# UNIVERSITY OF SWAZILAND SUPPLEMENTARY EXAMINATION 2011/12 

TITLE OF PAPER: PHYSICAL CHEMISTRY

COURSE NUMBER: C302

TIME: THREE (3) HOURS

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INSTRUCTIONS:
There are six questions. Each question is worth 25 marks. Answer any four questions.
A data sheet and a periodic table are attached
Non-programmable electronic calculators may be used.
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## Question 1 (25 marks)

(a) Calculate the radial nodes for the 2 s orbital of $\mathrm{aC}^{5+}$ ion

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\begin{equation*}
\psi_{2 s}=\frac{1}{4 \sqrt{2 \pi}}\left(\frac{Z}{a_{0}}\right)^{3 / 2}(2-\rho) e^{-\rho / 2}, \quad \rho=\frac{Z r}{a_{0}} \tag{3}
\end{equation*}
$$

(b) The ground state spectroscopic term symbols for some elements are given below:

| Element | Term symbol |
| :--- | :--- |
| Nb | ${ }^{6} D_{1 / 2}$ |
| Mo | ${ }^{7} S_{3}$ |
| Rh | ${ }^{4} F_{9 / 2}$ |
| W | ${ }^{5} D_{0}$ |

(i) Write the electron configuration of each atom that is compatible with the spectroscopic term.
(ii) Which of the atoms above are not following the building up principle?
(c) Derive the ground state term symbol for a zirconium atom given that its electron configuration is $[\mathrm{Kr}] 4 \mathrm{~d}^{2} 5 \mathrm{~s}^{2}$.
(d) How many lines will be observed in the fine structure of the transition ${ }^{2} \mathrm{D} \rightarrow{ }^{2} \mathrm{P}$ ? Clearly show your reasons.

## Question 2 (25 marks)

(a) Explain why Einstein's introduction of quantization accounted for the heat capacities of metals at low temperatures.
(b) When lithium is irradiated with light, the kinetic energy of ejected electrons is 2.935 x $10^{-19} \mathrm{~J}$ for $\lambda=300.0 \mathrm{~nm}$ and $1.280 \times 10^{-19} \mathrm{~J}$ for $\lambda=400.0 \mathrm{~nm}$. Calculate
(i) the Planck constant
(ii) the threshold frequency
(iii) the work function for lithium.
(c) For the following functions and operators show that $\mathrm{f}(\mathrm{x})$ is an eigen-function of the operator $\hat{\Omega}$ and determine the eigen-value
(i) $\hat{\Omega}=\frac{\partial}{\partial y}$
$f(x)=x^{2} e^{6 y}$
(ii) $\hat{\Omega}=\frac{d^{2}}{d x^{2}}+4 \frac{d}{d x}+3 \quad \mathrm{f}(\mathrm{x})=\mathrm{e}^{3 \mathrm{x}}$
(d) There is an uncertainty principle for energy and time; $\Delta \mathrm{E} \Delta \mathrm{t} \geq \mathrm{h}$. One application of this relationship has to do with the excited state energies and lifetimes of atoms and molecules. If we know that the lifetime of an excited state is $10^{-9} \mathrm{~s}$, then what is the uncertainty in the energy of this state?
(e) Calculate the de Broglie wavelength of a neutron moving at $6.0 \times 10^{6} \mathrm{~cm} / \mathrm{s}$.

## Question 3 ( 25 marks)

(a). A particle is in a state described by the function $\psi(\mathrm{x})=0.632 \mathrm{e}^{2 \mathrm{ix}}+0.775 \mathrm{e}^{-2 \mathrm{ix}}$. What is the probability that the particle will be found with momentum $2 \hbar$
(b). Consider the energy eigenvalues of a particle in a one dimensional box $E_{n}=\frac{h^{2} n^{2}}{8 m L^{2}}, \mathrm{n}$ $=1,2,3,$. as a function of $n, m$ and $L$.
(i) By what factor do you need to change the box length $L$ to decrease the zero point energy by a factor of 400 for a fixed value of $m$ ?
(ii) By what factor would you have to change $n$ for fixed values of $L$ and m to increase the energy by a factor of 400 ?
(iii) By what factor would you have to increase $L$ to have the zero point energy of an electron be equal to the zero point energy of a proton?
(c) The function $\Psi(\mathrm{x})=x\left(1-\frac{x}{L}\right)$, is an acceptable function for a particle in a one dimensional box of length $L$ and with infinitely high walls.
(i) Normalize $\Psi(x)$
(ii) Calculate the expectation value $<\mathrm{x}>$
$\left[\int x^{n} d x=\frac{1}{(n+1)} x^{n+1}, \quad \mathrm{n} \neq-1\right]$

## Question 4 (25 marks)

(a) Give the gross and specific selection rules for pure rotational spectroscopy.
(b) Which of the following molecules show pure rotational spectra?
$\begin{array}{llllll}\mathrm{H}_{2} & \mathrm{HCl} & \mathrm{CH}_{3} \mathrm{Cl} & \mathrm{CH}_{2} \mathrm{Cl}_{2} & \mathrm{H}_{2} \mathrm{O} & \mathrm{NH}_{3}\end{array}$
Explain your choices.
(c) The average spacing between adjacent lines in the rotational spectra of ${ }^{1} \mathrm{H}^{19} \mathrm{~F}$ is $41.912 \mathrm{~cm}^{-1}$. Calculate the bond length of ${ }^{1} \mathrm{H}^{19} \mathrm{~F}$.
(Atomic masses: ${ }^{1} \mathrm{H} 1.0078 \mathrm{u},{ }^{19} \mathrm{~F} 18.9984$ )
(d) Assuming the bond length is independent of isotopic substitution; calculate the spacing between adjacent lines in the rotational spectra of ${ }^{2} \mathrm{H}^{19} \mathrm{~F}$.
(Atomic mass ${ }^{2} \mathrm{H} 2.0140 \mathrm{u}$ )

Question 5 ( 25 marks)
(a) Give a brief description of the valence bond description of a $\mathrm{CCl}_{4}$ molecule.
(b) Give the ground state electron configuration of:
(i) NO
(ii) CS ,
(iii) $\mathrm{Be}_{2}$
(iv) $\mathrm{C}_{2}$
(c) Which of the species in (b) would you expect to be stabilized by
(i) the addition of an electron to form $\mathrm{AB}^{-}$
(ii) the removal of an electron to form $\mathrm{AB}^{+}$

In each case give the basis of your expectation.
(d) Use the ground state electron configurations of CIF and OF to predict which molecule will have
(i) a greater bond dissociation energy
(ii) a longer bond length

## Question 6 (25 marks)

(a) What is the Doppler shift and how can it be minimized?
(b) Which of the following molecules may show infrared absorption spectra? In each case give the basis of your decision
(i) $\mathrm{CH}_{4}$
(ii) $\mathrm{CH}_{3} \mathrm{Cl}$
(iii) $\mathrm{CO}_{2}$
(iv) $\mathrm{Cl}_{2}$
(c) How many normal modes of vibration are there for the following molecules (i) $\mathrm{HC}=\mathrm{C}-\mathrm{C} \equiv \mathrm{CH}$
(ii) $\mathrm{C}_{6} \mathrm{H}_{6} \quad$ (iii) $\mathrm{O}_{3}$
(d) The HCl molecule is well described by the Morse potential with $\mathrm{D}_{\mathrm{e}}=5.33 \mathrm{eV}, \bar{v}=$ $2989.7 \mathrm{~cm}^{-1}$, and $\chi_{e} \bar{\nu}=52.05 \mathrm{~cm}^{-1}$. Assuming the potential is unchanged on deuteration, predict the dissociation energies ( $\mathrm{D}_{0}$ ) of
(i) HCl and
(ii) DCl .
(Isotopic masses are $\mathrm{H}: 1.0078 \mathrm{u} ; \mathrm{D}: 2.0140 \mathrm{u}$ and ${ }^{35} \mathrm{Cl}: 34.9688 \mathrm{u}$ )

## General data and fundamental constants

| Quantity | Symbol | Value |
| :---: | :---: | :---: |
| Speed of light | c | $2.99792458 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$ |
| Elementary charge | e | $1.602177 \times 10^{-19} \mathrm{C}$ |
| Faraday constant | $\mathrm{F}=\mathrm{N}_{\mathrm{A}} \mathrm{e}$ | $9.6485 \times 10^{4} \mathrm{C} \mathrm{mol}^{-1}$ |
| Boltzmann constant | k | $1.38066 \times 10^{-23} \mathrm{~J} \mathrm{~K}^{-1}$ |
| Gas constant | $R=N_{A} k$ | $8.31451 \mathrm{~J} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}$ <br> $8.20578 \times 10^{-2} \mathrm{dm}^{3} \mathrm{~atm} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}$ <br> $6.2364 \times 10 \mathrm{~L}^{\text {Torr K }}{ }^{-1} \mathrm{~mol}^{-1}$ |
| Planck constant | h | $6.62608 \times 10^{-34} \mathrm{~J} \mathrm{~s}$ |
|  | $h=\mathrm{h} / 2 \pi$ | $1.05457 \mathrm{X}-10^{-34} \mathrm{~J} \mathrm{~s}$ |
| Avogadro constant | $\mathrm{N}_{\text {A }}$ | $6.02214 \times 10^{23} \mathrm{~mol}^{-1}$ |
| Atomic mass unit | u | $1.66054 \times 10^{-27} \mathrm{Kg}$ |
| Mass |  |  |
| electron | $\mathrm{m}_{\text {c }}$ | $9.10939 \times 10^{-31} \mathrm{Kg}$ |
| proton | $\mathrm{m}_{\mathrm{p}}$ | $1.67262 \times 10^{-27} \mathrm{Kg}$ |
| neutron | $\mathrm{m}_{\mathrm{s}}$ | $1.67493 \times 10^{-27} \mathrm{Kg}$ |
| Vacuum permittivity | $\varepsilon_{\mathrm{o}}=1 / \mathrm{c}^{2} \mu_{0}$ | $8.85419 \times 10^{-12} \mathrm{~J}^{-1} \mathrm{C}^{2} \mathrm{~m}^{-1}$ |
|  | $4 \pi \varepsilon_{\text {。 }}$ 。 | $1.11265 \times 10^{-10} \mathrm{~J}^{-1} \mathrm{C}^{2} \mathrm{~m}^{-1}$ |
| Vacuum permeability | $\mu_{0}$ | $\begin{aligned} & 4 \pi \times 10^{-7} \mathrm{~J} \mathrm{~s}^{2} \mathrm{C}^{-2} \mathrm{~m}^{-1} \\ & 4 \pi \times 10^{-7} \mathrm{~T}^{2} \mathrm{~J}^{-1} \mathrm{~m}^{3} \end{aligned}$ |
| Magneton |  |  |
| Bohr | $\mu_{B}=\mathrm{e} \hbar / 2 \mathrm{~m}_{\mathrm{c}}$ | $9.27402 \times 10^{-24} \mathrm{~J} \mathrm{~T}^{-1}$ |
| nuclear | $\mu_{N}=e^{7} / 2 m_{p}$ | $5.05079 \times 10^{-27} \mathrm{~J} \mathrm{~T}^{4}$ |
| $g$ value | $g e$ | 2.00232 |
| Bohr radius | $\mathrm{a}_{0}=4 \pi \varepsilon_{0} h / \mathrm{m}_{e} \mathrm{e}^{2}$. | $5.29177 \times 10^{-11} \mathrm{~m}$ |
| Fine-structure constant | $\alpha=\mu_{0} e^{2} c / 2 h$ | $7.29735 \times 10^{3}$ |
| Rydberg constant | $\mathrm{R}_{\mathrm{m}}=\mathrm{m}_{\mathrm{e}} \mathrm{e}^{4} / 8 \mathrm{~h}^{3} \varepsilon_{0}{ }^{2}$ | $1.09737 \times 10^{7} \mathrm{~m}^{-1}$ |
| Standard acceleration |  |  |
| Ofree favilational constant | $\stackrel{\mathrm{g}}{\mathrm{G}}$ | $9.80665 \mathrm{~m} \mathrm{~s}^{-1}$ $6.67259 \times 10^{-14} \mathrm{Nm}^{2} \mathrm{Kg}^{-2}$ |

## Conversion factors

| 1 cal | 4.184 joules (J) |  |  | 1 erg |  |  | $=$ | $1 \times 10^{-7} \mathrm{~J}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 eV | $1.6022 \times 10^{-19} \mathrm{~J}$ |  |  | $1 \mathrm{eV} /$ molecule |  |  |  | $96485 \mathrm{kr} \mathrm{mol}^{-1}$ |  |  |
| Prefixes | f | p | n | $\mu$ | m. | c | d | k | M | G |
|  | fermto | pico | nano | micro | milli | centi | deci | kilo | mega | giga |
|  | $10^{-15}$ | $10^{-12}$ | $10^{9}$ | $10^{-6}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | $10^{3}$ | $10^{6}$ | $10^{9}$ |


() indicates the mass number of the isotope with the longest half-life.

