UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION May 2006

TITLE OF PAPER: **CONTROL SYSTEMS**

COURSE NUMBER: **E430**

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER **QUESTION 1** AND ANY OTHER **THREE QUESTIONS**

QUESTION 1 CARRIES 40 MARKS

QUESTIONS 2, 3, 4, AND 5 CARRY 20 MARKS EACH.

MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE

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Partial Table of z- and s-Transforms

	f(t)	F(s)	F(z)	f(kt)
1.	$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$	$u(kT)$
2.	t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$	kT
3.	t^n	$\frac{n!}{s^{n+1}}$	$\lim_{a\to 0}(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n$
4.	e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$	e^{-akT}
5.	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n e^{-akT}$
6.	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z\sin\omega T}{z^2-2z\cos\omega T+1}$	$\sin \omega kT$
7.	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$\frac{z(z-\cos\omega T)}{z^2-2z\cos\omega T+1}$	$\cos \omega kT$
8.	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{ze^{-aT}\sin\omega T}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$	$e^{-akT}\sin\omega kT$
9.	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{z^2-ze^{-aT}\cos\omega T}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$	$e^{-akT}\cos\omega kT$
			$\frac{z}{z+a}$	$a^K \cos K\pi$

z-Transform Theorems

	Theorem	Name
1.	$z\{af(t)\} = aF(z)$	Linearity theorem
2.	$z\{f_1(t)+f_2(t)\} = F_1(z)+F_2(z)$	Linearity theorem
3.	$z\{e^{-at}f(t)\} = F(e^{aT}z)$	Complex differentiation
4.	$z\{f(t-nT)\} = z^{-n}F(z)$	Real translation
5.	$z\{tf(t)\} = -Tz\frac{dF(z)}{dz}$	Complex differentiation
6.	$f(0) = \lim_{z\to\infty} F(z)$	Initial value theorem
7.	$f(\infty) = \lim_{z\to 1}(1-z^{-1})F(z)$	Final value theorem

Note: kT may be substituted for t in the table.

Question 1

a) A motor and load system is represented by the following equations:

$$e_a(t) = i_a(t)R_a + k_b\frac{d\theta_m}{dt} \quad \ldots\ldots\ (1)$$

$$k_t i_a = J_m\frac{d^2\theta_m}{dt^2} + D_m\frac{d\theta_m}{dt} \quad \ldots\ldots\ (2)$$

$$\theta_o(t) = 0.1\theta_m \quad \ldots\ldots\ (3)$$

where $e_a(t)$ is the armature input voltage

$i_a(t)$ is the output angular displacement

R_a is the armature resistance

k_b is the armature constant

θ_m is the armature angular displacement

J_m is the equivalent inertia seen by the armature

D_m is the equivalent viscous damping seen by the armature

Define the state variables as $x_1 = \theta_m$ and $x_2 = \frac{d\theta_m}{dt}$ and find the state space representation of this system. [14 marks]

b) For the system shown in Figure 1.A it is desired that the steady state error should be 0.02 when the input is a unit step. Determine the value of the gain K which would allow the system to give the desired steady state error. [7 marks]

c) Obtain the characteristic equation of the system shown in Figure 1.A and then find the range of K for which the system is stable? [6 marks]

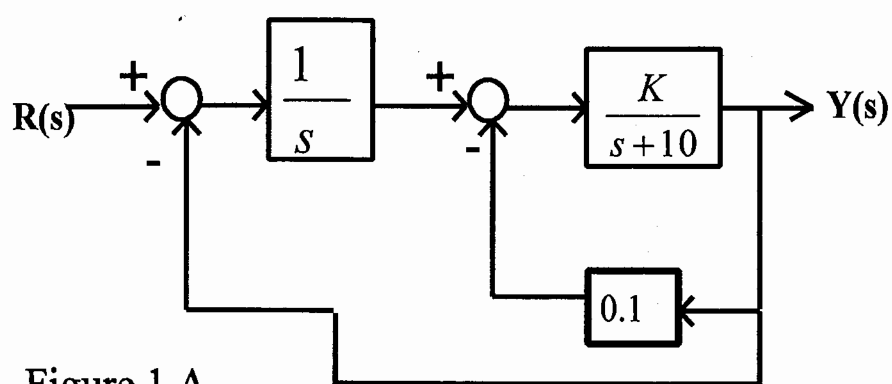


Figure 1.A

Question 1 Continued

d) i) State the condition for stability for a digital control system.

ii) Obtain the pulse transfer function C(z)/R(z) and then with a unit step input obtain the response C(kT) for the system shown in Figure 1.B [13 marks]

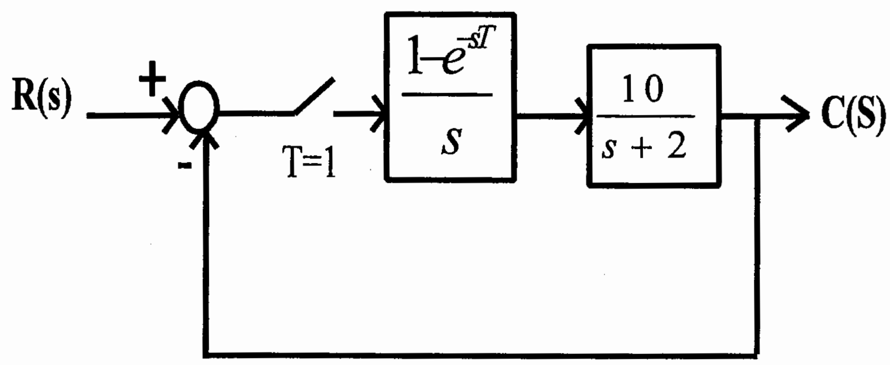


Figure 1.B

Question 2

For a control system having a transfer function $\frac{Y(s)}{R(s)} = \frac{s^2 + 2s + 1}{3s^3 + 5s^2 + 5s + 1}$

a) Use the phase variable format to obtain a signal flow graph [8 marks]

b) Obtain the system matrix, input matrix and output matrix for the state space representation [8 marks]

c) Determine the stability . [4 marks]

Question 3

Given a unity feedback system with the forward transfer

$$G(s)=\frac{100(s+1)^2}{s(s+10)}$$

draws the Bode diagrams (Magnitude and Phase). [20 marks]

Question 4

Plot the root locus of a unity feedback system having $G(s) = \dfrac{k_1(0.1s+1)}{(2s+1)(0.5s+1)(0.25s+1)}$

and a breakaway point at s = - 1.18. [20 marks]

Question 5

An ES 151 Educational servo system is to be used in a position control experiment to investigate the open loop time constant. The connection are as shown in Figure 5.
For this system obtain

a) the transfer function $\frac{\theta_o}{\theta_i}$ (5 marks)

b) the differential equation describing this system (3 marks)

c) the equations natural underdamped frequency and damping ratio and the values of the natural frequency and damping ration when k = 80 revolutions/minute/volt, k_{pot} = 0.1volt/degree) and τ = 0.25 seconds. . (12 marks)

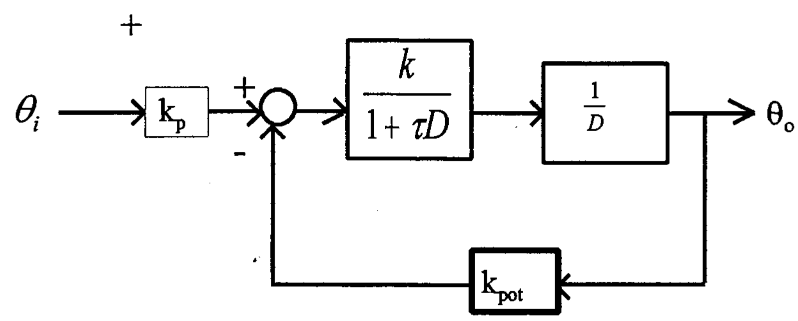
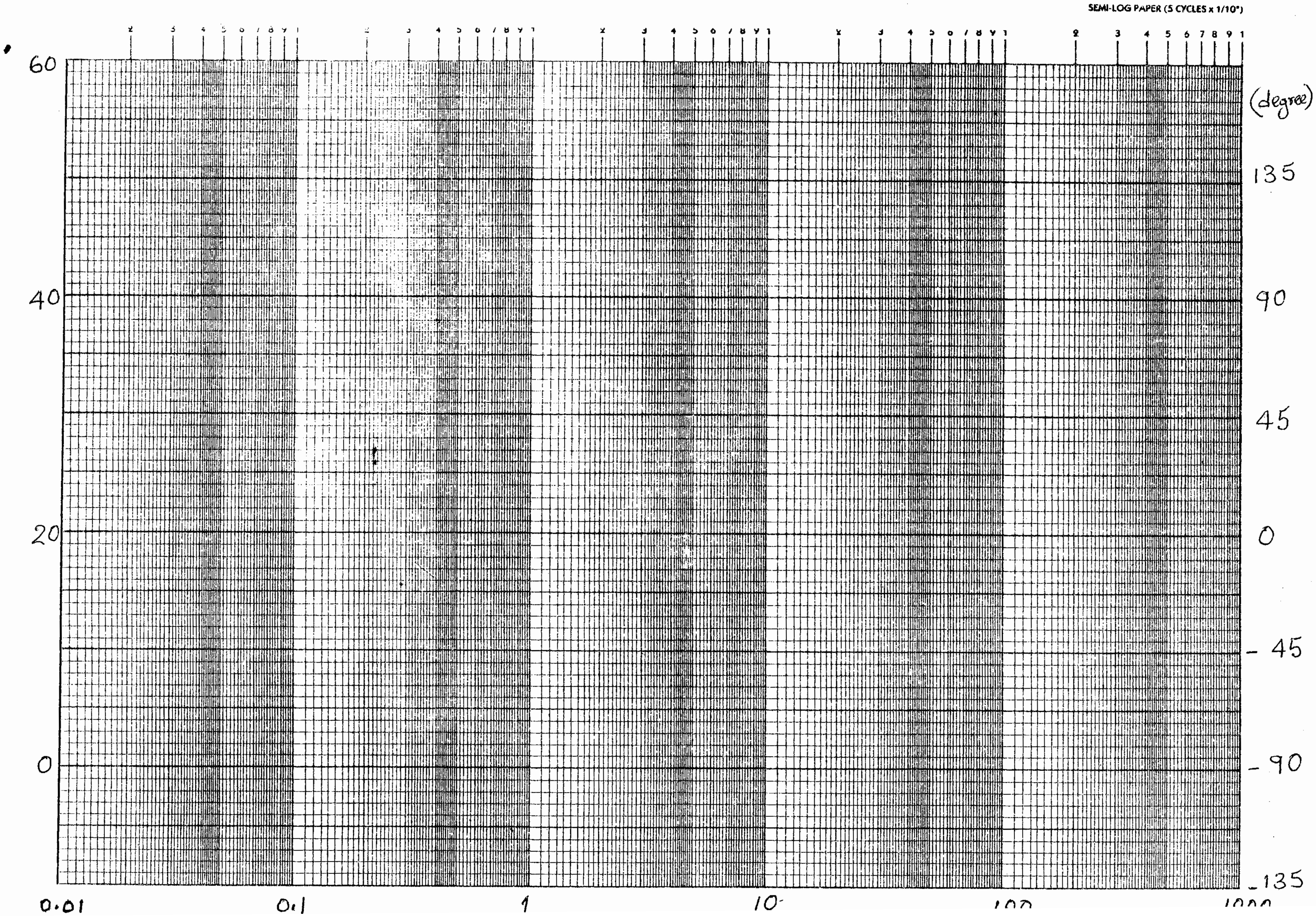


Figure 5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- 45
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