 | 10.271 |
| 6 | 6 | 1 | 4.000 | 4.945 | 6.286 | 7.121 | 8.165 | 9.077 | 9.692 |
| 6 | 6 | 2 | 4.438 | 5.410 | 6.667 | 7.467 | 8.210 | 9.219 | 9.752 |
| 6 | 6 | 3 | 4.558 | 5.625 | 6.900 | 7.725 | 8.458 | 9.458 | 10.150 |
| 6 | 6 | 4 | 4.548 | 5.724 | 7.107 | 8.000 | 8.754 | 9.662 | 10.342 |
| 6 | 6 | 5 | 4.542 | 5.765 | 7.152 | 8.124 | 8.987 | 9.948 | 10.524 |
| 6 | 6 | 6 | 4.643 | 5.801 | 7.240 | 8.222 | 9.170 | 10.187 | 10.889 |

## TABLE B.12 (cont.) CRITICAL VALUES OF THE KRUSKAL-WALLIS $H$ DISTRIBUTION

| $n_1$ | $n_2$ | $n_3$ | $n_4$ | $n_5$ | $\alpha$ = 0.10 | 0.05 | 0.02 | 0.01 | 0.005 | 0.002 | 0.001 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 7 | 7 | | | 4.594 | 5.819 | 7.332 | 8.378 | 9.373 | 10.516 | 11.310 |
| 8 | 8 | 8 | | | 4.595 | 5.805 | 7.355 | 8.468 | 9.495 | 10.805 | 11.705 |
| 2 | 2 | 1 | 1 | | ----- | | | | | | |
| 2 | 2 | 2 | 1 | | 5.357 | 5.679 | | | | | |
| 2 | 2 | 2 | 2 | | 5.667 | 6.167 | (6.667) | 6.667 | | | |
| 3 | 1 | 1 | 1 | | ----- | | | | | | |
| 3 | 2 | 1 | 1 | | 5.143 | | | | | | |
| 3 | 2 | 2 | 1 | | 5.556 | 5.833 | 6.500 | | | | |
| 3 | 2 | 2 | 2 | | 5.511 | 5.333 | 6.978 | 7.133 | 7.533 | | |
| 3 | 3 | 1 | 1 | | 5.333 | 6.333 | | | | | |
| 3 | 3 | 2 | 1 | | 5.689 | 6.244 | 6.689 | 7.200 | 7.400 | | |
| 3 | 3 | 2 | 2 | | 5.745 | 6.527 | 7.182 | 7.636 | 7.873 | 8.018 | 8.455 |
| 3 | 3 | 3 | 1 | | 5.655 | 6.600 | 7.109 | 7.400 | 8.055 | 8.345 | |
| 3 | 3 | 3 | 2 | | 5.879 | 6.727 | 7.636 | 8.105 | 8.379 | 8.803 | 9.030 |
| 3 | 3 | 3 | 3 | | 6.026 | 7.000 | 7.872 | 8.538 | 8.897 | 9.462 | 9.513 |
| 4 | 1 | 1 | 1 | | ----- | | | | | | |
| 4 | 2 | 1 | 1 | | 5.250 | 5.833 | | | | | |
| 4 | 2 | 2 | 1 | | 5.533 | 6.133 | 6.667 | 7.000 | | | |
| 4 | 2 | 2 | 2 | | 5.755 | 6.545 | 7.091 | 7.391 | 7.964 | 8.291 | |
| 4 | 3 | 1 | 1 | | 5.067 | 6.178 | 6.711 | 7.067 | | | |
| 4 | 3 | 2 | 1 | | 5.591 | 6.309 | 7.018 | 7.455 | 7.773 | 8.182 | |
| 4 | 3 | 2 | 2 | | 5.750 | 6.621 | 7.530 | 7.871 | 8.273 | 8.689 | 8.909 |
| 4 | 3 | 3 | 1 | | 5.533 | 6.545 | 7.485 | 7.758 | 8.212 | 8.697 | 9.182 |
| 4 | 3 | 3 | 2 | | 5.872 | 6.795 | 7.763 | 8.333 | 8.718 | 9.167 | 9.455 |
| 4 | 3 | 3 | 3 | | 6.016 | 6.984 | 7.995 | 8.659 | 9.253 | 9.709 | 10.016 |
| 4 | 4 | 1 | 1 | | 5.182 | 5.945 | 7.091 | 7.909 | 7.909 | | |
| 4 | 4 | 2 | 1 | | 5.568 | 6.386 | 7.364 | 7.886 | 8.341 | 8.591 | 8.909 |
| 4 | 4 | 2 | 2 | | 5.808 | 6.731 | 7.750 | 8.346 | 8.692 | 9.269 | 9.462 |
| 4 | 4 | 3 | 1 | | 5.692 | 6.635 | 7.660 | 8.231 | 8.583 | 9.038 | 9.327 |
| 4 | 4 | 3 | 2 | | 5.901 | 6.874 | 7.951 | 8.621 | 9.165 | 9.615 | 9.945 |
| 4 | 4 | 3 | 3 | | 6.019 | 7.038 | 8.181 | 8.876 | 9.495 | 10.105 | 10.467 |
| 4 | 4 | 4 | 1 | | 5.564 | 6.725 | 7.879 | 8.588 | 9.000 | 9.478 | 9.758 |
| 4 | 4 | 4 | 2 | | 5.914 | 6.957 | 8.157 | 8.871 | 9.486 | 10.043 | 10.429 |
| 4 | 4 | 4 | 3 | | 6.042 | 7.142 | 8.350 | 9.075 | 9.742 | 10.542 | 10.929 |
| 4 | 4 | 4 | 4 | | 6.088 | 7.235 | 8.515 | 9.287 | 9.971 | 10.809 | 11.338 |
| 2 | 1 | 1 | 1 | 1 | ----- | | | | | | |
| 2 | 2 | 1 | 1 | 1 | 5.786 | | | | | | |
| 2 | 2 | 2 | 1 | 1 | 6.250 | 6.750 | | | | | |
| 2 | 2 | 2 | 2 | 1 | 6.600 | 7.133 | (7.533) | 7.533 | | | |
| 2 | 2 | 2 | 2 | 2 | 6.982 | 7.418 | 8.073 | 8.291 | (8.727) | 8.727 | |
| 3 | 1 | 1 | 1 | 1 | ----- | | | | | | |
| 3 | 2 | 1 | 1 | 1 | 6.139 | 6.583 | | | | | |
| 3 | 2 | 2 | 1 | 1 | 6.511 | 6.800 | 7.400 | 7.600 | | | |
| 3 | 2 | 2 | 2 | 1 | 6.709 | 7.309 | 7.836 | 8.127 | 8.327 | 8.618 | |
| 3 | 2 | 2 | 2 | 2 | 6.955 | 7.682 | 8.303 | 8.682 | 8.985 | 9.273 | 9.364 |
| 3 | 3 | 1 | 1 | 1 | 6.311 | 7.111 | 7.467 | | | | |
| 3 | 3 | 2 | 1 | 1 | 6.600 | 7.200 | 7.892 | 8.073 | 8.345 | | |
| 3 | 3 | 2 | 2 | 1 | 6.788 | 7.591 | 8.258 | 8.576 | 8.924 | 9.167 | 9.303 |
| 3 | 3 | 2 | 2 | 2 | 7.026 | 7.910 | 8.667 | 9.115 | 9.474 | 9.769 | 10.026 |
| 3 | 3 | 3 | 1 | 1 | 6.788 | 7.576 | 8.242 | 8.424 | 8.848 | (9.455) | 9.455 |
| 3 | 3 | 3 | 2 | 1 | 6.910 | 7.759 | 8.590 | 9.051 | 9.410 | 9.769 | 9.974 |
| 3 | 3 | 3 | 2 | 2 | 7.121 | 8.044 | 9.011 | 9.505 | 9.890 | 10.330 | 10.037 |
| 3 | 3 | 3 | 3 | 1 | 7.077 | 8.000 | 8.879 | 9.451 | 9.846 | 10.286 | 10.549 |
| 3 | 3 | 3 | 3 | 2 | 7.210 | 8.200 | 9.267 | 9.876 | 10.333 | 10.838 | 11.171 |
| 3 | 3 | 3 | 3 | 3 | 7.333 | 8.333 | 9.467 | 10.200 | 10.733 | 10.267 | 11.667 |



*Examples:*

$H_{0.05,4,3,2} = 5.444$ and $H_{0.01,4,4,3} = H_{0.01,\ldots} = 7.760$

## TABLE B.10 CRITICAL VALUES OF THE MANN-WHITNEY *U* DISTRIBUTION

| $n_1$ | $n_2$ | α(2): 0.20 / α(1): 0.10 | 0.10 / 0.05 | 0.05 / 0.025 | 0.02 / 0.01 | 0.01 / 0.005 | 0.005 / 0.0025 | 0.002 / 0.001 | 0.001 / 0.0005 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | -- | -- | -- | -- | -- | -- | -- | -- |
| | 2 | -- | -- | -- | -- | -- | -- | -- | -- |
| | 3 | -- | -- | -- | -- | -- | -- | -- | -- |
| | 4 | -- | -- | -- | -- | -- | -- | -- | -- |
| | 5 | -- | -- | -- | -- | -- | -- | -- | -- |
| | 6 | -- | -- | -- | -- | -- | -- | -- | -- |
| | 7 | -- | -- | -- | -- | -- | -- | -- | -- |
| | 8 | -- | -- | -- | -- | -- | -- | -- | -- |
| | 9 | 9 | -- | -- | -- | -- | -- | -- | -- |
| | 10 | 10 | -- | -- | -- | -- | -- | -- | -- |
| | 11 | 11 | -- | -- | -- | -- | -- | -- | -- |
| | 12 | 12 | -- | -- | -- | -- | -- | -- | -- |
| | 13 | 13 | -- | -- | -- | -- | -- | -- | -- |
| | 14 | 14 | -- | -- | -- | -- | -- | -- | -- |
| | 15 | 15 | -- | -- | -- | -- | -- | -- | -- |
| | 16 | 16 | -- | -- | -- | -- | -- | -- | -- |
| | 17 | 17 | -- | -- | -- | -- | -- | -- | -- |
| | 18 | 18 | -- | -- | -- | -- | -- | -- | -- |
| | 19 | 18 | 19 | -- | -- | -- | -- | -- | -- |
| | 20 | 19 | 20 | -- | -- | -- | -- | -- | -- |
| | 21 | 20 | 21 | -- | -- | -- | -- | -- | -- |
| | 22 | 21 | 22 | -- | -- | -- | -- | -- | -- |
| | 23 | 22 | 23 | -- | -- | -- | -- | -- | -- |
| | 24 | 23 | 24 | -- | -- | -- | -- | -- | -- |
| | 25 | 24 | 25 | -- | -- | -- | -- | -- | -- |
| | 26 | 25 | 26 | -- | -- | -- | -- | -- | -- |
| | 27 | 26 | 27 | -- | -- | -- | -- | -- | -- |
| | 28 | 27 | 28 | -- | -- | -- | -- | -- | -- |
| | 29 | 27 | 29 | -- | -- | -- | -- | -- | -- |
| | 30 | 28 | 30 | -- | -- | -- | -- | -- | -- |
| | 31 | 29 | 31 | -- | -- | -- | -- | -- | -- |
| | 32 | 30 | 32 | -- | -- | -- | -- | -- | -- |
| | 33 | 31 | 33 | -- | -- | -- | -- | -- | -- |
| | 34 | 32 | 34 | -- | -- | -- | -- | -- | -- |
| | 35 | 33 | 35 | -- | -- | -- | -- | -- | -- |
| | 36 | 34 | 36 | -- | -- | -- | -- | -- | -- |
| | 37 | 35 | 37 | -- | -- | -- | -- | -- | -- |
| | 38 | 36 | 38 | -- | -- | -- | -- | -- | -- |
| | 39 | 36 | 38 | 39 | -- | -- | -- | -- | -- |
| 1 | 40 | 37 | 39 | 40 | -- | -- | -- | -- | -- |
| 2 | 2 | -- | -- | -- | -- | -- | -- | -- | -- |
| | 3 | 6 | -- | -- | -- | -- | -- | -- | -- |
| | 4 | 8 | -- | -- | -- | -- | -- | -- | -- |
| | 5 | 9 | 10 | -- | -- | -- | -- | -- | -- |
| | 6 | 11 | 12 | -- | -- | -- | -- | -- | -- |
| | 7 | 13 | 14 | -- | -- | -- | -- | -- | -- |
| | 8 | 14 | 15 | 16 | -- | -- | -- | -- | -- |
| | 9 | 16 | 17 | 18 | -- | -- | -- | -- | -- |
| 2 | 10 | 17 | 19 | 20 | -- | -- | -- | -- | -- |

## TABLE B.10 (cont.) CRITICAL VALUES OF THE MANN-WHITNEY *U* DISTRIBUTION

| $n_1$ | $n_2$ | α(2): 0.20 / α(1): 0.10 | 0.10 / 0.05 | 0.05 / 0.025 | 0.02 / 0.01 | 0.01 / 0.005 | 0.005 / 0.0025 | 0.002 / 0.001 | 0.001 / 0.0005 |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 11 | 19 | 21 | 22 | -- | -- | -- | -- | -- |
| | 12 | 20 | 22 | 23 | -- | -- | -- | -- | -- |
| | 13 | 22 | 24 | 25 | 26 | -- | -- | -- | -- |
| | 14 | 23 | 25 | 27 | 28 | -- | -- | -- | -- |
| | 15 | 25 | 27 | 29 | 30 | -- | -- | -- | -- |
| | 16 | 27 | 29 | 31 | 32 | -- | -- | -- | -- |
| | 17 | 28 | 31 | 32 | 34 | -- | -- | -- | -- |
| | 18 | 30 | 32 | 34 | 36 | -- | -- | -- | -- |
| | 19 | 31 | 34 | 36 | 37 | 38 | -- | -- | -- |
| | 20 | 33 | 36 | 38 | 39 | 40 | -- | -- | -- |
| | 21 | 34 | 37 | 39 | 41 | 42 | -- | -- | -- |
| | 22 | 36 | 39 | 41 | 43 | 44 | -- | -- | -- |
| | 23 | 37 | 41 | 43 | 45 | 46 | -- | -- | -- |
| | 24 | 39 | 42 | 45 | 47 | 48 | -- | -- | -- |
| | 25 | 41 | 44 | 47 | 49 | 50 | -- | -- | -- |
| | 26 | 42 | 46 | 48 | 51 | 52 | -- | -- | -- |
| | 27 | 44 | 47 | 50 | 52 | 53 | 54 | -- | -- |
| | 28 | 45 | 49 | 52 | 54 | 55 | 56 | -- | -- |
| | 29 | 47 | 51 | 54 | 56 | 57 | 58 | -- | -- |
| | 30 | 48 | 53 | 55 | 58 | 59 | 60 | -- | -- |
| | 31 | 50 | 54 | 57 | 60 | 61 | 62 | -- | -- |
| | 32 | 51 | 56 | 59 | 62 | 63 | 64 | -- | -- |
| | 33 | 53 | 58 | 61 | 64 | 65 | 66 | -- | -- |
| | 34 | 55 | 59 | 63 | 65 | 67 | 68 | -- | -- |
| | 35 | 56 | 61 | 64 | 67 | 69 | 70 | -- | -- |
| | 36 | 58 | 63 | 66 | 69 | 71 | 72 | -- | -- |
| | 37 | 59 | 64 | 68 | 71 | 73 | 74 | -- | -- |
| | 38 | 61 | 66 | 70 | 73 | 75 | 76 | -- | -- |
| | 39 | 62 | 68 | 71 | 75 | 76 | 77 | -- | -- |
| 2 | 40 | 64 | 69 | 73 | 77 | 78 | 79 | -- | -- |
| 3 | 3 | 8 | 9 | -- | -- | -- | -- | -- | -- |
| | 4 | 11 | 12 | -- | -- | -- | -- | -- | -- |
| | 5 | 13 | 14 | 15 | -- | -- | -- | -- | -- |
| | 6 | 15 | 16 | 17 | -- | -- | -- | -- | -- |
| | 7 | 17 | 19 | 20 | 21 | -- | -- | -- | -- |
| | 8 | 19 | 21 | 22 | 24 | -- | -- | -- | -- |
| | 9 | 21 | 23 | 25 | 26 | 27 | -- | -- | -- |
| | 10 | 24 | 26 | 27 | 29 | 30 | -- | -- | -- |
| | 11 | 26 | 28 | 30 | 32 | 33 | -- | -- | -- |
| | 12 | 28 | 31 | 32 | 34 | 35 | 36 | -- | -- |
| | 13 | 30 | 33 | 35 | 37 | 38 | 39 | -- | -- |
| | 14 | 32 | 35 | 37 | 40 | 41 | 42 | -- | -- |
| | 15 | 35 | 38 | 40 | 42 | 43 | 44 | -- | -- |
| | 16 | 37 | 40 | 42 | 45 | 46 | 47 | -- | -- |
| | 17 | 39 | 42 | 45 | 47 | 49 | 50 | 51 | -- |
| | 18 | 41 | 45 | 47 | 50 | 52 | 53 | 54 | -- |
| | 19 | 43 | 47 | 50 | 53 | 54 | 56 | 57 | -- |
| 3 | 20 | 45 | 49 | 52 | 55 | 57 | 58 | 60 | -- |

## TABLE B.10 (cont.) CRITICAL VALUES OF THE MANN-WHITNEY *U* DISTRIBUTION

| $n_1$ | $n_2$ | α(2): 0.20 / α(1): 0.10 | 0.10 / 0.05 | 0.05 / 0.025 | 0.02 / 0.01 | 0.01 / 0.005 | 0.005 / 0.0025 | 0.002 / 0.001 | 0.001 / 0.0005 |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 21 | 48 | 52 | 55 | 58 | 60 | 61 | 62 | 63 |
| | 22 | 50 | 54 | 57 | 60 | 63 | 64 | 65 | 66 |
| | 23 | 52 | 56 | 60 | 63 | 65 | 67 | 68 | 69 |
| | 24 | 54 | 59 | 62 | 66 | 68 | 69 | 71 | 72 |
| | 25 | 56 | 61 | 65 | 68 | 70 | 72 | 74 | 75 |
| | 26 | 58 | 63 | 67 | 71 | 73 | 75 | 77 | 78 |
| | 27 | 60 | 66 | 70 | 74 | 76 | 78 | 79 | 80 |
| | 28 | 63 | 68 | 72 | 76 | 79 | 80 | 82 | 83 |
| | 29 | 65 | 70 | 74 | 79 | 81 | 83 | 85 | 86 |
| | 30 | 67 | 73 | 77 | 81 | 84 | 86 | 88 | 89 |
| | 31 | 69 | 75 | 79 | 84 | 87 | 89 | 91 | 92 |
| | 32 | 71 | 77 | 82 | 87 | 89 | 92 | 94 | 95 |
| | 33 | 73 | 80 | 84 | 89 | 92 | 94 | 96 | 98 |
| | 34 | 76 | 82 | 87 | 92 | 95 | 97 | 99 | 101 |
| | 35 | 78 | 84 | 89 | 94 | 97 | 100 | 102 | 103 |
| | 36 | 80 | 87 | 92 | 97 | 100 | 103 | 105 | 106 |
| | 37 | 82 | 89 | 94 | 100 | 103 | 105 | 108 | 109 |
| | 38 | 84 | 91 | 97 | 102 | 105 | 108 | 111 | 112 |
| | 39 | 86 | 94 | 99 | 105 | 108 | 111 | 113 | 115 |
| 3 | 40 | 89 | 96 | 102 | 107 | 111 | 114 | 116 | 118 |
| 4 | 4 | 13 | 15 | 16 | -- | -- | -- | -- | -- |
| | 5 | 16 | 18 | 19 | 20 | -- | -- | -- | -- |
| | 6 | 19 | 21 | 22 | 23 | 24 | -- | -- | -- |
| | 7 | 22 | 24 | 25 | 27 | 28 | -- | -- | -- |
| | 8 | 25 | 27 | 28 | 30 | 31 | 32 | -- | -- |
| | 9 | 27 | 30 | 32 | 33 | 35 | 36 | -- | -- |
| | 10 | 30 | 33 | 35 | 37 | 38 | 39 | 40 | -- |
| | 11 | 33 | 36 | 38 | 40 | 42 | 43 | 44 | -- |
| | 12 | 36 | 39 | 41 | 43 | 45 | 46 | 48 | -- |
| | 13 | 39 | 42 | 44 | 47 | 49 | 50 | 51 | 52 |
| | 14 | 41 | 45 | 47 | 50 | 52 | 53 | 55 | 56 |
| | 15 | 44 | 48 | 50 | 53 | 55 | 57 | 59 | 60 |
| | 16 | 47 | 50 | 53 | 57 | 59 | 60 | 62 | 63 |
| | 17 | 50 | 53 | 57 | 60 | 62 | 64 | 66 | 67 |
| | 18 | 52 | 56 | 60 | 63 | 66 | 67 | 69 | 71 |
| | 19 | 55 | 59 | 63 | 67 | 69 | 71 | 73 | 74 |
| | 20 | 58 | 62 | 66 | 70 | 72 | 75 | 77 | 78 |
| | 21 | 61 | 65 | 69 | 73 | 76 | 78 | 80 | 82 |
| | 22 | 63 | 68 | 72 | 77 | 79 | 82 | 84 | 85 |
| | 23 | 66 | 71 | 75 | 80 | 83 | 85 | 88 | 89 |
| | 24 | 69 | 74 | 78 | 83 | 86 | 89 | 91 | 93 |
| | 25 | 72 | 77 | 82 | 87 | 90 | 92 | 95 | 97 |
| | 26 | 74 | 80 | 85 | 90 | 93 | 96 | 98 | 100 |
| | 27 | 77 | 83 | 88 | 93 | 96 | 99 | 102 | 104 |
| | 28 | 80 | 86 | 91 | 96 | 100 | 103 | 106 | 108 |
| | 29 | 83 | 89 | 94 | 100 | 103 | 106 | 109 | 111 |
| | 30 | 85 | 92 | 97 | 103 | 107 | 110 | 113 | 115 |
| 4 | 31 | 88 | 95 | 100 | 106 | 110 | 113 | 117 | 119 |

## TABLE B.10 (cont.) CRITICAL VALUES OF THE MANN-WHITNEY *U* DISTRIBUTION

| $n_1$ | $n_2$ | α(2): 0.20 / α(1): 0.10 | 0.10 / 0.05 | 0.05 / 0.025 | 0.02 / 0.01 | 0.01 / 0.005 | 0.005 / 0.0025 | 0.002 / 0.001 | 0.001 / 0.0005 |
|---|---|---|---|---|---|---|---|---|---|
| 4 | 32 | 91 | 98 | 104 | 110 | 114 | 117 | 120 | 122 |
| | 33 | 94 | 101 | 107 | 113 | 117 | 120 | 124 | 126 |
| | 34 | 96 | 104 | 110 | 116 | 120 | 124 | 127 | 130 |
| | 35 | 99 | 107 | 113 | 120 | 124 | 127 | 131 | 133 |
| | 36 | 102 | 110 | 116 | 123 | 127 | 131 | 135 | 137 |
| | 37 | 105 | 113 | 119 | 126 | 131 | 134 | 138 | 141 |
| | 38 | 107 | 116 | 122 | 130 | 134 | 138 | 142 | 144 |
| | 39 | 110 | 118 | 125 | 133 | 137 | 141 | 145 | 148 |
| 4 | 40 | 113 | 121 | 129 | 136 | 141 | 145 | 149 | 152 |
| 5 | 5 | 20 | 21 | 23 | 24 | 25 | -- | -- | -- |
| | 6 | 23 | 25 | 27 | 28 | 29 | 30 | -- | -- |
| | 7 | 26 | 29 | 30 | 32 | 34 | 35 | -- | -- |
| | 8 | 30 | 32 | 34 | 36 | 38 | 38 | 40 | -- |
| | 9 | 33 | 35 | 38 | 40 | 42 | 43 | 44 | 45 |
| | 10 | 37 | 39 | 42 | 44 | 46 | 47 | 49 | 50 |
| | 11 | 40 | 43 | 46 | 48 | 50 | 52 | 53 | 54 |
| | 12 | 43 | 47 | 49 | 52 | 54 | 56 | 58 | 59 |
| | 13 | 47 | 50 | 53 | 56 | 58 | 60 | 62 | 63 |
| | 14 | 50 | 54 | 57 | 60 | 63 | 64 | 67 | 68 |
| | 15 | 53 | 57 | 61 | 64 | 67 | 69 | 71 | 72 |
| | 16 | 57 | 61 | 65 | 68 | 71 | 73 | 75 | 77 |
| | 17 | 60 | 65 | 68 | 72 | 75 | 77 | 80 | 81 |
| | 18 | 63 | 68 | 72 | 76 | 79 | 81 | 84 | 86 |
| | 19 | 67 | 72 | 76 | 80 | 83 | 86 | 88 | 90 |
| | 20 | 70 | 75 | 80 | 84 | 87 | 90 | 93 | 95 |
| | 21 | 73 | 79 | 83 | 88 | 91 | 94 | 97 | 99 |
| | 22 | 77 | 82 | 87 | 92 | 96 | 98 | 102 | 104 |
| | 23 | 80 | 86 | 91 | 96 | 100 | 103 | 106 | 108 |
| | 24 | 84 | 90 | 95 | 100 | 104 | 107 | 110 | 113 |
| | 25 | 87 | 93 | 98 | 104 | 108 | 111 | 115 | 117 |
| | 26 | 90 | 97 | 102 | 108 | 112 | 115 | 119 | 121 |
| | 27 | 94 | 100 | 106 | 112 | 116 | 120 | 123 | 126 |
| | 28 | 97 | 104 | 110 | 116 | 120 | 124 | 128 | 130 |
| | 29 | 100 | 107 | 114 | 120 | 124 | 128 | 132 | 135 |
| | 30 | 104 | 111 | 117 | 124 | 128 | 132 | 136 | 139 |
| | 31 | 107 | 115 | 121 | 128 | 133 | 136 | 141 | 144 |
| | 32 | 110 | 118 | 125 | 132 | 137 | 141 | 145 | 148 |
| | 33 | 114 | 122 | 129 | 136 | 141 | 145 | 150 | 153 |
| | 34 | 117 | 125 | 132 | 140 | 145 | 149 | 154 | 157 |
| | 35 | 120 | 129 | 136 | 144 | 149 | 153 | 158 | 161 |
| | 36 | 124 | 132 | 140 | 148 | 153 | 158 | 163 | 166 |
| | 37 | 127 | 136 | 144 | 152 | 157 | 162 | 167 | 170 |
| | 38 | 130 | 140 | 147 | 156 | 161 | 166 | 171 | 175 |
| | 39 | 134 | 143 | 151 | 160 | 165 | 170 | 176 | 179 |
| 5 | 40 | 137 | 147 | 155 | 164 | 169 | 174 | 180 | 184 |
| 6 | 6 | 27 | 29 | 31 | 33 | 34 | 35 | -- | -- |

## TABLE B.10 (cont.) CRITICAL VALUES OF THE MANN-WHITNEY *U* DISTRIBUTION

| $n_1$ | $n_2$ | α(2): 0.20 / α(1): 0.10 | 0.10 / 0.05 | 0.05 / 0.025 | 0.02 / 0.01 | 0.01 / 0.005 | 0.005 / 0.0025 | 0.002 / 0.001 | 0.001 / 0.0005 |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 9 | 39 | 42 | 44 | 47 | 49 | 50 | 52 | 53 |
| | 10 | 43 | 46 | 49 | 52 | 54 | 55 | 57 | 58 |
| | 11 | 47 | 50 | 53 | 57 | 59 | 60 | 62 | 64 |
| | 12 | 51 | 55 | 58 | 61 | 63 | 65 | 68 | 69 |
| | 13 | 55 | 59 | 62 | 66 | 68 | 70 | 73 | 74 |
| | 14 | 59 | 63 | 67 | 71 | 73 | 75 | 78 | 79 |
| | 15 | 63 | 67 | 71 | 75 | 78 | 80 | 83 | 85 |
| | 16 | 67 | 71 | 75 | 80 | 83 | 85 | 88 | 90 |
| | 17 | 71 | 76 | 80 | 84 | 87 | 90 | 93 | 95 |
| | 18 | 74 | 80 | 84 | 89 | 92 | 95 | 98 | 100 |
| | 19 | 78 | 84 | 89 | 94 | 97 | 100 | 103 | 105 |
| | 20 | 82 | 88 | 93 | 98 | 102 | 105 | 108 | 111 |
| | 21 | 86 | 92 | 97 | 103 | 107 | 110 | 114 | 116 |
| | 22 | 90 | 96 | 102 | 108 | 111 | 115 | 119 | 121 |
| | 23 | 94 | 101 | 106 | 112 | 116 | 120 | 124 | 126 |
| | 24 | 98 | 105 | 111 | 117 | 121 | 125 | 129 | 132 |
| | 25 | 102 | 109 | 115 | 121 | 126 | 130 | 134 | 137 |
| | 26 | 105 | 113 | 119 | 126 | 131 | 134 | 139 | 142 |
| | 27 | 110 | 117 | 124 | 131 | 135 | 139 | 144 | 147 |
| | 28 | 114 | 122 | 128 | 135 | 140 | 144 | 149 | 152 |
| | 29 | 118 | 126 | 132 | 140 | 145 | 149 | 154 | 157 |
| | 30 | 122 | 130 | 137 | 145 | 150 | 154 | 159 | 163 |
| | 31 | 125 | 134 | 141 | 149 | 154 | 159 | 164 | 168 |
| | 32 | 129 | 138 | 146 | 154 | 159 | 164 | 169 | 173 |
| | 33 | 133 | 142 | 150 | 158 | 164 | 169 | 174 | 178 |
| | 34 | 137 | 147 | 154 | 163 | 169 | 174 | 179 | 183 |
| | 35 | 141 | 151 | 159 | 168 | 173 | 179 | 185 | 188 |
| | 36 | 145 | 155 | 163 | 172 | 178 | 184 | 190 | 194 |
| | 37 | 149 | 159 | 167 | 177 | 183 | 188 | 195 | 199 |
| | 38 | 153 | 163 | 172 | 182 | 188 | 193 | 200 | 204 |
| | 39 | 157 | 167 | 176 | 186 | 193 | 198 | 205 | 209 |
| 6 | 40 | 161 | 172 | 181 | 191 | 197 | 203 | 210 | 214 |
| 7 | 7 | 36 | 38 | 41 | 43 | 45 | 46 | 48 | 49 |
| | 8 | 40 | 43 | 46 | 49 | 50 | 52 | 54 | 55 |
| | 9 | 45 | 48 | 51 | 54 | 56 | 58 | 60 | 61 |
| | 10 | 49 | 53 | 56 | 59 | 61 | 63 | 65 | 67 |
| | 11 | 54 | 58 | 61 | 65 | 67 | 69 | 71 | 73 |
| | 12 | 58 | 63 | 66 | 70 | 72 | 75 | 77 | 79 |
| | 13 | 63 | 67 | 71 | 75 | 78 | 80 | 83 | 85 |
| | 14 | 67 | 72 | 76 | 81 | 83 | 86 | 89 | 91 |
| | 15 | 72 | 77 | 81 | 86 | 89 | 92 | 95 | 97 |
| | 16 | 76 | 82 | 86 | 91 | 94 | 97 | 101 | 103 |
| | 17 | 81 | 86 | 91 | 96 | 100 | 103 | 106 | 109 |
| | 18 | 85 | 91 | 96 | 102 | 105 | 108 | 112 | 115 |
| | 19 | 90 | 96 | 101 | 107 | 111 | 114 | 118 | 120 |
| | 20 | 94 | 101 | 106 | 112 | 116 | 120 | 124 | 126 |

## TABLE B.10 (cont.) CRITICAL VALUES OF THE MANN-WHITNEY *U* DISTR

| $n_1$ | $n_2$ | α(2): 0.20 / α(1): 0.10 | 0.10 / 0.05 | 0.05 / 0.025 | 0.02 / 0.01 | 0.01 / 0.005 | 0.005 / 0.0025 | 0.002 / 0.001 | 0.00[illegible] / 0.00[illegible] |
|---|---|---|---|---|---|---|---|---|---|
| 7 | 23 | 108 | 115 | 121 | 128 | 132 | 136 | 141 | 144 |
| | 24 | 112 | 120 | 126 | 133 | 138 | 142 | 147 | 150 |
| | 25 | 117 | 125 | 131 | 139 | 143 | 148 | 153 | 156 |
| | 26 | 121 | 129 | 136 | 144 | 148 | 153 | 158 | 162 |
| | 27 | 126 | 134 | 141 | 149 | 154 | 159 | 164 | 168 |
| | 28 | 130 | 138 | 146 | 154 | 160 | 164 | 170 | 174 |
| | 29 | 135 | 144 | 151 | 160 | 165 | 170 | 176 | 179 |
| | 30 | 139 | 148 | 156 | 165 | 170 | 176 | 181 | 185 |
| | 31 | 144 | 153 | 161 | 170 | 176 | 181 | 187 | 191 |
| | 32 | 148 | 158 | 166 | 175 | 181 | 187 | 193 | 197 |
| | 33 | 153 | 163 | 171 | 180 | 187 | 192 | 199 | 203 |
| | 34 | 157 | 168 | 176 | 186 | 192 | 198 | 204 | 209 |
| | 35 | 162 | 172 | 181 | 191 | 198 | 203 | 210 | 215 |
| | 36 | 166 | 177 | 186 | 196 | 203 | 209 | 216 | 221 |
| | 37 | 171 | 182 | 191 | 201 | 208 | 215 | 222 | 227 |
| | 38 | 175 | 187 | 196 | 207 | 214 | 220 | 227 | 233 |
| | 39 | 180 | 191 | 201 | 212 | 219 | 226 | 233 | 238 |
| 7 | 40 | 184 | 196 | 206 | 217 | 225 | 231 | 239 | 244 |
| 8 | 8 | 45 | 48 | 51 | 55 | 57 | 58 | 60 | 62 |
| | 9 | 50 | 54 | 57 | 61 | 63 | 65 | 67 | 68 |
| | 10 | 56 | 60 | 63 | 67 | 69 | 71 | 74 | 75 |
| | 11 | 61 | 65 | 69 | 73 | 75 | 77 | 80 | [illegible] |
| | 12 | 66 | 70 | 74 | 79 | 81 | 84 | 87 | [illegible] |
| | 13 | 71 | 76 | 80 | 84 | 87 | 90 | 93 | [illegible] |
| | 14 | 75 | 81 | 86 | 90 | 94 | 96 | 100 | [illegible] |
| | 15 | 81 | 87 | 91 | 96 | 100 | 103 | 106 | [illegible] |
| | 16 | 86 | 92 | 97 | 102 | 106 | 109 | 113 | [illegible] |
| | 17 | 91 | 97 | 102 | 108 | 112 | 115 | 119 | [illegible] |
| | 18 | 96 | 103 | 108 | 114 | 118 | 122 | 126 | [illegible] |
| | 19 | 101 | 108 | 114 | 120 | 124 | 128 | 132 | [illegible] |
| | 20 | 106 | 113 | 119 | 126 | 130 | 134 | 139 | [illegible] |
| | 21 | 112 | 119 | 125 | 132 | 136 | 140 | 145 | [illegible] |
| | 22 | 117 | 124 | 131 | 138 | 142 | 147 | 152 | [illegible] |
| | 23 | 122 | 130 | 136 | 144 | 149 | 153 | 158 | [illegible] |
| | 24 | 127 | 135 | 142 | 150 | 155 | 159 | 165 | [illegible] |
| | 25 | 132 | 140 | 147 | 155 | 161 | 165 | 171 | [illegible] |
| | 26 | 137 | 146 | 153 | 161 | 167 | 172 | 177 | [illegible] |
| | 27 | 142 | 151 | 158 | 167 | 173 | 178 | 184 | [illegible] |
| | 28 | 147 | 156 | 164 | 173 | 179 | 184 | 190 | [illegible] |
| | 29 | 152 | 162 | 170 | 179 | 185 | 190 | 197 | [illegible] |
| | 30 | 157 | 167 | 175 | 185 | 191 | 197 | 203 | [illegible] |
| | 31 | 162 | 172 | 181 | 191 | 197 | 203 | 210 | [illegible] |
| | 32 | 167 | 178 | 186 | 197 | 203 | 209 | 216 | [illegible] |
| | 33 | 172 | 183 | 192 | 203 | 209 | 215 | 223 | [illegible] |
| | 34 | 177 | 188 | 198 | 208 | 215 | 222 | 229 | [illegible] |
| | 35 | 182 | 194 | 203 | 214 | 221 | 228 | 235 | [illegible] |
| | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

TABLE B.10 (cont.) CRITICAL VALUES OF THE MANN-WHITNEY U DISTRIBUTION

TABLE B.10 (cont.) CRITICAL VALUES OF THE MANN-WHITNEY U DISTRIBUTION

TABLE B.10 (cont.) CRITICAL VALUES OF THE MANN-WHITNEY U DISTRIBUTION

TABLE B.10 (cont.) CRITICAL VALUES OF THE MANN-WHITNEY U DISTRIBUTION

| $n_1$ | $n_2$ | α(2): 0.20 α(1): 0.10 | 0.10 0.05 | 0.05 0.025 | 0.02 0.01 | 0.01 0.005 | 0.005 0.0025 | 0.002 0.001 | 0.001 0.0005 |
|---|---|---|---|---|---|---|---|---|---|
| 19 | 29 | 334 | 355 | 368 | 386 | 397 | 407 | 420 | 428 |
| | 30 | 348 | 366 | 381 | 398 | 410 | 421 | 433 | 442 |
| | 31 | 360 | 378 | 395 | 411 | 423 | 436 | 447 | 456 |
| | 32 | 371 | 390 | 405 | 423 | 436 | 447 | 460 | 469 |
| | 33 | 382 | 401 | 417 | 434 | 448 | 460 | 474 | 483 |
| | 34 | 393 | 413 | 428 | 446 | 461 | 473 | 487 | 497 |
| | 35 | 405 | 424 | 441 | 458 | 474 | 486 | 500 | 512 |
| | 36 | 416 | 436 | 453 | 470 | 487 | 499 | 514 | 524 |
| | 37 | 427 | 448 | 465 | 482 | 500 | 512 | 527 | 538 |
| | 38 | 438 | 459 | 477 | 494 | 512 | 525 | 541 | 552 |
| | 39 | 449 | 471 | 489 | 506 | 525 | 538 | 554 | 565 |
| 19 | 40 | -- | 482 | 501 | 518 | 538 | 551 | 568 | 578 |
| 20 | 20 | 249 | 262 | 273 | 288 | 295 | 303 | 312 | 318 |
| | 21 | 260 | 274 | 286 | 299 | 308 | 317 | 326 | 333 |
| | 22 | 272 | 286 | 299 | 313 | 322 | 330 | 341 | 348 |
| | 23 | 284 | 298 | 311 | 326 | 335 | 344 | 355 | 362 |
| | 24 | 296 | 311 | 324 | 339 | 349 | 358 | 369 | 377 |
| | 25 | 307 | 323 | 337 | 352 | 362 | 372 | 383 | 391 |
| | 26 | 319 | 335 | 349 | 365 | 376 | 386 | 397 | 405 |
| | 27 | 331 | 348 | 362 | 378 | 389 | 399 | 411 | 419 |
| | 28 | 343 | 360 | 374 | 391 | 403 | 413 | 425 | 434 |
| | 29 | 354 | 372 | 387 | 404 | 416 | 427 | 440 | 448 |
| | 30 | 366 | 384 | 400 | 417 | 430 | 440 | 454 | 463 |
| | 31 | 378 | 396 | 412 | 430 | 443 | 454 | 468 | 477 |
| | 32 | 389 | 409 | 425 | 444 | 456 | 468 | 482 | 492 |
| | 33 | 401 | 421 | 438 | 457 | 470 | 482 | 496 | 506 |
| | 34 | 413 | 433 | 450 | 470 | 483 | 495 | 510 | 520 |
| | 35 | 425 | 445 | 463 | 483 | 497 | 509 | 524 | 534 |
| | 36 | 436 | 457 | 475 | 496 | 510 | 523 | 538 | 549 |
| | 37 | 448 | 469 | 488 | 509 | 525 | 536 | 552 | 563 |
| | 38 | -- | 482 | 501 | 522 | 537 | 550 | 566 | 577 |
| | 39 | -- | -- | 513 | 535 | 550 | 564 | 580 | 591 |
| 20 | 40 | -- | -- | 526 | 548 | 563 | 577 | 594 | 604 |

The preceding values were derived, with permission of the publisher, from the tal
(1964, J. Amer. Statist. Assoc. 59: 925-934).

*Examples*

$U_{0.05(2),3,8} = 34$ and $U_{0.05(1),10,8} = U_{0.05(1),8,10} = 60$.

For the Mann-Whitney test involving $n_1$ and/or $n_2$ larger than those in this tabl approximation (Section 9.9) may be used. This approximation is excellent for two-ta α = 0.10 or 0.05 (or one-tailed testing at α = 0.05 or 0.025, respectively), especially are similar in magnitude. The approximation becomes progressively poorer as we e extreme significance levels

**TABLE B.16 CRITICAL VALUES OF THE CORRELATION COEFFICIENT, $r$**

| $\alpha(2)$: | 0.50 | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.005 | 0.002 | 0.001 |
|---|---|---|---|---|---|---|---|---|---|
| $\alpha(1)$: | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0025 | 0.001 | 0.0005 |
| $\nu$ | | | | | | | | | |
| 1 | 0.707 | 0.951 | 0.988 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 2 | 0.500 | 0.800 | 0.900 | 0.950 | 0.980 | 0.990 | 0.995 | 0.998 | 0.999 |
| 3 | 0.404 | 0.687 | 0.805 | 0.878 | 0.934 | 0.959 | 0.974 | 0.986 | 0.991 |
| 4 | 0.347 | 0.608 | 0.729 | 0.811 | 0.882 | 0.917 | 0.942 | 0.963 | 0.974 |
| 5 | 0.309 | 0.551 | 0.669 | 0.755 | 0.833 | 0.875 | 0.906 | 0.935 | 0.951 |
| 6 | 0.281 | 0.507 | 0.621 | 0.707 | 0.789 | 0.834 | 0.870 | 0.905 | 0.925 |
| 7 | 0.260 | 0.472 | 0.582 | 0.666 | 0.750 | 0.798 | 0.836 | 0.875 | 0.898 |
| 8 | 0.242 | 0.443 | 0.549 | 0.632 | 0.715 | 0.765 | 0.805 | 0.847 | 0.872 |
| 9 | 0.228 | 0.419 | 0.521 | 0.602 | 0.685 | 0.735 | 0.776 | 0.820 | 0.847 |
| 10 | 0.216 | 0.398 | 0.497 | 0.576 | 0.658 | 0.708 | 0.750 | 0.795 | 0.823 |
| 11 | 0.206 | 0.380 | 0.476 | 0.553 | 0.634 | 0.684 | 0.726 | 0.772 | 0.801 |
| 12 | 0.197 | 0.365 | 0.457 | 0.532 | 0.612 | 0.661 | 0.703 | 0.750 | 0.780 |
| 13 | 0.189 | 0.351 | 0.441 | 0.514 | 0.592 | 0.641 | 0.683 | 0.730 | 0.760 |
| 14 | 0.182 | 0.338 | 0.426 | 0.497 | 0.574 | 0.623 | 0.664 | 0.711 | 0.742 |
| 15 | 0.176 | 0.327 | 0.412 | 0.482 | 0.558 | 0.606 | 0.647 | 0.694 | 0.725 |
| 16 | 0.170 | 0.317 | 0.400 | 0.468 | 0.542 | 0.590 | 0.631 | 0.678 | 0.708 |
| 17 | 0.165 | 0.308 | 0.389 | 0.456 | 0.529 | 0.575 | 0.616 | 0.662 | 0.693 |
| 18 | 0.160 | 0.279 | 0.378 | 0.444 | 0.515 | 0.561 | 0.602 | 0.648 | 0.679 |
| 19 | 0.156 | 0.291 | 0.369 | 0.433 | 0.503 | 0.549 | 0.589 | 0.635 | 0.665 |
| 20 | 0.152 | 0.284 | 0.360 | 0.423 | 0.492 | 0.537 | 0.576 | 0.622 | 0.652 |
| 21 | 0.148 | 0.277 | 0.352 | 0.413 | 0.482 | 0.526 | 0.565 | 0.610 | 0.640 |
| 22 | 0.145 | 0.271 | 0.344 | 0.404 | 0.472 | 0.515 | 0.554 | 0.599 | 0.629 |
| 23 | 0.141 | 0.265 | 0.337 | 0.396 | 0.462 | 0.505 | 0.543 | 0.588 | 0.618 |
| 24 | 0.138 | 0.260 | 0.330 | 0.388 | 0.453 | 0.496 | 0.534 | 0.578 | 0.607 |
| 25 | 0.136 | 0.255 | 0.323 | 0.381 | 0.445 | 0.487 | 0.524 | 0.568 | 0.597 |
| 26 | 0.133 | 0.250 | 0.317 | 0.374 | 0.437 | 0.479 | 0.515 | 0.559 | 0.588 |
| 27 | 0.131 | 0.245 | 0.311 | 0.367 | 0.430 | 0.471 | 0.507 | 0.550 | 0.579 |
| 28 | 0.128 | 0.241 | 0.306 | 0.361 | 0.423 | 0.463 | 0.499 | 0.541 | 0.570 |
| 29 | 0.126 | 0.237 | 0.301 | 0.355 | 0.416 | 0.456 | 0.491 | 0.533 | 0.562 |
| 30 | 0.124 | 0.233 | 0.296 | 0.349 | 0.409 | 0.449 | 0.484 | 0.526 | 0.554 |
| 31 | 0.122 | 0.229 | 0.291 | 0.344 | 0.403 | 0.442 | 0.477 | 0.518 | 0.546 |
| 32 | 0.120 | 0.225 | 0.287 | 0.339 | 0.397 | 0.436 | 0.470 | 0.511 | 0.539 |
| 33 | 0.118 | 0.222 | 0.283 | 0.334 | 0.392 | 0.430 | 0.464 | 0.504 | 0.532 |
| 34 | 0.116 | 0.219 | 0.279 | 0.329 | 0.386 | 0.424 | 0.458 | 0.498 | 0.525 |
| 35 | 0.115 | 0.216 | 0.275 | 0.325 | 0.381 | 0.418 | 0.452 | 0.492 | 0.519 |
| 36 | 0.113 | 0.213 | 0.271 | 0.320 | 0.376 | 0.413 | 0.446 | 0.486 | 0.513 |
| 37 | 0.111 | 0.210 | 0.267 | 0.316 | 0.371 | 0.408 | 0.441 | 0.480 | 0.507 |
| 38 | 0.110 | 0.207 | 0.264 | 0.312 | 0.367 | 0.403 | 0.435 | 0.474 | 0.501 |
| 39 | 0.108 | 0.204 | 0.261 | 0.308 | 0.362 | 0.398 | 0.430 | 0.469 | 0.495 |
| 40 | 0.107 | 0.202 | 0.257 | 0.304 | 0.358 | 0.393 | 0.425 | 0.463 | 0.490 |
| 41 | 0.106 | 0.199 | 0.254 | 0.301 | 0.354 | 0.389 | 0.420 | 0.458 | 0.484 |
| 42 | 0.104 | 0.197 | 0.251 | 0.297 | 0.350 | 0.384 | 0.416 | 0.453 | 0.479 |
| 43 | 0.103 | 0.195 | 0.248 | 0.294 | 0.346 | 0.380 | 0.411 | 0.449 | 0.474 |
| 44 | 0.102 | 0.192 | 0.246 | 0.291 | 0.342 | 0.376 | 0.407 | 0.444 | 0.469 |
| 45 | 0.101 | 0.190 | 0.243 | 0.288 | 0.3[illegible] | 0.372 | 0.403 | 0.439 | 0.465 |
| 46 | 0.100 | 0.188 | 0.240 | 0.285 | 0.335 | 0.368 | 0.399 | 0.435 | 0.460 |
| 47 | 0.099 | 0.186 | 0.238 | 0.282 | 0.331 | 0.365 | 0.395 | 0.431 | 0.456 |
| 48 | 0.098 | 0.184 | 0.235 | 0.279 | 0.328 | 0.361 | 0.391 | 0.427 | 0.451 |
| 49 | 0.097 | 0.182 | 0.233 | 0.276 | 0.325 | 0.358 | 0.387 | 0.423 | 0.447 |
| 50 | 0.096 | 0.181 | 0.231 | 0.273 | 0.322 | 0.354 | 0.384 | 0.419 | 0.443 |

## ABLE B.3 CRITICAL VALUES OF THE $t$ DISTRIBUTION

| ν | α(2): 0.50<br>α(1): 0.25 | 0.20<br>0.10 | 0.10<br>0.05 | 0.05<br>0.025 | 0.02<br>0.01 | 0.01<br>0.005 | 0.005<br>0.0025 | 0.002<br>0.001 | 0.001<br>0.000 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 127.321 | 318.309 | 636.619 |
| 2 | 0.816 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 14.089 | 22.327 | 31.599 |
| 3 | 0.765 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 7.453 | 10.215 | 12.924 |
| 4 | 0.741 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5 | 0.727 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 6 | 0.718 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7 | 0.711 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8 | 0.706 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9 | 0.703 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10 | 0.700 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11 | 0.697 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12 | 0.695 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13 | 0.694 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14 | 0.692 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15 | 0.691 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16 | 0.690 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17 | 0.689 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18 | 0.688 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.197 | 3.610 | 3.922 |
| 19 | 0.688 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20 | 0.687 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21 | 0.686 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22 | 0.686 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23 | 0.685 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24 | 0.685 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25 | 0.684 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26 | 0.684 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27 | 0.684 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 28 | 0.683 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29 | 0.683 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 30 | 0.683 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 31 | 0.682 | 1.309 | 1.696 | 2.040 | 2.453 | 2.744 | 3.022 | 3.375 | 3.633 |
| 32 | 0.682 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 3.015 | 3.365 | 3.622 |
| 33 | 0.682 | 1.308 | 1.692 | 2.035 | 2.445 | 2.733 | 3.008 | 3.356 | 3.611 |
| 34 | 0.682 | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 | 3.002 | 3.348 | 3.601 |
| 35 | 0.682 | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 | 2.996 | 3.340 | 3.591 |
| 36 | 0.681 | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 | 2.990 | 3.333 | 3.582 |
| 37 | 0.681 | 1.305 | 1.687 | 2.026 | 2.431 | 2.715 | 2.985 | 3.326 | 3.574 |
| 38 | 0.681 | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 | 2.980 | 3.319 | 3.566 |
| 39 | 0.681 | 1.304 | 1.685 | 2.023 | 2.426 | 2.708 | 2.976 | 3.313 | 3.558 |
| 40 | 0.681 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 41 | 0.681 | 1.303 | 1.683 | 2.020 | 2.421 | 2.701 | 2.967 | 3.301 | 3.544 |
| 42 | 0.680 | 1.302 | 1.682 | 2.018 | 2.418 | 2.698 | 2.963 | 3.296 | 3.538 |
| 43 | 0.680 | 1.302 | 1.681 | 2.017 | 2.416 | 2.695 | 2.959 | 3.291 | 3.532 |
| 44 | 0.680 | 1.301 | 1.680 | 2.015 | 2.414 | 2.692 | 2.956 | 3.286 | 3.526 |
| 45 | 0.680 | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 | 2.952 | 3.281 | 3.520 |
| 46 | 0.680 | 1.300 | 1.679 | 2.013 | 2.410 | 2.687 | 2.949 | 3.277 | 3.515 |
| 47 | 0.680 | 1.300 | 1.678 | 2.012 | 2.408 | 2.685 | 2.946 | 3.273 | 3.510 |
| 48 | 0.680 | 1.299 | 1.677 | 2.011 | 2.407 | 2.682 | 2.943 | 3.269 | 3.505 |
| 49 | 0.680 | 1.299 | 1.677 | 2.010 | 2.405 | 2.680 | 2.940 | 3.265 | 3.500 |
| 50 | 0.679 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |

**TABLE B.5 (cont.)** CRITICAL VALUES OF THE $q$ DISTRIBUTION

$\alpha = 0.05$

| $\nu$ | $k$(or $p$) = 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 17.97 | 26.98 | 32.82 | 37.08 | 40.41 | 43.12 | 45.40 | 47.36 | 49.07 |
| 2 | 6.085 | 8.331 | 9.798 | 10.88 | 11.74 | 12.44 | 13.03 | 13.54 | 13.99 |
| 3 | 4.501 | 5.910 | 6.825 | 7.502 | 8.037 | 8.478 | 8.853 | 9.177 | 9.462 |
| 4 | 3.927 | 5.040 | 5.757 | 6.287 | 6.707 | 7.053 | 7.347 | 7.602 | 7.826 |
| 5 | 3.635 | 4.602 | 5.218 | 5.673 | 6.033 | 6.330 | 6.582 | 6.802 | 6.995 |
| 6 | 3.461 | 4.339 | 4.896 | 5.305 | 5.628 | 5.895 | 6.122 | 6.319 | 6.493 |
| 7 | 3.344 | 4.165 | 4.681 | 5.060 | 5.359 | 5.606 | 5.815 | 5.998 | 6.158 |
| 8 | 3.261 | 4.041 | 4.529 | 4.886 | 5.167 | 5.399 | 5.597 | 5.767 | 5.918 |
| 9 | 3.199 | 3.949 | 4.415 | 4.756 | 5.024 | 5.244 | 5.432 | 5.595 | 5.739 |
| 10 | 3.151 | 3.877 | 4.327 | 4.654 | 4.912 | 5.124 | 5.305 | 5.461 | 5.599 |
| 11 | 3.113 | 3.820 | 4.256 | 4.574 | 4.823 | 5.028 | 5.202 | 5.353 | 5.487 |
| 12 | 3.082 | 3.773 | 4.199 | 4.508 | 4.751 | 4.950 | 5.119 | 5.265 | 5.395 |
| 13 | 3.055 | 3.735 | 4.151 | 4.453 | 4.690 | 4.885 | 5.049 | 5.192 | 5.318 |
| 14 | 3.033 | 3.702 | 4.111 | 4.407 | 4.639 | 4.829 | 4.990 | 5.131 | 5.254 |
| 15 | 3.014 | 3.674 | 4.076 | 4.367 | 4.595 | 4.782 | 4.940 | 5.077 | 5.198 |
| 16 | 2.998 | 3.649 | 4.046 | 4.333 | 4.557 | 4.741 | 4.897 | 5.031 | 5.150 |
| 17 | 2.984 | 3.628 | 4.020 | 4.303 | 4.524 | 4.705 | 4.858 | 4.991 | 5.108 |
| 18 | 2.971 | 3.609 | 3.997 | 4.277 | 4.495 | 4.673 | 4.824 | 4.956 | 5.071 |
| 19 | 2.960 | 3.593 | 3.977 | 4.253 | 4.469 | 4.645 | 4.794 | 4.924 | 5.038 |
| 20 | 2.950 | 3.578 | 3.958 | 4.232 | 4.445 | 4.620 | 4.768 | 4.896 | 5.008 |
| 24 | 2.919 | 3.532 | 3.901 | 4.166 | 4.373 | 4.541 | 4.684 | 4.807 | 4.915 |
| 30 | 2.888 | 3.486 | 3.845 | 4.102 | 4.302 | 4.464 | 4.602 | 4.720 | 4.824 |
| 40 | 2.858 | 3.442 | 3.791 | 4.039 | 4.232 | 4.389 | 4.521 | 4.635 | 4.735 |
| 60 | 2.829 | 3.399 | 3.737 | 3.977 | 4.163 | 4.314 | 4.441 | 4.550 | 4.646 |
| 120 | 2.800 | 3.356 | 3.685 | 3.917 | 4.096 | 4.241 | 4.363 | 4.468 | 4.560 |
| ∞ | 2.772 | 3.314 | 3.633 | 3.858 | 4.030 | 4.170 | 4.286 | 4.387 | 4.474 |

| $\nu$ | $k$(or $p$) = 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 50.59 | 51.96 | 53.20 | 54.33 | 55.36 | 56.32 | 57.22 | 58.04 | 58.83 |
| 2 | 14.39 | 14.75 | 15.08 | 15.38 | 15.65 | 15.91 | 16.14 | 16.37 | 16.57 |
| 3 | 9.717 | 9.946 | 10.15 | 10.35 | 10.53 | 10.69 | 10.84 | 10.98 | 11.11 |
| 4 | 8.027 | 8.208 | 8.373 | 8.525 | 8.664 | 8.794 | 8.914 | 9.028 | 9.134 |
| 5 | 7.168 | 7.324 | 7.466 | 7.596 | 7.717 | 7.828 | 7.932 | 8.030 | 8.122 |
| 6 | 6.649 | 6.789 | 6.917 | 7.034 | 7.143 | 7.244 | 7.338 | 7.426 | 7.508 |
| 7 | 6.302 | 6.431 | 6.550 | 6.658 | 6.759 | 6.852 | 6.939 | 7.020 | 7.097 |
| 8 | 6.054 | 6.175 | 6.287 | 6.389 | 6.483 | 6.571 | 6.653 | 6.729 | 6.802 |
| 9 | 5.867 | 5.983 | 6.089 | 6.186 | 6.276 | 6.359 | 6.437 | 6.510 | 6.579 |
| 10 | 5.722 | 5.833 | 5.935 | 6.028 | 6.114 | 6.194 | 6.269 | 6.339 | 6.405 |
| 11 | 5.605 | 5.713 | 5.811 | 5.901 | 5.984 | 6.062 | 6.134 | 6.202 | 6.265 |
| 12 | 5.511 | 5.615 | 5.710 | 5.798 | 5.878 | 5.953 | 6.023 | 6.089 | 6.151 |
| 13 | 5.431 | 5.533 | 5.625 | 5.711 | 5.789 | 5.862 | 5.931 | 5.995 | 6.055 |
| 14 | 5.364 | 5.463 | 5.554 | 5.637 | 5.714 | 5.786 | 5.852 | 5.915 | 5.974 |
| 15 | 5.306 | 5.404 | 5.493 | 5.574 | 5.649 | 5.720 | 5.785 | 5.846 | 5.904 |
| 16 | 5.256 | 5.352 | 5.439 | 5.520 | 5.593 | 5.662 | 5.727 | 5.786 | 5.843 |
| 17 | 5.212 | 5.307 | 5.392 | 5.471 | 5.544 | 5.612 | 5.675 | 5.734 | 5.790 |
| 18 | 5.174 | 5.267 | 5.352 | 5.429 | 5.501 | 5.568 | 5.630 | 5.688 | 5.743 |
| 19 | 5.140 | 5.231 | 5.315 | 5.391 | 5.462 | 5.528 | 5.589 | 5.647 | 5.701 |
| 20 | 5.108 | 5.199 | 5.282 | 5.357 | 5.427 | 5.493 | 5.553 | 5.610 | 5.663 |
| 24 | 5.012 | 5.099 | 5.179 | 5.251 | 5.319 | 5.381 | 5.439 | 5.494 | 5.545 |
| 30 | 4.917 | 5.001 | 5.077 | 5.147 | 5.211 | 5.271 | 5.327 | 5.379 | 5.429 |
| 40 | 4.824 | 4.904 | 4.977 | 5.044 | 5.106 | 5.163 | 5.216 | 5.266 | 5.313 |
| 60 | 4.732 | 4.808 | 4.878 | 4.942 | 5.001 | 5.056 | 5.107 | 5.154 | 5.199 |
| 120 | 4.641 | 4.714 | 4.781 | 4.842 | 4.898 | 4.950 | 4.998 | 5.044 | 5.086 |
| ∞ | 4.552 | 4.622 | 4.685 | 4.743 | 4.796 | 4.845 | 4.891 | 4.934 | 4.974 |

## TABLE B.2 PROPORTIONS OF THE NORMAL CURVE (ONE-TAILED)

This table gives the proportion of the normal curve that lies beyond (i.e., is more extreme than) a given normal deviate, (e.g., $Z = (X_i - \mu)/\sigma$ or $Z = (\bar{X} - \mu)/\sigma_{\bar{X}}$). For example, the proportion of a normal distribution for which $Z \geq 1.51$ is 0.0655.

| Z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 | 0.0 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 | 0.1 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 | 0.2 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 | 0.3 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 | 0.4 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 | 0.5 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 | 0.6 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2297 | 0.2266 | 0.2236 | 0.2207 | 0.2177 | 0.2148 | 0.7 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 | 0.8 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 | 0.9 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 | 1.0 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 | 1.1 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 | 1.2 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 | 1.3 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 | 1.4 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 | 1.5 |
| 1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 | 1.6 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 | 1.7 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 | 1.8 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 | 1.9 |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 | 2.0 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 | 2.1 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 | 2.2 |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 | 2.3 |
| 2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 | 2.4 |
| 2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 | 2.5 |
| 2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 | 2.6 |
| 2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 | 2.7 |
| 2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 | 2.8 |
| 2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 | 2.9 |
| 3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 | 3.0 |
| 3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 | 3.1 |
| 3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 | 3.2 |
| 3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 | 3.3 |
| 3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 | 3.4 |
| 3.5 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 3.5 |
| 3.6 | 0.0002 | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 3.6 |
| 3.7 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 3.7 |
| 3.8 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 3.8 |

Table B.2 was prepared using an algorithm of Hastings (1955: 187).
Probabilities for values of $Z$ in between those shown in this table may be obtained by either linear or harmonic interpolation.