132

# UNIVERSITY OF SWAZILAND

## MAIN EXAMINATION 2004/2005

## FACULTY OF SCIENCE

### DEPARTMENT OF ELECTRONIC ENGINEERING

| | |
|---|---|
| TITLE OF PAPER: | SIGNALS II |
| COURSE NUMBER: | E462 |
| TIME ALLOWED: | THREE HOURS |

INSTRUCTIONS:

1. **Answer any FOUR (4) of the following seven questions.**
2. **Each question carries 25 marks.**
3. **Tables of selected Fourier transform pairs and Fourier transform properties are attached.**

**THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR**

**THIS PAPER CONTAINS TEN (10) PAGES INCLUDING THIS PAGE**

**Question One (25 marks)**

(a) (i) Define the Fourier transform of a signal and state the conditions for its existence. *(3 marks)*

(ii) Briefly explain the importance of the Fourier representation of a signal *(3marks)*

(b) (i) From the definition of the Fourier transform obtain the Fourier transform of a rectangular pulse of unit amplitude and duration T sec. *(4 marks)*

(ii) Use the result in b(i) to obtain the bandwidth in Hz between the first nulls of the spectrum of a pulse of duration 0.25 μs. *(6 marks)*

(c) If the Fourier transform of a signal $x(t)$ is $X(\omega) = 2\sin 6\omega$. Deduce the Fourier transform of:

(i) $x(2t)$ *(4 marks)*

(ii) $x(t_o - 2t)$ *(5 marks)*

**Question Two (25 marks)**

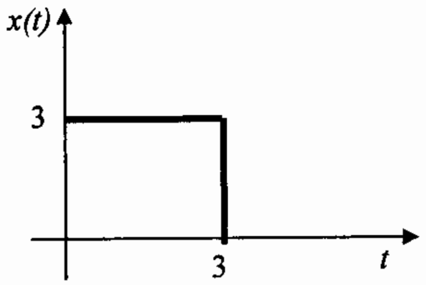
**(a)** A signal of 35 dBm is passed through an amplifier with a power gain of 10. What is the output signal level in dBm? *(4 marks)*

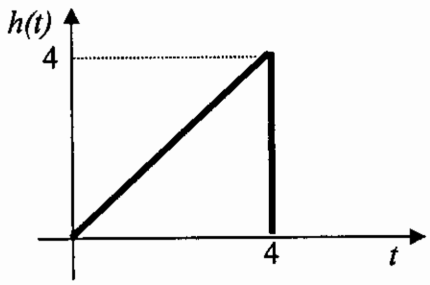
**(b)** A signal $x(t)$ sketched below is passed through a system with impulse response $h(t)$ also shown below:

**(i)** Find expressions the output signal $y(t)$. The signal $y(t)$ may be divided into clearly defined intervals. *(15 marks)*

**(ii)** Find the maximum value of the response and the time at which it occurs. *(3 marks)*

**(iii)** By assuming that $x(t)$ was a dc signal of unit amplitude, find the dc gain of the system. *(3 marks)*



**Question Three (25 marks)**

A chirp signal $x(t) = \cos(\alpha t^2 + \beta t + \phi)$ sweeps in frequency from $f_1$ to $f_2$ in time T sec.

(a) Find α and β in terms of $f_1$, $f_2$ and T. *(7 marks)*

(b) Determine the formula for a chirp signal that sweeps from 1200 Hz to 400 Hz in 2 sec with initial phase of 60°. *(7 marks)*

(c) A chirp signal is given by $\text{Re}\left\{e^{j2\pi(40t^2-10t)}\right\}$

(i) Derive an expression for its instantaneous frequency. *(3 marks)*

(ii) Can its frequency be negative? Explain your answer. *(4 marks)*

(iii) Plot the instantaneous frequency vs time for the period $0 \le t \le 1$ sec. *(4 marks)*

**Question Four (25 marks)**

**(a)** (i) Distinguish between the power and energy of a signal. *(2 marks)*

(ii) Distinguish between an energy signal and a power signal. *(3 marks)*

**(b)** For each of the following signals find the power and energy of the signal and classify it as one of: energy signal, power signal, neither power or energy signal, or both energy and power signal:

(i) $x(t) = 3u(t)$ *(4 marks)*

(ii) $x(t) = e^{-a|t|}, \quad a > 0$ *(6 marks)*

(iii) $x(t) = 4[u(t+a) - u(t-a)], \quad a > 0$ *(5 marks)*

(iv) $x(t) = t\,u(t)$ *(5 marks)*

**Question 5 (25 marks)**

A periodic signal is given by

$$x(t) = 4\sin 3t + 3\cos 10t - 2\sin 15t + \cos 20t \quad \text{volts.}$$

**(a)** What is the reading of an ac (rms) voltmeter measuring it? Hint use Parseval's theorem. *(4 marks)*

**(b)** Sketch its amplitude and phase spectrum. *( 7 marks)*

**(c)** If the above signal is passed through an ideal low pass filter with a cut off frequency of 12 rad/s, find the rms value of the filter output. *(4 marks)*

**(d)** Assuming a cut-off frequency of twice the fundamental frequency, design, a single pole filter to filter the signal and if possible produce a sinewave. In your design you only need to specify the transfer function. *(3 marks)*

**(e)** Test your design in (d) by checking to see if the ratio of the power in the



**Question Six (25 marks)**

**(a)** Distinguish between the responses of following filter approximations: Butterworth, Chebyshev. *(5 marks)*

**(b)** Without using tables determine the location of poles of a 5-pole Butterworth filter. *( 8 marks)*

**(c)** Determine the order of a Chebyshev filter that has a 0.5 dB ripple in the pass band and is down at least 40 dB at twice its cut-off frequency. *(12 marks)*

**Question Seven (25 marks)**

**(a)** State the properties of a probability density function. *(3 marks)*

**(b)** Which of the following functions satisfies the properties of a pdf ? Justify your answer in each case.

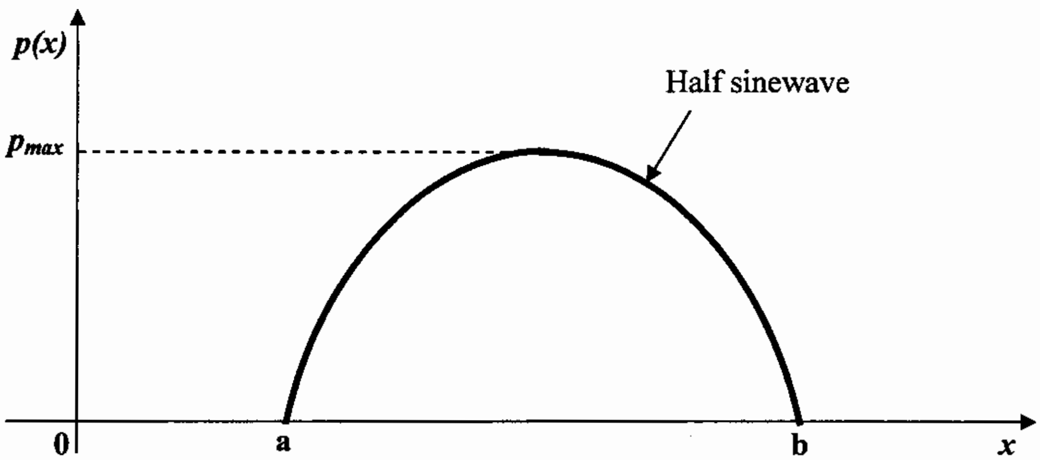
(i) $f(x) = \frac{1}{\pi}\left(\frac{1}{1+x^2}\right)$ *(6 marks)*

(ii) $f(x) = \left| \begin{matrix} |x| , & |x|<1 \\ 0 , & x \text{ otherwise} \end{matrix} \right.$ *(6 marks)*

Note that $\int \frac{1}{1+x^2} dx = \tan^{-1} x$ .

**(c)** Suppose the probability density function (pdf) of a signal varying randomly between the amplitude intervals [a, b] is as illustrated below. The 'dome' shape of the pdf is that of a half cycle of a sinusoid.
Using properties of the pdf determine the value of $p_{max}$ in terms of a and b.

*(10 marks)*




**== END OF EXAMINATION PAPER. ATTACHMENTS FOLLOW ==**

# TABLE OF BASIC FOURIER TRANSFORM PAIRS

| # | Time Domain: x(t) | Frequency Domain: X(jω) |
|---|---|---|
| 1 | $\delta(t)$ | $1$ |
| 2 | $1$ | $2\pi\delta(\omega)$ |
| 3 | $\delta(t-t_d)$ | $e^{-j\omega t_d}$ |
| 4 | $e^{j\omega_o t}$ | $2\pi\delta(\omega-\omega_o)$ |
| 5 | $e^{-at}u(t),\ (a>0)$ | $\frac{1}{a+j\omega}$ |
| 6 | $e^{bt}u(-t),\ (b>0)$ | $\frac{1}{b-j\omega}$ |
| 7 | $u(t)$ | $\pi\delta(\omega)+\frac{1}{j\omega}$ |
| 8 | $u(t+\frac{1}{2}T)-u(t-\frac{1}{2}T)$ | $\frac{\sin(\omega T/2)}{\omega/2}$ |
| 9 | $\frac{\sin(\omega_b t)}{\pi t}$ | $u(\omega+\omega_b)-u(\omega-\omega_b)$ |
| 10 | $A\cos(\omega_o t+\phi)$ | $\pi Ae^{j\phi}\delta(\omega-\omega_o)+\pi Ae^{-j\phi}\delta(\omega+\omega_o)$ |
| 11 | $\cos(\omega_o t)$ | $\pi\delta(\omega-\omega_o)+\pi\delta(\omega+\omega_o)$ |
| 12 | $\sin(\omega_o t)$ | $-j\pi\delta(\omega-\omega_o)+j\pi\delta(\omega+\omega_o)$ |
| 13 | $\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$ | $\sum_{k=-\infty}^{\infty} 2\pi a_k\delta(\omega-k\omega_o)$ |
| 14 | $\sum_{k=-\infty}^{\infty}\delta(t-nT)$ | $\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega-\frac{2\pi}{T}k)$ |

## TABLE OF BASIC FOURIER TRANSFORM PROPERTIES

| # | PROPERTY NAME | TIME DOMAIN: x(t) | FREQUENCY DOMAIN: X(jω) |
|---|---|---|---|
| 1 | Linearity | $ax_1(t)+bx_2(t)$ | $aX_1(j\omega)+bX_2(j\omega)$ |
| 2 | Conjugation | $x^*(t)$ | $X^*(-j\omega)$ |
| 3 | Time-Reversal | $x(-t)$ | $X(-j\omega)$ |
| 4 | Time Scaling | $x(at)$ | $\frac{1}{\lvert a\rvert}X(j\frac{\omega}{a})$ |
| 5 | Time Delay | $x(t-t_d)$ | $e^{-j\omega t_d}X(j\omega)$ |
| 6 | Modulation | $x(t)e^{j\omega_o t}$ | $X[j(\omega-\omega_o)]$ |
| 7 | Modulation | $x(t)\cos(\omega_o t)$ | $\frac{1}{2}X[j(\omega-\omega_o)]+\frac{1}{2}X[j(\omega+\omega_o)]$ |
| 8 | Differentiation | $\frac{d^n x(t)}{dt^n}$ | $(j\omega)^n X(j\omega)$ |
| 9 | Convolution | $x(t)*h(t)$ | $X(j\omega)H(j\omega)$ |
| 10 | Multiplication | $x(t)y(t)$ | $\frac{1}{2\pi}X(j\omega)*Y(j\omega)$ |