UNIVERSITY OF SWAZILAND

MAIN EXAMINATION, SECOND SEMESTER 2008

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

TITLE OF PAPER:	DIGITAL SIGNAL PROCESSING
COURSE CODE:	E420
TIME ALLOWED:	THREE HOURS

INSTRUCTIONS:

1. **Answer any <u>FOUR</u> (4) of the following five questions.**
2. **Each question carries 25 marks.**
3. **Tables of selected window functions and selected Z-transform pairs are attached.**

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THIS PAPER CONTAINS EIGHT (8) PAGES INCLUDING THIS PAGE

QUESTION ONE ***(25 marks)***

(a) (i) Find the closed form inverse z-transform of

$$X(z) = \frac{1+2z^{-1}+z^{-2}}{1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}}, \qquad |z|>1$$

(8 marks)

(ii) From your inverse z-transform evaluate the first three samples of the resulting sequence., *(3 marks)*

(iii) Does the sequence converge? If so to what value? *(3 marks)*

(iv) Check you answer to (ii) using the long division method. *(4 marks)*

(b) A sequence $x(n) = \left(\frac{1}{2}\right)^n u(n)$ is passed through a system whose impulse response is *u(n)*, where *u(n)* is the unit step function. Use a z-transform technique or otherwise to find a closed form expression for the output sequence. *(7 marks)*

QUESTION TWO (25 *marks*)

(a) A filter has an impulse response sequence [1, 2, 3, 2, 1].

(i) Obtain its phase response function and magnitude response function. *(5 marks)*

(ii) At which digital (radian) frequency is its magnitude zero? And what is the phase response at this frequency? *(4 marks)*

(iii) A signal of 3 kHz sampled at 16 kHz is passed through the filter. What will be the change in amplitude (in dB), change in phase (in degrees) and group delay (in sec) for the signal coming out of the filter? *(6 marks)*

(b) A uniform 8-level quantizer is designed for an input signal with a dynamic range of ± 1.6 V

(i) Calculate the variance of the quantization error vector foran input signal of $x(n)$ = [-1.35, -0.85, 0.15, 1.15] V. *(5 marks)*

(ii) Calculate the variance of the quantization error for the same input signal if the quantizer is preceded by a $\mu = 255$ compander (compressor/expander). The compression μ-Law is given by $F_{\mu}(n) = \dfrac{\ln\left(1+\mu\left|F(n)\right|\right)}{\ln\left(1+\mu\right)}\operatorname{sgn}\left[F(n)\right]$.

(5 marks)

QUESTION THREE **(25 *marks)***

(a) (i) Draw a clearly labeled signal flow diagram of the Radix-2 decimation-in-time algorithm for obtaining the DFT of a 8-point sequence. *(7 marks)*

(ii) How many complex multiplications and complex additions are involved? *(2 marks)*

(b) (i) Using a signal flow diagram of a Radix-2 decimation-in-time algorithm for obtaining the DFT of a 4-point sequence, find the DFT of the sequence $x(n)$ = [3, 1, 4, -2], specifying all the intermediate outputs of your flow diagram. *(13 marks)*

(ii) What simple checks can you perform to check if your answer is correct? *(3 marks)*

QUESTION FOUR **(25 *marks*)**

A low pass linear-phase audio FIR filter with the following specification is to be designed:

Filter length, N =11

Cut off frequency = 3 kHz

Sampling rate = 20 kHz

Window to be applied = Hamming Window

(a) Calculate the coefficients of the filter. Your calculations should be accurate to 4 decimal places. *(13 marks)*

(b) What is the transition bandwidth of your filter? *(2 marks)*

(c) Calculate the magnitude and phase of the filter at 2 kHz. *(10 marks)*

QUESTION FIVE **(25 *marks)***

A signal is to be digitally filtered using a digital filter based on an analogue second-order Butterworth filter with a normalized transfer function $H(s) = \frac{1}{s^2 + \sqrt{2}\,s + 1}$.

The signal sampling frequency is 2 kHz and the filter cutoff frequency is 600 Hz.

(a) Using the Bilinear Transformation, obtain the transfer function of the digital filter.

(15 marks)

(b) Draw a realization structure of the filter using a minimum number of delay elements.

(3 marks)

(c) Briefly discuss some of the problems that could be encountered when trying to implement this filter in hardware. *(7 marks)*

TABLE OF Z-TRANSFORMS OF SOME COMMON SEQUENCES

Discrete-time sequence $x(n),\ n \geq 0$	**Z-transform** $H(z)$
$k\delta(n)$	k
k	$\frac{kz}{z-1}$
$ke^{-\alpha n}$	$\frac{kz}{z-e^{-\alpha}}$
$k\alpha^n$	$\frac{kz}{z-\alpha}$
kn	$\frac{kz}{(z-1)^2}$
kn^2	$\frac{kz(z+1)}{(z-1)^3}$
$kn\alpha^n$	$\frac{k\alpha z}{(z-\alpha)^2}$

SUMMARY OF IMPORTANT FEATURES OF SELECTED WINDOW FUNCTIONS

Name of Widow	Normalized Transition Width	Passband Ripple (dB)	Main lobe relative to Sidelobe (dB)	Max. Stopband attenuation (dB)	6 dB normalized bandwidth (bins)	Window Function $\omega(n)$, $\lvert n\rvert \le (N-1)/2$
Rectangular	0.9/N	0.7416	13	21	1.21	1
Hanning	3.1/N	0.0546	31	44	2.00	$0.5+0.5\cos\left(\frac{2\pi n}{N}\right)$
Hamming	3.3/N	0.0194	41	53	1.81	$0.54+0.46\cos\left(\frac{2\pi n}{N}\right)$
Blackman	5.5/N	0.0017	57	74	2.35	$0.42+0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{4\pi n}{N-1}\right)$
Kaiser	2.93/N (β=4.54)	0.0274		50		$\frac{I_o\left(\beta\left\{1-\left[\frac{2n}{N-1}\right]^2\right\}^{\frac{1}{2}}\right)}{I_o(\beta)}$
	4.32/N (β=6.76	0.00275		70		
	5.71/N (β=8.96)	0.000275		90		

$$\text{Bin width} = \frac{f_s}{N} \text{ Hz}$$