UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION 2007/2008

TITLE OF PAPER	**:**	**ORDINARY DIFFERENTIAL EQUATIONS, PROBABILITY AND STATISTICS**
COURSE NUMBER	**:**	**E371**
TIME ALLOWED	**:**	**THREE HOURS**
INSTRUCTIONS	**:**	**ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS.** **MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.** **STUDENTS ARE PERMITTED TO USE MAPLE TO ANSWER THE QUESTIONS.**

THIS PAPER HAS SIX PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

E371 Ordinary Differential Equations, Probability and Statistics

Question one

Given the following non-homogeneous ordinary differential equation as

$$\frac{d^2\,y(t)}{dt^2} + 2\,\frac{d\,y(t)}{dt} + 9\,y(t) = 7\,\sin(t) - 3\,\sin(5t)$$

(a) find its particular solution $y_p(t)$, **(9 marks)**

(b) find the general solution $y_h(t)$ for the homogeneous part of the given differential equation, **(4 marks)**

(c) find the general solution $y_g(t)$ for the above given non-homogeneous differential equation, **(2 marks)**

(d) if given initial conditions as $y(0) = 4$ *and* $\left.\frac{d\,y(t)}{dt}\right|_{t=0} = -2$,

find its specific solution of $y(t)$ and plot it for $t = 0$ *to* 5 . **(10 marks)**

Question two

(a) If the inverse laplace transform of $F(s)$ and $G(s)$ are $5\,e^{-3t}$ and $7\cos(2t)$ respectively, utilize the convolution theorem to find the inverse laplace transform of $F(s)\times G(s)$. **(6 marks)**

(b) Given the following differential equation as

$$\frac{d^2\,y(t)}{dt^2} + 3\,\frac{d\,y(t)}{dt} + 7\,y(t) = f(t)$$

where $$f(t) = \begin{cases} 0 & if \quad t \le 0 \\ 2t & if \quad 0 \le t \le 1 \\ 2 & if \quad 1 \le t \le 5 \\ -t+7 & if \quad 5 \le t \le 7 \\ 0 & if \quad t \ge 7 \end{cases}$$

(i) find the laplace transform of the above given $f(t)$, **(6 marks)**

(ii) if given the initial conditions as $y(0) = 9$ *and* $\left.\frac{d\,y(t)}{dt}\right|_{t=0} = 5$, find the laplace transform of $y(t)$, **(8 marks)**

(iii) find the specific solution of $y(t)$ through inverse laplace transform of your answer in (b) (ii). Plot this $y(t)$ for $t = 0$ *to* 10. **(5 marks)**

Question three

Given the following differential equation as

$$\frac{d^2\,y(x)}{d\,x^2} + 3\,\frac{d\,y(t)}{d\,t} + 8\,y(t) = 0$$

set $y(x) = \sum_{n=0}^{\infty} a_n\, x^{n+s}$ *and* $a_0 \neq 0$, utilize the power series method and

(a) write down the indicial equations and find the values of s and possibly the value of a_1 (if a_1 is in terms of a_0 and s , then find the possible values of a_1 by setting $a_0 = 1$) **(7 marks)**

(b) write down the recurrence relation. Set $a_0 = 1$ and use the recurrence relation to find the values of a_n ($n = 2$ *to* 10) for each value of s found in (a). Write down two independent series solutions truncated up to a_{10} term.

(8 marks)

(c) (i) write the general solution for the above given differential equation,

(2 marks)

(ii) if given initial conditions as $y(0) = 6$ *and* $\left.\frac{d\,y(x)}{d\,x}\right|_{x=0} = 0$, find the specific solution and plot it for $x = 0$ *to* 2 . **(8 marks)**

Question four

(a) Given a set of data of x as [10 , 3 , 7 , 13 , 8 , 9 , 11 , 6], find the values of its mean value, variance and standard deviation. **(6 marks)**

(b) Given a probability function $f(x) = \frac{x^2}{14}$ *and* $x = 1, 2, 3$, find its distributive probability function G(x) , i.e., find the values of G(1) , G(2) and G(3). Plot a bar chart of G(x) for $x = 0$ *to* 3 . **(8 marks)**

(c) (i) Use the random number generator to generate an ensemble of 25 data of x with its values ranging from 30 to 68 , **(3 marks)**

(ii) using the interval of 10 starting with 29.5 , i.e., (29.5 to 39.5) , (39.5 to 49.5) , ……. , (59.5 to 69.5) , plot its histogram. **(8 marks)**

Question five

(a) Six identical coins are tossed simultaneously and each coin has its probability of "head up" in a toss as 0.49 ,

(i) find the probability of precisely 2 heads up , **(4 marks)**

(ii) find the probability of at least 2 heads up. **(6 marks)**

(b) If the defect rate for a skew production is 1 out of 80 and one picks up a handful of 200 skews, find the probability of no more than 3 defected skews being picked up. **(6 marks)**

(c) Given an ensemble of data which follows a normal distribution with its mean value of 8 and a standard deviation of 1.2 , find the confidence range of these data if the confidence level is set as 90% . **(9 marks)**