University of Swaziland

Faculty of Science

Department of Electrical and Electronic Engineering

Examinations: May 2009

Title of Paper: **Digital Signal Processing**

Course Code: **E420**

Duration: **Three hours**

Instructions:
1. Answer any **FOUR QUESTIONS.**
2. Each question carries 25 marks.
3. Tables of Z-transforms pairs and window functions are attached.

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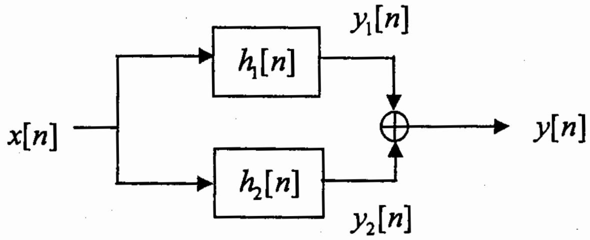
Question 1

(a) A system is expressed by a causal, second-order difference equation:

$$y[n] = -0.3y[n-1] + 0.18y[n-2] + 3x[n-1] + 0.9x[n-2]$$

The above system can be implemented in parallel and is shown in below.



The upper first-order system has impulse response $h_1[n]$ and is given by the following difference equation:

$$y_1[n] = a_1\, y_1[n-1] + b_1 x[n-1]$$

The lower first-order system has impulse response $h_2[n]$ and is given by the following difference equation:

$$y_2[n] = a_2 y_1[n-1] + b_2 x[n-1]$$

Determine the numerical values of a_i and b_i, $i = 1,2$-four values total.

[11 marks]

(b) A transfer function of an FIR filter proposed for the recovery of vertical details in the structure employed for the separation of the luminance and chrominance components in composite video is given by

$$H_{BS}(z) = \tfrac{1}{16}(1+z^{-2})^2(-1+6z^{-2}-z^{-4})$$

Realize the above transfer function with two sections in cascade using any of the Direct form structures.

[4 marks]

(c) The transfer function of a certain digital filter is given by

$$\mathbf{H(z)} = \frac{0.1432(1+3\mathbf{z}^{-1}+3\mathbf{z}^{-2}+\mathbf{z}^{-3})}{1-0.180\mathbf{z}^{-1}+0.3419\mathbf{z}^{-2}-0.0165\mathbf{z}^{-3}}$$

$$= \frac{1.2916-0.08407\mathbf{z}^{-1}}{1-0.131\mathbf{z}^{-1}+0.3355\mathbf{z}^{-2}} + \frac{10.1764}{1-0.049\mathbf{z}^{-1}} - 8.7107$$

Develop the structures for

(i) the canonic direct form realization, and
(ii) parallel form realization.

[10 marks]

Question 2

(a) The Z-transform $X(z)$ of a sequence $x[n]$ is defined by $X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}$. Find the inverse Z-transform of the following

$$H(z)=\frac{z}{(z-0.75)(z+0.5)}.$$

$H(z)$ has three ROCs. Find the discrete-time sequence $h[n]$ to each ROC using partial fraction expansion.

[12 marks]

(b) Two digital signals are given as $x(n)=\delta(n)-\delta(n-2)+\delta(n-3)$, and $h(n)=2\delta(n-1)+\delta(n-2)-\delta(n-3)$.
Determine

(i) the linear convolution of x(n) and h(n),
(ii) their Z-transforms X(z) and H(z),
(iii) the product Y(z) = H(z)X(z), and
(iv) the inverse Z-transform of Y(z).

[13 marks]

Question 3

(a) The transfer function of a discrete-time system has poles at z = 0.5, z = 0.1+j0.2, z = 0.1-j0.2 and zeros at z = -1 and z = +1.
For the system

(i) sketch the pole-zero diagram,
(ii) derive the transfer function from the pole-zero diagram,
(iii) develop the difference equation, and
(iv) draw the realization diagram.

[15 marks]

(b) A linear-phase FIR lowpass filter is to be designed using the windowed Fourier series method. The filter is to be of the lowest order with the following specifications: passband edge at 0.3π and the stopband edge at 0.5π with a minimum stopband attenuation of 40 dB.

(i) Which window function is appropriate for the design?
(ii) Determine the order of the filter.
(iii) Compute the first three coefficients of the filter.

[10 marks]

Question 4

(a) Five samples of a 9-point DFT X(k) of a real length-9 sequence are given as $X(0)=11$, $X(2)=1.2-j2.3$, $X(3)=-7.2-j4.1$, $X(5)=-3.1+j8.2$, *and* $X(8)=4.5+j1.6$.
Determine the other missing four samples.

[6 marks]

(b) Compute the circular convolution of the following two sequences:

$$g(n) = \{\underset{\uparrow}{1} \quad 2 \quad 3 \quad 4\}, \text{ and } \quad h(n) = \{\underset{\uparrow}{4} \quad 3 \quad 2 \quad 1\}.$$

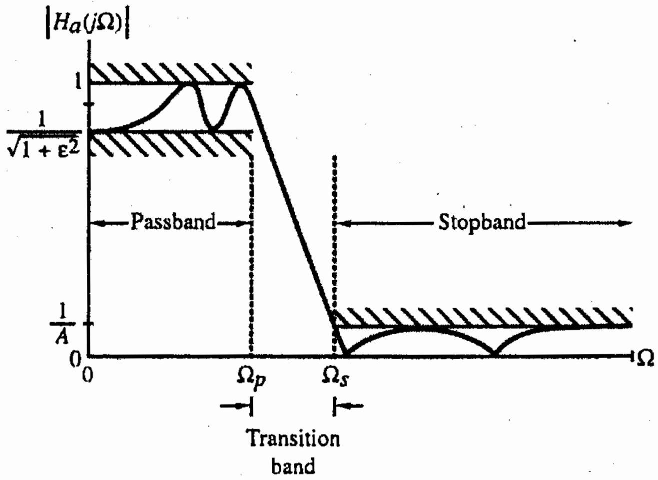
[6 marks]

(c) A data sequence is given as v(n) = {1 0 0 1}
(i) Calculate the DFT V(k) of v(n) (work from definition of DFT).
(ii) Calculate V(k) using the decimation-in-time FFT algorithm

[13 marks]

Question 5

(a) In analog lowpass filter design, magnitude specification may alternatively be given in a normalized form as indicated below.



Where $1/\sqrt{1+\varepsilon^2}$ is the maximum passband deviation. $1/A$ is the maximum stopband magnitude. Ω_p is the passband edge frequency. Ω_s is the stopband edge frequency.
The magnitude square response of an N-th order analog lowpass Butterworth filter is given by

$$|H_a(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}},$$

where Ω_c is the 3-dB cutoff frequency.

Derive the lowest order N of a lowpass Butterworth filter to meet the above specification in terms of A, ε, Ω_p, and Ω_s.

[10 marks]

(b) Explain the difference between a Discrete-Time Fourier Transform (DTFT) and a Discrete Fourier Transform (DFT).

[5 marks]

(c) Determine the DTFT of the sequence $x(n) = -\alpha^n u(-n-1)$, for $|\alpha|>1$. where u (n) is a unit step sequence.

[10 marks]

Question 6

(a) A digital IIR filter is designed from an analog Butterworth filter via the bilinear Transformation method through the transformation $s = \frac{z-1}{z+1}$. A canonical analog Butterworth filter with a 3-dB cutoff frequency of $\Omega = 1$ rads/sec has three poles at the following locations in the s-plane: $s_1 = e^{j2\pi/3}$, $s_2 = -1$, and $s_3 = e^{-j2\pi/3}$.

The analog Butterworth filter has a gain of one (unity) at DC $(\omega = 0)$.

(i) Obtain the transfer function H(z) of the digital filter.

(ii) Determine the locations of the three poles of the resulting digital IIR filter in z-plane.

(iii) Plot a pole-zero diagram for the resulting digital filter in z-plane. Be sure that both zeros and poles are shown in the pole-zero diagram.

(iv) Is the resulting digital filter stable or unstable? Explain your answer.

[16 marks]

(b) In the design of FIR filters:

(i) Why is a rectangular window function not recommended in the design of FIR filters using the window method?

(ii) Normally a window function with a small main lobe width is desirable if the intention, in the design of FIR filters, is to approximate the ideal magnitude response as much as possible. What is the problem with this requirement?

(iii) What is unique about the Kaiser window function compared to other window functions?

[9 marks]

TABLE OF Z-TRANSFORMS OF SOME COMMON SEQUENCES

Discrete-time sequence $x(n),\ n \geq 0$	**Z-transform** $H(z)$
$k\delta(n)$	k
k	$\frac{kz}{z-1}$
$ke^{-\alpha n}$	$\frac{kz}{z-e^{-\alpha}}$
$k\alpha^n$	$\frac{kz}{z-\alpha}$
kn	$\frac{kz}{(z-1)^2}$
kn^2	$\frac{kz(z+1)}{(z-1)^3}$
$kn\alpha^n$	$\frac{k\alpha z}{(z-\alpha)^2}$

SUMMARY OF IMPORTANT FEATURES OF SELECTED WINDOW FUNCTIONS

Name of Widow	Normalized Transition Width	Passband Ripple (dB)	Main lobe relative to Sidelobe (dB)	Max. Stopband attenuation (dB)	6 dB normalized bandwidth (bins)	Window Function $\omega(n)$, $\lvert n\rvert \le (N-1)/2$
Rectangular	0.9/N	0.7416	13	21	1.21	1
Hanning	3.1/N	0.0546	31	44	2.00	$0.5+0.5\cos\left(\frac{2\pi n}{N}\right)$
Hamming	3.3/N	0.0194	41	53	1.81	$0.54+0.46\cos\left(\frac{2\pi n}{N}\right)$
Blackman	5.5/N	0.0017	57	74	2.35	$0.42+0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{4\pi n}{N-1}\right)$
Kaiser	2.93/N (β=4.54)	0.0274		50		$\dfrac{I_o\left(\beta\left\{1-\left[\frac{2n}{N-1}\right]^2\right\}^{\frac{1}{2}}\right)}{I_o(\beta)}$
	4.32/N (β=6.76	0.00275		70		
	5.71/N (β=8.96)	0.000275		90		

$$\text{Bin width} = \frac{f_s}{N}\ \text{Hz}$$