# UNIVERSITY OF SWAZILAND FACULTY OF SCIENCE DEPARTMENT OF ELECTRONIC ENGINEERING

### **MAIN EXAMINATION MAY 2008**

TITLE OF PAPER: LINEAR SYSTEMS

COURSE CODE: E352

TIME ALLOWED: THREE HOURS

### **INSTRUCTIONS:**

- 1. Answer question one and any other three questions.
- 2. Question one carries 40 marks.
- 3. Questions 2, 3, 4, and 5 carry 20 marks each.
- 4. Marks for different sections are shown in the right-hand margin

This paper has 6 pages including this page.

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(a) Is the system shown in Figure 1A linear? Show how you derive your conclusion.

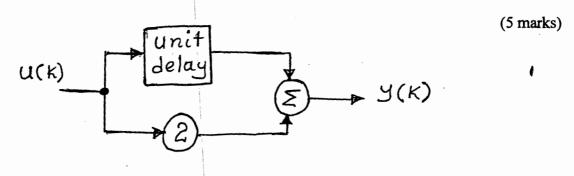


Figure 1A

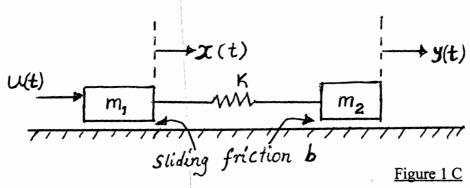
(b) The output, y, and input, x, of a device are related by

$$y = x + 1.4x^3$$

obtain a linearized model for two operating points  $x_0=1$  and  $x_0=2$ .

(6 marks)

(c) For the mechanical system shown in Figure 1C determine the transfer function  $\frac{Y(s)}{U(s)}$ (15 marks)



(d) The transfer function of a linear system is  $\frac{Y(s)}{R(s)} = \frac{10(s+6)}{s^2 + 7s + 12}$ 

If r(t) is a unit step input, determine

- (i) the response y(t)
- (ii) the rise time, and
- (iii) the settling time.

(14 marks)

Question 2
Use Mason's gain rule to find the transfer function of the system shown in Figure 2

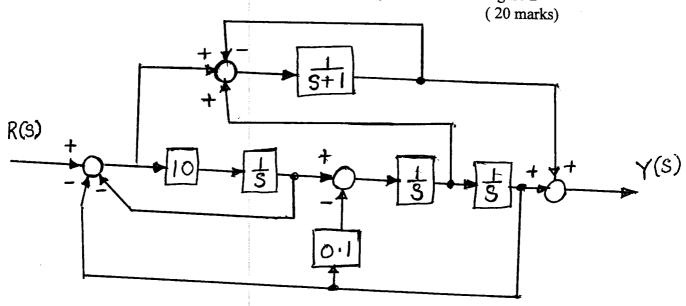


Figure 2

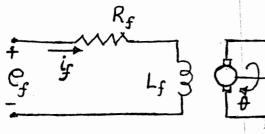
The linear system shown in Figure 1 can be represented by equations of the form

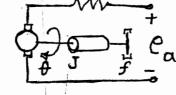
$$\dot{x} = Ax + Bu$$

$$y = Cx$$

Determine the matrices A, B, and  $C_4$ 

[ 20 marks]





### Figure 1

where  $T = K_2 i_f$  with  $K_2$  being a constant

 $\theta$  = angular displacement of the motor shaft

 $e_f$  = field voltage

 $i_a$  = armature current and it is a constant

Note: The state variables are defined as follows  $i_f = x_1$  and  $\omega = x_2$ .

The field voltage is the input and the angular velocity of the motor shaft  $\omega = d\theta /dt$  is the output.

Obtain state variable equations, using phase variable format only, for a linear system whose

transfer function is 
$$\frac{Y(s)}{R(s)} = \frac{8s^3 + 10s + 210}{s^4 + 19s^3 + 7s^2 + 30s}.$$

For pendulum oscillator Figure 5, the torque on the mass is given by  $T = MgLSin\theta$ 

(a) determine the linear approximation using the equilibrium condition for the mass

[ 10 marks ]

(b) determine the range of  $\theta$  for which the approximation is within 1% of the actual nonlinear pendulum. [10 marks]

Length L

Mass M

Mass M g is the gravity constant.

Figure 5