

**UNIVERSITY OF ESWATINI**  
**MAIN EXAMINATION 2018/2019**

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**TITLE OF PAPER: INORGANIC CHEMISTRY**

**COURSE NUMBER: C301**

**TIME ALLOWED: THREE (3) HOURS**

**INSTRUCTIONS: THERE ARE SIX (6) QUESTIONS EACH WORTH 25 MARKS. SECTION A CONTAINS TWO (2) QUESTIONS WHILE SECTION B HAS FOUR (4) QUESTIONS. ANSWER ANY FOUR (4) QUESTIONS WITH AT LEAST ONE QUESTION FROM EACH SECTION. EACH SECTION SHOULD BE ANSWERED IN SEPARATE ANSWER FOLDER**

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**THE FOLLOWING HAVE BEEN PROVIDED WITH THIS EXAMINATION PAPER:**

- ❖ Periodic Table of the Elements
- ❖ Table of Universal Constants
- ❖ Tanabe-Sugano diagrams for octahedral complexes
- ❖ Character tables for  $C_{2v}$  and  $D_{3h}$  point groups
- ❖ Decision Tree ( Flow chart) for point groups
- ❖ Tables of contributions by various symmetry operations on unshifted atom to the character and transformation of spectroscopic terms into mulliken symbols

*“Marks will be awarded for method, clearly labelled diagrams, organization and presentation of thoughts in clear and concise language”*

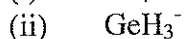
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## SECTION A

### CHEMICAL APPLICATIONS OF GROUP THEORY

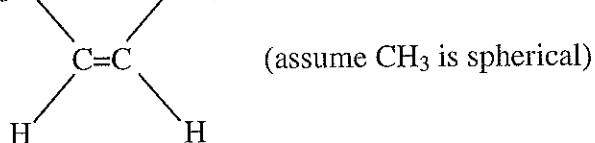
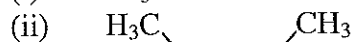
#### QUESTION ONE

(a) Draw the shapes of the following species and state the number of electron lone pairs:



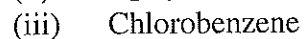
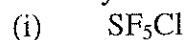
[6]

(b) Identify all the symmetry elements of the following molecules:



[4]

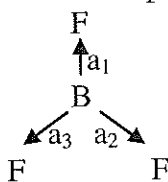
(c) Classify the following species into their point groups:



[9]

(d) (i) Using the following set of three equivalent B-F bonds (treated as vectors) as a basis, derive the matrix representations for the symmetry operations  $E$ ,  $C_3$ ,  $C_2$ ,  $\sigma_h$ ,  $S_3$  and  $\sigma_v$ .

(ii) Find the character of each representations derived in (d)(i) above.



## QUESTION TWO

- (a) Using appropriate examples explain the following:
- (i) an inverse symmetry operation. [6]
  - (ii) commutation of symmetry operations. [6]
- (b) Reduce the following representation [4]
- |           |   |                 |                 |                 |                 |
|-----------|---|-----------------|-----------------|-----------------|-----------------|
| <b>Td</b> | E | 8C <sub>3</sub> | 3C <sub>2</sub> | 6S <sub>4</sub> | 6σ <sub>d</sub> |
|           | 4 | 1               | 0               | 0               | 2               |
- (c) Sketch a qualitative molecular orbital energy level diagram for NH<sub>3</sub> using group theory methods. [6]
- (d) With the help of group theory methods, determine the number of IR and Raman peaks expected for CH<sub>4</sub>. [9]

## SECTION B

### COORDINATION AND TRANSITION METAL CHEMISTRY

#### QUESTION THREE

- (a) Give the IUPAC name for each of the following:
- (i)  $(\text{NH}_4)_3[\text{Fe}(\text{SCN})_6]$
  - (ii)  $[\text{Cr}(\text{OH}_2)_4\text{Cl}_2]\text{Cl}$
  - (iii)  $[\text{Cu}(\text{NH}_3)_4][\text{Fe}(\text{CN})_5\text{OH}]$  [3]
- (b) Give the formula of each of the following:
- (i) Sodium hexafluoroaluminate
  - (ii) Hexaammineruthenium(III) tetrachloronickelate(II)
  - (iii) Tetraammineaquacobalt(III)- $\mu$ -cyanobromotetracyanocobaltate(III) [3]
- (c) Draw the structures of the following species:
- (i) *trans*-dichlorobis(ethylenediamine)cobalt (III) chlorate.
  - (ii) *mer*- $[\text{Fe}(\text{NC})_3(\text{ONO})_3]^{-4}$
  - (iii)  $\mu$ -hydroxobis[pentaamminechromium(III)] ion [6]
- (d) Draw all the isomers, geometrical and optical of  $[\text{Co}(\text{en})(\text{NH}_3)_2\text{Cl}_2]$ . [7]
- (e) What is the *chelate effect*? Give two ways of explaining how the chelate effect leads to greater stability of complexes. [6]

#### QUESTION FOUR

- (a) Which of the following complexes are chiral?
- (i)  $[\text{Cr}(\text{ox})_3]^{3-}$ ;
  - (ii) *cis*- $[\text{PtCl}_2(\text{en})]$ ;
  - (iii) *cis*- $[\text{RhCl}_2(\text{NH}_3)_4]^+$ ;
  - (iv)  $[\text{Ru}(\text{bipy})_3]^{4+}$ ;
  - (v)  $[\text{Co}(\text{edta})]^-$ ;
  - (vi) *fac*- $[\text{Co}(\text{NO}_2)_3(\text{dien})]$ ;
  - (vii) *mer*- $[\text{Co}(\text{NO}_2)_3(\text{dien})]$ .
- Draw the enantiomers of the complexes identified as chiral. [9]
- (b)  $[\text{Cu}(\text{H}_2\text{O})_6]^{2+}$  and  $[\text{Cu}(\text{NH}_3)_4]^{2+}$  both appear blue in solution because of the presence of copper ions. However, the two solutions are not identical. How would the appearance of these solutions differ? If given an unlabeled sample of each, how could the two solutions be distinguished without collecting any spectra? [6]

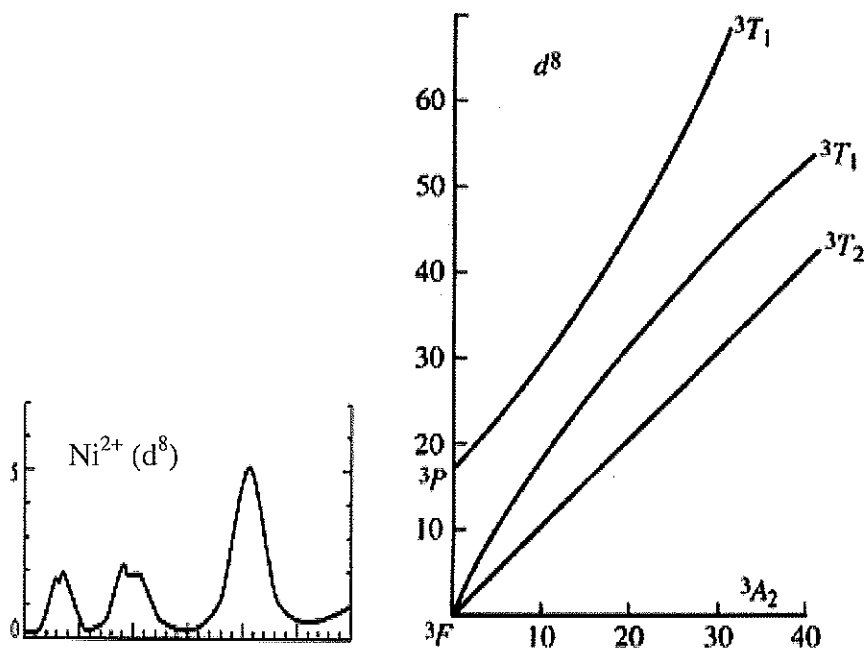
- (c) Explain why?
- (i)  $[\text{Cr}(\text{NH}_3)_6]^{3+}$  is paramagnetic while  $[\text{Ni}(\text{CN})_4]^{2-}$  is diamagnetic.
  - (ii) A solution of  $[\text{Ni}(\text{H}_2\text{O})_6]^{2+}$  is green but a solution of  $[\text{Ni}(\text{CN})_4]^{2-}$  is colourless.
  - (iii)  $[\text{Fe}(\text{CN})_6]^{4-}$  and  $[\text{Fe}(\text{H}_2\text{O})_6]^{2+}$  are of different colours in dilute solutions.
- [10]**

### QUESTION FIVE

- (a) Which of the following complexes obey the 18-electron rule?
- (i)  $[\text{Cu}(\text{NH}_3)_4]^{2+}$
  - (ii)  $[\text{Fe}(\text{CN})_6]^{4-}$
  - (iii)  $[\text{Fe}(\text{CN})_6]^{3-}$
  - (iv)  $[\text{Cr}(\text{NH}_3)_6]^{3+}$
  - (v)  $[\text{Cr}(\text{CO})_6]$
  - (vi)  $[\text{Fe}(\text{CO})_5]$
- [6]**
- (b) How and why does the pairing energy change when a first series transition element is replaced by a second series transition element? **[4]**
- (c) Explain why the purple colour of  $\text{MnO}_4^-$  ions cannot arise from a ligand field transition? **[5]**
- (d) The ion  $[\text{CoCl}_4]^{2-}$  is a regular tetrahedron but  $[\text{CuCl}_4]^{2-}$  is a flattened tetrahedron. Discuss. **[5]**
- (e) How complex anions are separated from by-products and isolated in crystalline form? **[5]**

## QUESTION SIX

- (a) Explain why  $[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$  has magnetic moment value of 5.92 BM whereas  $[\text{Fe}(\text{CN})_6]^{3-}$  has a value of only 1.74 BM. [4]
- (b) For a nickel(II) complex explain the following electronic spectrum with the help of the adjacent Tanabe-Sugano diagram. [6]



- (c) Explain the Jahn-Teller distortion in  $[\text{Cu}(\text{H}_2\text{O})_6]^{2+}$ . [7]
- (d) Classify the following configurations as A, E, or T in complexes having  $O_h$  symmetry. Some of these configurations represent excited states. [8]
- $t_{2g}^4 e_g^2$
  - $t_{2g}^3 e_g^3$
  - $t_{2g}^5 e_g^3$
  - $e_g$



# PERIODIC TABLE OF ELEMENTS

## GROUPS

PERIODS	1		2		3		4		5		6		7		8		9		10		11		12		13		14		15		16		17		18	
	IA	IIA	IIIB	IVB	VB	VIB	VIB	VIB	VIB	VIB	VIB	VIB	VIB	VIB	VIB	VIB	VIB	VIB	IIIB	IIIB	IIIB	IIIB	IIIB	IIIB	IIIA	IIIA	IIIA	IIIA	IIIA	IIIA	IIIA	IIIA	VIIA	VIIA	VIIA	
1	1.008 H																																		4.003 He	
2	6.941 Li	9.012 Be																																	20.180 Ne	
3	22.990 Na	24.305 Mg																																	39.948 Ar	
<b>TRANSITION ELEMENTS</b>																																				
4	39.098 K	40.078 Ca	44.956 Sc	47.88 Ti	50.942 V	51.996 Cr	54.938 Mn	55.847 Fe	58.933 Co	58.69 Ni	63.546 Cu	65.39 Zn	69.723 Ga	72.61 Ge	74.922 As	78.96 Se	79.904 Br	83.80 Kr																	131.29 Xe	
5	85.468 Rb	87.62 Sr	88.906 Y	91.224 Zr	92.906 Nb	95.94 Mo	98.907 Tc	101.07 Ru	102.91 Rh	106.42 Pd	107.87 Ag	112.41 Cd	114.82 In	118.71 Sn	121.75 Sb	127.60 Te	126.90 I	131.29 Xe																	54	
6	132.91 Cs	137.33 Ba	138.91 *La	178.49 Hf	180.95 Ta	183.85 W	186.21 Re	190.2 Os	192.22 Ir	195.08 Pt	196.97 Au	200.59 Hg	204.38 Tl	207.2 Pb	208.98 Bi	(209) Po	(210) At	(222) Rn																		
7	223 Fr	226.03 Ra	(227) **Ac	(261) Rf	(262) Ha	(263) Unh	(262) Uns	(265) Uno	(266) Une	(267) Uun																										

\*Lanthanide Series

\*\*Actinide Series

140.12 Ce	140.91 Pr	144.24 Nd	(145) Pm	150.36 Sm	151.96 Eu	157.25 Gd	158.93 Tb	162.50 Dy	164.93 Ho	167.26 Er	168.93 Tm	173.04 Yb	174.97 Lu
232.04 Th	231.04 Pa	238.03 U	237.05 Np	(244) Pu	(243) Am	(247) Cm	(247) Bk	(251) Cf	(252) Es	(257) Fm	(258) Md	(259) No	(260) Lr
58	59	60	61	62	63	64	65	66	67	68	69	70	71
90	91	92	93	94	95	96	97	98	99	100	101	102	103

( ) indicates the mass number of the isotope with the longest half-life.



## General data and fundamental constants

Quantity	Symbol	Value
Speed of light	$c$	$2.997\ 924\ 58 \times 10^8 \text{ m s}^{-1}$
Elementary charge	$e$	$1.602\ 177 \times 10^{-19} \text{ C}$
Faraday constant	$F = N_A e$	$9.6485 \times 10^4 \text{ C mol}^{-1}$
Boltzmann constant	$k$	$1.380\ 66 \times 10^{-23} \text{ J K}^{-1}$
Gas constant	$R = N_A k$	$8.314\ 51 \text{ J K}^{-1} \text{ mol}^{-1}$ $8.205\ 78 \times 10^{-2} \text{ dm}^3 \text{ atm K}^{-1} \text{ mol}^{-1}$ $6.2364 \times 10 \text{ L Torr K}^{-1} \text{ mol}^{-1}$
Planck constant	$h$ $\hbar = h/2\pi$	$6.626\ 08 \times 10^{-34} \text{ J s}$ $1.054\ 57 \times 10^{-34} \text{ J s}$
Avogadro constant	$N_A$	$6.022\ 14 \times 10^{23} \text{ mol}^{-1}$
Atomic mass unit	$u$	$1.660\ 54 \times 10^{-27} \text{ Kg}$
Mass		
electron	$m_e$	$9.109\ 39 \times 10^{-31} \text{ Kg}$
proton	$m_p$	$1.672\ 62 \times 10^{-27} \text{ Kg}$
neutron	$m_n$	$1.674\ 93 \times 10^{-27} \text{ Kg}$
Vacuum permittivity	$\epsilon_0 = 1/c^2 \mu_0$ $4\pi\epsilon_0$	$8.854\ 19 \times 10^{-12} \text{ J}^{-1} \text{ C}^2 \text{ m}^{-1}$ $1.112\ 65 \times 10^{-10} \text{ J}^{-1} \text{ C}^2 \text{ m}^{-1}$
Vacuum permeability	$\mu_0$	$4\pi \times 10^{-7} \text{ J s}^2 \text{ C}^{-2} \text{ m}^{-1}$ $4\pi \times 10^{-7} \text{ T}^2 \text{ J}^{-1} \text{ C}^{-2} \text{ m}^3$
Magneton		
Bohr	$\mu_B = e\hbar/2m_e$	$9.274\ 02 \times 10^{-24} \text{ J T}^{-1}$
nuclear	$\mu_N = e\hbar/2m_p$	$5.050\ 79 \times 10^{-27} \text{ J T}^{-1}$
g value	$g_e$	2.002 32
Bohr radius	$a_0 = 4\pi\epsilon_0\hbar/m_e e^2$	$5.291\ 77 \times 10^{-11} \text{ m}$
Fine-structure constant	$\alpha = \mu_0 e^2 c/2h$	$7.297\ 35 \times 10^{-3}$
Rydberg constant	$R_\infty = m_e e^4/8h^3 c \epsilon_0^2$	$1.097\ 37 \times 10^7 \text{ m}^{-1}$
Standard acceleration of free fall	$g$	$9.806\ 65 \text{ m s}^{-2}$
Gravitational constant	$G$	$6.672\ 59 \times 10^{-11} \text{ N m}^2 \text{ Kg}^{-2}$

### Conversion factors

1 cal	4.184 joules (J)	1 erg	$1 \times 10^{-7} \text{ J}$
1 eV	$1.602\ 2 \times 10^{-19} \text{ J}$	1 eV/molecule	$96\ 485 \text{ kJ mol}^{-1}$ $23.061 \text{ kcal mol}^{-1}$

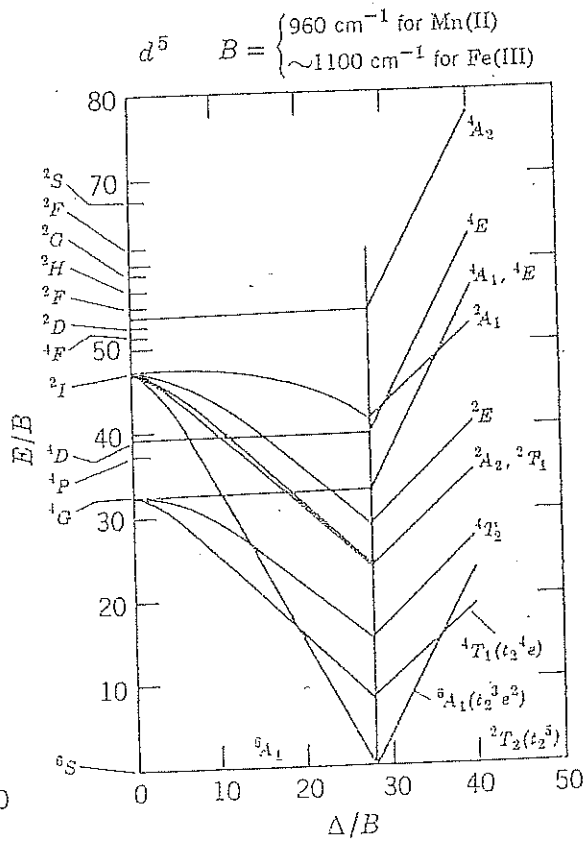
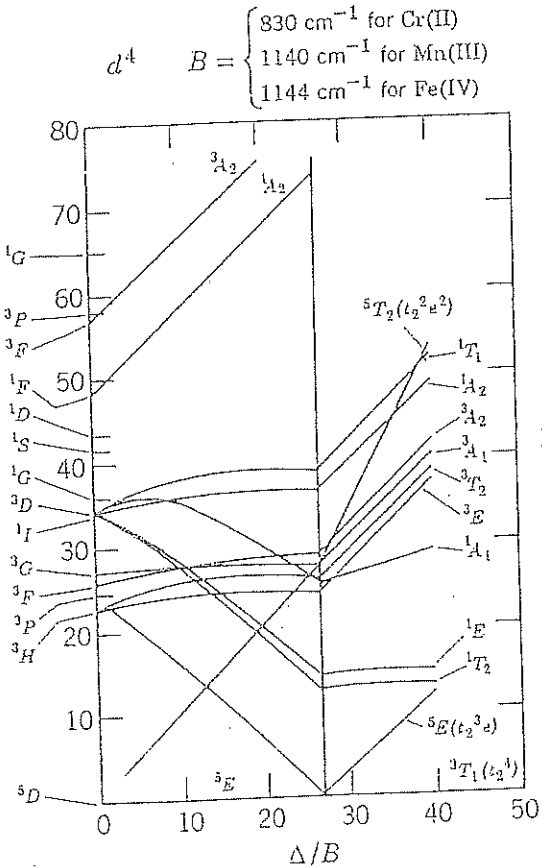
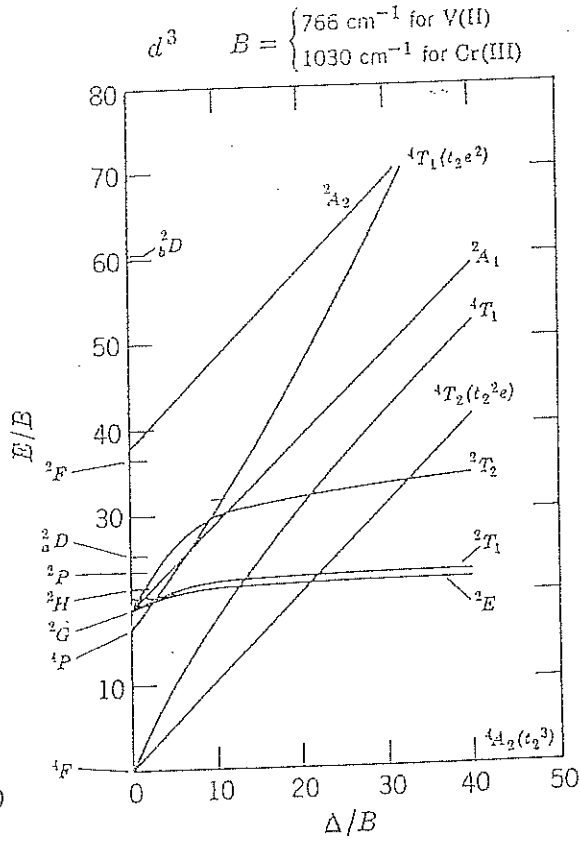
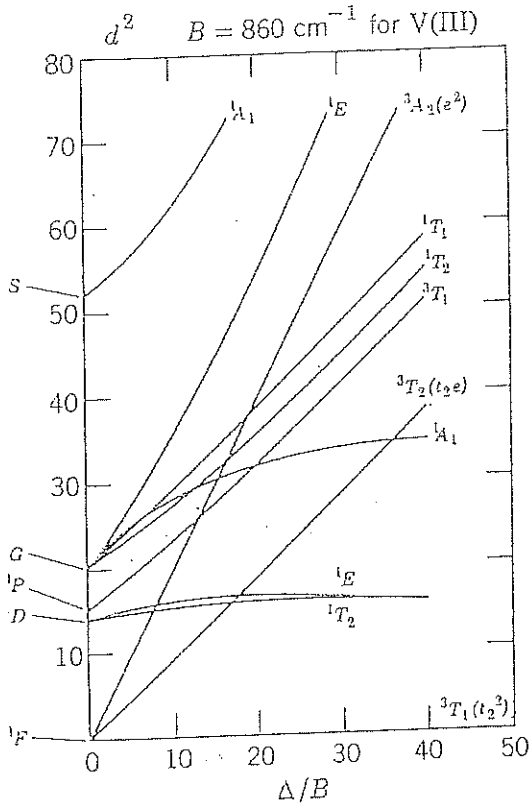
f	p	n	$\mu$	m	c	d	k	M	G	Prefixes
femto	pico	nano	micro	milli	centi	deci	kilo	mega	giga	
$10^{-15}$	$10^{-12}$	$10^{-9}$	$10^{-6}$	$10^{-3}$	$10^{-2}$	$10^{-1}$	$10^3$	$10^6$	$10^9$	

### Spectrochemical Series

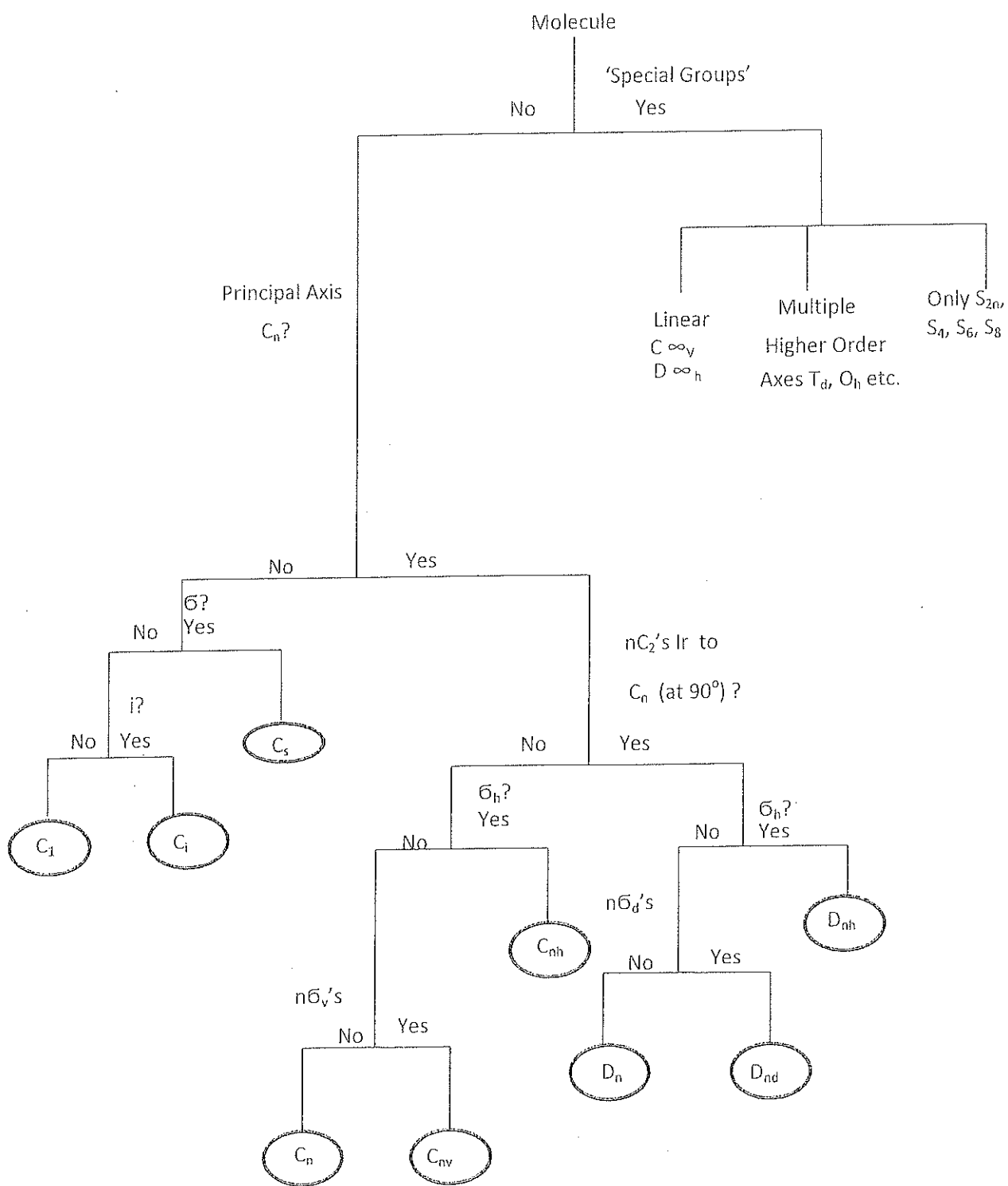
$\Gamma^- < \text{Br}^- < \text{S}^{2-} < \text{Cl}^- < \text{NO}_3^- < \text{F}^- < \text{OH}^- < \text{EtOH} < \text{C}_2\text{O}_4^{2-} < \text{H}_2\text{O} < \text{EDTA} < (\text{NH}_3, \text{py}) < \text{en} < \text{dipy} < \text{NO}_2^- < \text{CN}^- < \text{CO}$ .



Tanabe and Sugano Diagram



FLOW CHART FOR CLASSIFICATION OF POINT GROUPS.



Note:  $C_{\infty v}$ : Anti-symmetrical molecules e.g. HCN  
 $D_{\infty h}$ : Symmetrical molecules e.g.  $CO_2$   
 $C_1$ : No  $C_n$  or  $S_n$ , No  $\sigma$  and No  $i$ .  
 $C_s$ : No  $C_n$  or  $S_n$ , but has  $\sigma$ .  
 $C_i$ : No  $C_n$  or  $S_n$ , No  $\sigma$  but has  $i$ .

**CONTRIBUTIONS BY VARIOUS SYMMETRY  
OPERATIONS ON UNSHIFTED ATOM TO THE  
CHARACTER**

E	$\sigma$	i	$C_n$	$S_n$
3	1	-3	$2\cos\theta + 1$	$2\cos\theta - 1$
$C_2$	$C_3$	$C_4$	$C_5$	$C_6$
-1	0	1	1.618	2
$S_3$	$S_4$	$S_5$	$S_6$	$S_8$
-2	-1	-0.382	0	0.414

**TRANSFORMATION OF SPECTROSCOPIC TERMS  
INTO MULLIKEN SYMBOLS**

Term	$O_h$	$T_d$
S	$A_{1g}$	$A_1$
P	$T_{1g}$	$T_1$
D	$E_g + T_{2g}$	$E + T_2$
F	$A_{2g} + T_{1g} + T_{2g}$	$A_2 + T_1 + T_2$
G	$A_{1g} + E_g + T_{1g} + T_{2g}$	$A_1 + E + T_1 + T_2$

# Character Tables for Chemically Important Symmetry Groups

## 1. The Nonaxial Groups

$C_1$	$E$
$A$	1

$C_s$	$E$	$\sigma_h$		
$A'$	1	1	$x, y, R_z$	$x^2, y^2, z^2, xy$
$A''$	1	-1	$z, R_x, R_y$	$yz, xz$

$C_i$	$E$	$i$		
$A_g$	1	1	$R_x, R_y, R_z$	$x^2, y^2, z^2, xy, xz, yz$
$A_u$	1	-1	$x, y, z$	

## 2. The $C_n$ Groups

$C_2$	$E$	$C_2$		
$A$	1	1	$z, R_z$	$x^2, y^2, z^2, xy$
$B$	1	-1	$x, y, R_x, R_y$	$yz, xz$

$C_3$	$E$	$C_3$	$C_3^2$		$\epsilon = \exp(2\pi i/3)$
$A$	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
$E$	$\begin{Bmatrix} 1 & \epsilon & \epsilon^* \\ 1 & \epsilon^* & \epsilon \end{Bmatrix}$			$(x, y)(R_x, R_y)$	$(x^2 - y^2, xy)(yz, xz)$

$C_4$	$E$	$C_4$	$C_2$	$C_4^3$		
$A$	1	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
$B$	1	-1	1	-1		$x^2 - y^2, xy$
$E$	$\begin{Bmatrix} 1 & i & -1 & -i \\ 1 & -i & -1 & i \end{Bmatrix}$				$(x, y)(R_x, R_y)$	$(yz, xz)$

The  $C_n$  Groups (continued)

$C_5$	$E$	$C_5$	$C_5^2$	$C_5^3$	$C_5^4$		$\epsilon = \exp(2\pi i/5)$
$A$	1	1	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
$E_1$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^2 \\ \epsilon^{2*} \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^{2*} \\ \epsilon^2 \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right\}$	$(x, y)$ $(R_x, R_y)$	$(yz, xz)$
$E_2$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^2 \\ \epsilon^{2*} \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^{2*} \\ \epsilon^2 \end{array} \right\}$		$(x^2 - y^2, xy)$

$C_6$	$E$	$C_6$	$C_3$	$C_2$	$C_3^2$	$C_6^5$		$\epsilon = \exp(2\pi i/6)$
$A$	1	1	1	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
$B$	1	-1	1	-1	1	-1		
$E_1$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right\}$	$\left\{ \begin{array}{l} -\epsilon^* \\ -\epsilon \end{array} \right\}$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right\}$	$\left\{ \begin{array}{l} -\epsilon \\ -\epsilon^* \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right\}$	$(x, y)$ $(R_x, R_y)$	$(xz, yz)$
$E_2$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} -\epsilon^* \\ -\epsilon \end{array} \right\}$	$\left\{ \begin{array}{l} -\epsilon \\ -\epsilon^* \end{array} \right\}$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} -\epsilon^* \\ -\epsilon \end{array} \right\}$	$\left\{ \begin{array}{l} -\epsilon \\ -\epsilon^* \end{array} \right\}$		$(x^2 - y^2, xy)$

$C_7$	$E$	$C_7$	$C_7^2$	$C_7^3$	$C_7^4$	$C_7^5$	$C_7^6$		$\epsilon = \exp(2\pi i/7)$
$A$	1	1	1	1	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
$E_1$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^2 \\ \epsilon^{2*} \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^3 \\ \epsilon^{3*} \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^{3*} \\ \epsilon^3 \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^{2*} \\ \epsilon^2 \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right\}$	$(x, y)$ $(R_x, R_y)$	$(xz, yz)$
$E_2$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^2 \\ \epsilon^{2*} \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^{3*} \\ \epsilon^3 \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^3 \\ \epsilon^{3*} \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^{2*} \\ \epsilon^2 \end{array} \right\}$		$(x^2 - y^2, xy)$
$E_3$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^3 \\ \epsilon^{3*} \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^2 \\ \epsilon^{2*} \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^{2*} \\ \epsilon^2 \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^{3*} \\ \epsilon^3 \end{array} \right\}$		

$C_8$	$E$	$C_8$	$C_4$	$C_2$	$C_4^3$	$C_8^3$	$C_8^5$	$C_8^7$		$\epsilon = \exp(2\pi i/8)$
$A$	1	1	1	1	1	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
$B$	1	-1	1	1	1	-1	-1	-1		
$E_1$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right\}$	$\left\{ \begin{array}{l} i \\ -i \end{array} \right\}$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right\}$	$\left\{ \begin{array}{l} -i \\ i \end{array} \right\}$	$\left\{ \begin{array}{l} -\epsilon^* \\ -\epsilon \end{array} \right\}$	$\left\{ \begin{array}{l} -\epsilon \\ -\epsilon^* \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right\}$	$(x, y)$ $(R_x, R_y)$	$(xz, yz)$
$E_2$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} i \\ -i \end{array} \right\}$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right\}$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right\}$	$\left\{ \begin{array}{l} -i \\ i \end{array} \right\}$	$\left\{ \begin{array}{l} i \\ -i \end{array} \right\}$	$\left\{ \begin{array}{l} -i \\ i \end{array} \right\}$		$(x^2 - y^2, xy)$
$E_3$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} -\epsilon \\ -\epsilon^* \end{array} \right\}$	$\left\{ \begin{array}{l} i \\ -i \end{array} \right\}$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right\}$	$\left\{ \begin{array}{l} -i \\ i \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right\}$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right\}$	$\left\{ \begin{array}{l} -\epsilon^* \\ -\epsilon \end{array} \right\}$		

### 3. The $D_n$ Groups

$D_2$	$E$	$C_2(z)$	$C_2(y)$	$C_2(x)$		
$A$	1	1	1	1		$x^2, y^2, z^2$
$B_1$	1	1	-1	-1	$z, R_z$	$xy$
$B_2$	1	-1	1	-1	$y, R_y$	$xz$
$B_3$	1	-1	-1	1	$x, R_x$	$yz$

$D_3$	$E$	$2C_3$	$3C_2$			
$A_1$	1	1	1			$x^2 + y^2, z^2$
$A_2$	1	1	-1	$z, R_z$		
$E$	2	-1	0	$(x, y)(R_x, R_y)$		$(x^2 - y^2, xy)(xz, yz)$

$D_4$	$E$	$2C_4$	$C_2(=C_4^2)$	$2C_2'$	$2C_2''$		
$A_1$	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	-1	$z, R_z$	
$B_1$	1	-1	1	1	-1		$x^2 - y^2$
$B_2$	1	-1	1	-1	1		$xy$
$E$	2	0	-2	0	0	$(x, y)(R_x, R_y)$	$(xz, yz)$

$D_5$	$E$	$2C_5$	$2C_5^2$	$5C_2$		
$A_1$	1	1	1	1		$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	$z, R_z$	
$E_1$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	$(x, y)(R_x, R_y)$	$(xz, yz)$
$E_2$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0		$(x^2 - y^2, xy)$

$D_6$	$E$	$2C_6$	$2C_3$	$C_2$	$3C_2'$	$3C_2''$		
$A_1$	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2$	1	1	1	1	-1	-1	$z, R_z$	
$B_1$	1	-1	1	-1	1	-1		
$B_2$	1	-1	1	-1	-1	1		
$E_1$	2	1	-1	-2	0	0	$(x, y)(R_x, R_y)$	$(xz, yz)$
$E_2$	2	-1	-1	2	0	0		$(x^2 - y^2, xy)$



4. The  $C_{nv}$  Groups

$C_{2v}$	$E$	$C_2$	$\sigma_v(xz)$	$\sigma'_v(yz)$		
$A_1$	1	1	1	1	$z$	$x^2, y^2, z^2$
$A_2$	1	1	-1	-1	$R_z$	$xy$
$B_1$	1	-1	1	-1	$x, R_y$	$xz$
$B_2$	1	-1	-1	1	$y, R_x$	$yz$

$C_{3v}$	$E$	$2C_3$	$3\sigma_v$			
$A_1$	1	1	1	$z$		$x^2 + y^2, z^2$
$A_2$	1	1	-1	$R_z$		
$E$	2	-1	0	$(x, y)(R_x, R_y)$		$(x^2 - y^2, xy)(xz, yz)$

$C_{4v}$	$E$	$2C_4$	$C_2$	$2\sigma_v$	$2\sigma_d$		
$A_1$	1	1	1	1	1	$z$	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	-1	$R_z$	
$B_1$	1	-1	1	1	-1		$x^2 - y^2$
$B_2$	1	-1	1	-1	1		$xy$
$E$	2	0	-2	0	0	$(x, y)(R_x, R_y)$	$(xz, yz)$

$C_{6v}$	$E$	$2C_3$	$2C_2$	$2C_6$	$5\sigma_v$		
$A_1$	1	1	1	1	1	$z$	$x^2 + y^2, z^2$
$A_2$	1	1	1	1	-1	$R_z$	
$E_1$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	$2 \cos 144^\circ$	0	$(x, y)(R_x, R_y)$	$(xz, yz)$
$E_2$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	$2 \cos 72^\circ$	0		$(x^2 - y^2, xy)$

$C_{6v}$	$E$	$2C_6$	$2C_3$	$C_2$	$3\sigma_v$	$3\sigma_d$		
$A_1$	1	1	1	1	1	1	$z$	$x^2 + y^2, z^2$
$A_2$	1	1	1	1	-1	-1	$R_z$	
$B_1$	1	-1	1	-1	1	-1		
$B_2$	1	-1	1	-1	-1	1		
$E_1$	2	1	-1	-2	0	0	$(x, y)(R_x, R_y)$	$(xz, yz)$
$E_2$	2	-1	-1	2	0	0		$(x^2 - y^2, xy)$

### 5. The $C_{nh}$ Groups

$C_{2h}$	$E$	$C_2$	$i$	$\sigma_h$		
$A_g$	1	1	1	1	$R_z$	$x^2, y^2, z^2, xy$
$B_g$	1	-1	1	-1	$R_x, R_y$	$xz, yz$
$A_u$	1	1	-1	-1	$z$	
$B_u$	1	-1	-1	1	$x, y$	

$C_{3h}$	$E$	$C_3$	$C_3^2$	$\sigma_h$	$S_3$	$S_3^5$		$\epsilon = \exp(2\pi i/3)$
$A'$	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$E'$	$\begin{Bmatrix} 1 & \epsilon & \epsilon^2 \\ 1 & \epsilon^2 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon & 1 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & \epsilon & 1 \\ \epsilon & 1 & \epsilon^2 \end{Bmatrix}$	1	$\epsilon$	$\epsilon^2$	$(x, y)$	$(x^2 - y^2, xy)$
$A''$	1	1	1	-1	-1	-1	$z$	
$E''$	$\begin{Bmatrix} 1 & \epsilon & \epsilon^2 \\ 1 & \epsilon^2 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon & 1 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & \epsilon & 1 \\ \epsilon & 1 & \epsilon^2 \end{Bmatrix}$	-1	$-\epsilon$	$-\epsilon^2$	$(R_x, R_y)$	$(xz, yz)$

$C_{4h}$	$E$	$C_4$	$C_2$	$C_4^3$	$i$	$S_4$	$\sigma_h$	$S_4^3$		
$A_g$	1	1	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$B_g$	1	-1	1	-1	1	-1	1	-1		$x^2 - y^2, xy$
$E_g$	$\begin{Bmatrix} 1 & i & -1 & -i \\ 1 & -i & -1 & i \end{Bmatrix}$	$\begin{Bmatrix} i & 1 & -i & -1 \\ -i & 1 & -i & -1 \end{Bmatrix}$	$\begin{Bmatrix} -1 & -i & 1 & i \\ -1 & -i & 1 & i \end{Bmatrix}$	1	$i$	$-i$	$(R_x, R_y)$	$(xz, yz)$		
$A_u$	1	1	1	1	-1	-1	-1	-1	$z$	
$B_u$	1	-1	1	-1	-1	1	-1	1		
$E_u$	$\begin{Bmatrix} 1 & i & -1 & -i \\ 1 & -i & -1 & i \end{Bmatrix}$	$\begin{Bmatrix} i & 1 & -i & -1 \\ -i & 1 & -i & -1 \end{Bmatrix}$	$\begin{Bmatrix} -1 & -i & 1 & i \\ -1 & -i & 1 & i \end{Bmatrix}$	-1	$-i$	$i$	$(x, y)$			

$C_{5h}$	$E$	$C_5$	$C_5^2$	$C_5^3$	$C_5^4$	$\sigma_h$	$S_5$	$S_5^7$	$S_5^3$	$S_5^9$		$\epsilon = \exp(2\pi i/5)$
$A'$	1	1	1	1	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$E'_1$	$\begin{Bmatrix} 1 & \epsilon & \epsilon^2 & \epsilon^3 & \epsilon^4 \\ 1 & \epsilon^2 & \epsilon^4 & \epsilon & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 & \epsilon^4 & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & \epsilon^4 & \epsilon & 1 & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & \epsilon^4 & \epsilon & 1 & \epsilon^3 \\ \epsilon^4 & \epsilon & 1 & \epsilon^2 & \epsilon^3 \end{Bmatrix}$	1	$\epsilon$	$\epsilon^2$	$\epsilon^4$	$\epsilon^3$	$\epsilon$	$(x, y)$		
$E'_2$	$\begin{Bmatrix} 1 & \epsilon^2 & \epsilon^4 & \epsilon & \epsilon^3 \\ 1 & \epsilon^4 & \epsilon & \epsilon^2 & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & 1 & \epsilon^3 & \epsilon^4 & \epsilon \\ \epsilon^4 & \epsilon & 1 & \epsilon^2 & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^4 & \epsilon & 1 & \epsilon^2 & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 & \epsilon^4 & \epsilon^3 \end{Bmatrix}$	1	$\epsilon^2$	$\epsilon^4$	$\epsilon$	$\epsilon^3$	$\epsilon^2$	$(x^2 - y^2, xy)$		
$A''$	1	1	1	1	1	-1	-1	-1	-1	-1	$z$	
$E''_1$	$\begin{Bmatrix} 1 & \epsilon & \epsilon^2 & \epsilon^3 & \epsilon^4 \\ 1 & \epsilon^2 & \epsilon^4 & \epsilon & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 & \epsilon^4 & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & \epsilon^4 & \epsilon & 1 & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & \epsilon^4 & \epsilon & 1 & \epsilon^3 \\ \epsilon^4 & \epsilon & 1 & \epsilon^2 & \epsilon^3 \end{Bmatrix}$	-1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^4$	$-\epsilon^3$	$-\epsilon$	$(R_x, R_y)$	$(xz, yz)$	
$E''_2$	$\begin{Bmatrix} 1 & \epsilon^2 & \epsilon^4 & \epsilon & \epsilon^3 \\ 1 & \epsilon^4 & \epsilon & \epsilon^2 & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & 1 & \epsilon^3 & \epsilon^4 & \epsilon \\ \epsilon^4 & \epsilon & 1 & \epsilon^2 & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^4 & \epsilon & 1 & \epsilon^2 & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 & \epsilon^4 & \epsilon^3 \end{Bmatrix}$	-1	$-\epsilon^2$	$-\epsilon^4$	$-\epsilon$	$-\epsilon^3$	$-\epsilon^2$			

$C_{6h}$	$E$	$C_6$	$C_3$	$C_2$	$C_3^2$	$C_6^5$	$i$	$S_6$	$S_6^5$	$\sigma_h$	$S_6^3$	$S_6^7$		$\epsilon = \exp(2\pi i/6)$
$A_g$	1	1	1	1	1	1	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$B_g$	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
$E_{1g}$	$\begin{Bmatrix} 1 & \epsilon & -\epsilon^2 & -1 & -\epsilon & \epsilon^3 \\ 1 & \epsilon^2 & -\epsilon & -1 & -\epsilon^2 & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 & \epsilon^5 & -\epsilon & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & \epsilon^5 & \epsilon & -\epsilon & \epsilon^2 & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} -1 & -\epsilon & -\epsilon^2 & 1 & -\epsilon & \epsilon^3 \\ -1 & -\epsilon & -\epsilon^2 & 1 & -\epsilon & \epsilon^3 \end{Bmatrix}$	1	$\epsilon$	$-\epsilon^2$	$-\epsilon$	$\epsilon^3$	$-\epsilon$	$-\epsilon^2$	$-\epsilon$	$(R_x, R_y)$	$(xz, yz)$	
$E_{2g}$	$\begin{Bmatrix} 1 & -\epsilon & -\epsilon^2 & 1 & -\epsilon^2 & -\epsilon^3 \\ 1 & -\epsilon^2 & -\epsilon^4 & 1 & -\epsilon & -\epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} -\epsilon & 1 & -\epsilon^5 & -\epsilon & -\epsilon^2 & -\epsilon^3 \\ -\epsilon^2 & -\epsilon^5 & -\epsilon & -\epsilon & -\epsilon^2 & -\epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} 1 & -\epsilon & -\epsilon^2 & 1 & -\epsilon & -\epsilon^3 \\ 1 & -\epsilon & -\epsilon^2 & 1 & -\epsilon & -\epsilon^3 \end{Bmatrix}$	1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$		$(x^2 - y^2, xy)$		
$A_u$	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	$z$	
$B_u$	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1		
$E_{1u}$	$\begin{Bmatrix} 1 & \epsilon & -\epsilon^2 & -1 & -\epsilon & \epsilon^3 \\ 1 & \epsilon^2 & -\epsilon & -1 & -\epsilon^2 & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 & \epsilon^5 & -\epsilon & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & \epsilon^5 & \epsilon & -\epsilon & \epsilon^2 & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} -1 & -\epsilon & -\epsilon^2 & 1 & -\epsilon & \epsilon^3 \\ -1 & -\epsilon & -\epsilon^2 & 1 & -\epsilon & \epsilon^3 \end{Bmatrix}$	-1	$\epsilon$	$-\epsilon^2$	$-\epsilon$	$\epsilon^3$	$\epsilon$	$-\epsilon^2$	$-\epsilon$	$(x, y)$		
$E_{2u}$	$\begin{Bmatrix} 1 & -\epsilon & -\epsilon^2 & 1 & -\epsilon^2 & -\epsilon^3 \\ 1 & -\epsilon^2 & -\epsilon^4 & 1 & -\epsilon & -\epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} -\epsilon & 1 & -\epsilon^5 & -\epsilon & -\epsilon^2 & -\epsilon^3 \\ -\epsilon^2 & -\epsilon^5 & -\epsilon & -\epsilon & -\epsilon^2 & -\epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} 1 & -\epsilon & -\epsilon^2 & 1 & -\epsilon & -\epsilon^3 \\ 1 & -\epsilon & -\epsilon^2 & 1 & -\epsilon & -\epsilon^3 \end{Bmatrix}$	-1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$				

### 6. The $D_{nh}$ Groups

$D_{2h}$	$E$	$C_2(z)$	$C_2(y)$	$C_2(x)$	$i$	$\sigma(xy)$	$\sigma(xz)$	$\sigma(yz)$		
$A_g$	1	1	1	1	1	1	1	1		$x^2, y^2, z^2$
$B_{1g}$	1	1	-1	-1	1	1	-1	-1	$R_x$	$xy$
$B_{2g}$	1	-1	1	-1	1	-1	1	-1	$R_y$	$xz$
$B_{3g}$	1	-1	-1	1	1	-1	-1	1	$R_z$	$yz$
$A_u$	1	1	1	1	-1	-1	-1	-1		
$B_{1u}$	1	1	-1	-1	-1	-1	1	1	$z$	
$B_{2u}$	1	-1	1	-1	-1	1	-1	1	$y$	
$B_{3u}$	1	-1	-1	1	-1	1	1	-1	$x$	

$D_{3h}$	$E$	$2C_3$	$3C_2$	$\sigma_h$	$2S_6$	$3\sigma_v$		
$A'_1$	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A'_2$	1	1	-1	1	1	-1	$R_x$	
$E'$	2	-1	0	2	-1	0	$(x, y)$	$(x^2 - y^2, xy)$
$A''_1$	1	1	1	-1	-1	-1		
$A''_2$	1	1	-1	-1	-1	1	$z$	
$E''$	2	-1	0	-2	1	0	$(R_x, R_y)$	$(xz, yz)$

$D_{4h}$	$E$	$2C_4$	$C_2$	$2C_2'$	$2C_2''$	$i$	$2S_4$	$\sigma_h$	$2\sigma_v$	$2\sigma_d$		
$A_{1g}$	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	-1	-1	1	1	1	-1	-1	$R_x$	
$B_{1g}$	1	-1	1	1	-1	1	-1	1	1	-1		$x^2 - y^2$
$B_{2g}$	1	-1	1	-1	1	1	-1	1	-1	1	$(R_x, R_y)$	$xy$
$E_g$	2	0	-2	0	0	2	0	-2	0	0		$(xz, yz)$
$A_{1u}$	1	1	1	1	1	-1	-1	-1	-1	-1		
$A_{2u}$	1	1	1	-1	-1	-1	-1	-1	1	1	$z$	
$B_{1u}$	1	-1	1	1	-1	-1	1	-1	-1	1		
$B_{2u}$	1	-1	1	-1	1	-1	1	-1	1	-1		
$E_u$	2	0	-2	0	0	-2	0	2	0	0	$(x, y)$	

$D_{5h}$	$E$	$2C_5$	$2C_5^2$	$5C_2$	$\sigma_h$	$2S_5$	$2S_5^3$	$5\sigma_v$		
$A'_1$	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A'_2$	1	1	1	-1	1	1	1	-1	$R_x$	
$E'_1$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	2	$2 \cos 72^\circ$	$-2 \cos 144^\circ$	0	$(x, y)$	
$E'_2$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0		$(x^2 - y^2, xy)$
$A''_1$	1	1	1	1	-1	-1	-1	-1		
$A''_2$	1	1	1	-1	-1	-1	-1	1	$z$	
$E''_1$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	-2	$-2 \cos 72^\circ$	$-2 \cos 144^\circ$	0	$(R_x, R_y)$	$(xz, yz)$
$E''_2$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	-2	$-2 \cos 144^\circ$	$-2 \cos 72^\circ$	0		

$D_{6h}$	$E$	$2C_6$	$2C_3$	$C_2$	$3C_2'$	$3C_2''$	$i$	$2S_6$	$2S_6^5$	$\sigma_h$	$3\sigma_d$	$3\sigma_v$		
$A_{1g}$	1	1	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	1	-1	-1	1	1	1	1	-1	-1	$R_x$	
$B_{1g}$	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
$B_{2g}$	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1	$(R_x, R_y)$	$(xz, yz)$
$E_{1g}$	2	1	-1	-2	0	0	2	1	-1	-2	0	0		$(x^2 - y^2, xy)$
$E_{2g}$	2	-1	-1	2	0	0	2	-1	-1	2	0	0		
$A_{1u}$	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1		
$A_{2u}$	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	$z$	
$B_{1u}$	1	-1	1	-1	1	-1	-1	-1	-1	1	-1	1		
$B_{2u}$	1	-1	1	-1	-1	1	-1	-1	1	1	1	-1		
$E_{1u}$	2	1	-1	-2	0	0	-2	-1	1	2	0	0	$(x, y)$	
$E_{2u}$	2	-1	-1	2	0	0	-2	1	1	-2	0	0		

### 7. The $D_{nd}$ Groups

$D_{2d}$	$E$	$2S_4$	$C_2$	$2C_2'$	$2\sigma_d$		
$A_1$	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	-1		$x^2 - y^2$
$B_1$	1	-1	1	1	-1	$z$ $(x, y)$ $(R_x, R_y)$	$xy$ $(xz, yz)$
$B_2$	1	-1	1	-1	1		
$E$	2	0	-2	0	0		

$D_{3d}$	$E$	$2C_3$	$3C_2$	$i$	$2S_6$	$3\sigma_d$		
$A_{1g}$	1	1	1	1	1	1	$R_z$ $(R_x, R_y)$	$x^2 + y^2, z^2$
$A_{2g}$	1	1	-1	1	1	-1		$(x^2 - y^2, xy)$
$E_g$	2	-1	0	2	-1	0		$(xz, yz)$
$A_{1u}$	1	1	1	-1	-1	-1	$z$ $(x, y)$	
$A_{2u}$	1	1	-1	-1	-1	1		
$E_u$	2	-1	0	-2	1	0		

$D_{4d}$	$E$	$2S_4$	$2C_4$	$2S_4^3$	$C_2$	$4C_2'$	$4\sigma_d$		
$A_1$	1	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$A_2$	1	1	1	1	1	-1	-1		
$B_1$	1	-1	1	-1	1	1	-1	$z$ $(x, y)$	
$B_2$	1	-1	1	-1	1	-1	1		
$E_1$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	0	$(R_x, R_y)$	$(x^2 - y^2, xy)$
$E_2$	2	0	-2	0	2	0	0		$(xz, yz)$
$E_3$	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	0		

$D_{5d}$	$E$	$2C_5$	$2C_5^2$	$5C_2$	$i$	$2S_{10}$	$2S_{10}^3$	$5\sigma_d$		
$A_{1g}$	1	1	1	1	1	1	1	1	$R_z$ $(R_x, R_y)$	$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	-1	1	1	1	-1		$(xz, yz)$
$E_{1g}$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0		$(x^2 - y^2, xy)$
$E_{2g}$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0		
$A_{1u}$	1	1	1	1	-1	-1	-1	-1	$z$ $(x, y)$	
$A_{2u}$	1	1	1	-1	-1	-1	-1	1		
$E_{1u}$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	-2	$-2 \cos 72^\circ$	$-2 \cos 144^\circ$	0		
$E_{2u}$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	-2	$-2 \cos 144^\circ$	$-2 \cos 72^\circ$	0		

$D_{6d}$	$E$	$2S_{12}$	$2C_6$	$2S_6$	$2C_3$	$2S_6^5$	$C_2$	$6C_2'$	$6\sigma_d$		
$A_1$	1	1	1	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$A_2$	1	1	1	1	1	1	1	-1	-1		
$B_1$	1	-1	1	-1	1	-1	1	1	-1	$z$ $(x, y)$	
$B_2$	1	-1	1	-1	1	-1	1	-1	1		
$E_1$	2	$\sqrt{3}$	1	0	-1	$-\sqrt{3}$	-2	0	0	$(R_x, R_y)$	$(x^2 - y^2, xy)$
$E_2$	2	1	-1	-2	-1	1	2	0	0		
$E_3$	2	0	-2	0	2	0	-2	0	0		
$E_4$	2	-1	-1	2	-1	$-\sqrt{3}$	2	0	0		
$E_5$	2	$-\sqrt{3}$	1	0	-1	$\sqrt{3}$	-2	0	0		

8: The  $S_n$  Groups

$S_4$	$E$	$S_4$	$C_2$	$S_4^3$		
$A$	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$B$	1	-1	1	-1	$z$	$x^2 - y^2, xy$
$E$	$\begin{Bmatrix} 1 & i & -1 & -i \\ 1 & -i & -1 & i \end{Bmatrix}$				$(x, y); (R_x, R_y)$	$(xz, yz)$

$S_6$	$E$	$C_3$	$C_3^2$	$i$	$S_6^5$	$S_6$		$\epsilon = \exp(2\pi i/3)$
$A_g$	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$E_g$	$\begin{Bmatrix} 1 & \epsilon & \epsilon^* & 1 & \epsilon & \epsilon^* \\ 1 & \epsilon^* & \epsilon & 1 & \epsilon^* & \epsilon \end{Bmatrix}$						$(R_x, R_y)$	$(x^2 - y^2, xy);$ $(xz, yz)$
$A_u$	1	1	1	-1	-1	-1	$z$	
$E_u$	$\begin{Bmatrix} 1 & \epsilon & \epsilon^* & -1 & -\epsilon & -\epsilon^* \\ 1 & \epsilon^* & \epsilon & -1 & -\epsilon^* & -\epsilon \end{Bmatrix}$						$(x, y)$	

$S_8$	$E$	$S_8$	$C_4$	$S_8^3$	$C_2$	$S_8^5$	$C_4^3$	$S_8^7$		$\epsilon = \exp(2\pi i/8)$
$A$	1	1	1	1	1	1	1	1	$R_x$	$x^2 + y^2, z^2$
$B$	1	-1	1	-1	1	-1	1	-1	$z$	
$E_1$	$\begin{Bmatrix} 1 & \epsilon & i & -\epsilon^* & -1 & -\epsilon & -i & \epsilon^* \\ 1 & \epsilon^* & -i & -\epsilon & -1 & -\epsilon^* & i & \epsilon \end{Bmatrix}$								$(x, y);$ $(R_x, R_y)$	
$E_2$	$\begin{Bmatrix} 1 & i & -1 & -i & 1 & i & -1 & -i \\ 1 & -i & -1 & i & 1 & -i & -1 & i \end{Bmatrix}$									$(x^2 - y^2, xy)$
$E_3$	$\begin{Bmatrix} 1 & -\epsilon^* & -i & \epsilon & -1 & \epsilon^* & i & -\epsilon \\ 1 & -\epsilon & i & \epsilon^* & -1 & \epsilon & -i & -\epsilon^* \end{Bmatrix}$									$(xz, yz)$



### 11. The Icosahedral Group

$J_A$	$E$	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$	$i$	$12S_6$	$12S_6^5$	$20S_4$	$15\sigma$	
$A_g$	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2 + z^2$
$T_{1g}$	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1	$(R_x, R_y, R_z)$
$T_{2g}$	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1	
$G_g$	4	-1	-1	1	0	4	-1	1	1	0	
$H_g$	5	0	0	-1	1	5	0	-1	-1	1	
$A_u$	1	1	1	1	1	-1	-1	-1	-1	-1	$(x, y, z)$
$T_{1u}$	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1	-3	$-\frac{1}{2}(1 - \sqrt{5})$	$-\frac{1}{2}(1 + \sqrt{5})$	0	1	
$T_{2u}$	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1	-3	$-\frac{1}{2}(1 + \sqrt{5})$	$-\frac{1}{2}(1 - \sqrt{5})$	0	1	
$G_u$	4	-1	-1	1	0	-4	1	1	-1	0	
$H_u$	5	0	0	-1	1	-5	0	1	1	-1	