UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

MAIN EXAMINATION 2009/2010

TITLE OF PAPER	:	ORDINARY DIFFERENTIAL EQUATIONS, PROBABILITY AND STATISTICS
COURSE NUMBER	:	E371
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS. MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN. STUDENTS ARE PERMITTED TO USE MAPLE TO ANSWER THE QUESTIONS.

THIS PAPER HAS SIX PAGES, INCLUDING THIS PAGE.

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E371 Ordinary Differential Equations, Probability and Statistics

Question one

(a) Given the following 2^{nd} order homogeneous differential equation as

$$\frac{d^2\, y(x)}{d\,x^2} + 2\,\frac{d\,y(x)}{d\,x} + 5\,y(x) = 0$$

(i) set $y(x) = e^{a\,x}$ and find the appropriate values of a . Write down its general solution. **(4 marks)**

(ii) if the initial conditions are given as $y(0) = -1$ & $\left.\frac{d\,y(x)}{d\,x}\right|_{x=0} = 2$, then find its specific solution and plot it for x = 0 to 5 . **(5 marks)**

(b) Given the following non-homogeneous differential equation as

$$\frac{d^2\, y(t)}{d\,t^2} + 4\,\frac{d\,y(t)}{d\,t} + 10\,y(t) = 5\,e^{-4\,t} - 2\,t$$

(i) find its particular solution $y_p(t)$, **(6 marks)**

(ii) find the general solution to the homogeneous part of the given equation $y_h(t)$ and then write down the general solution to the given non-homogeneous differential equation $y_g(t)$ **(5 marks)**

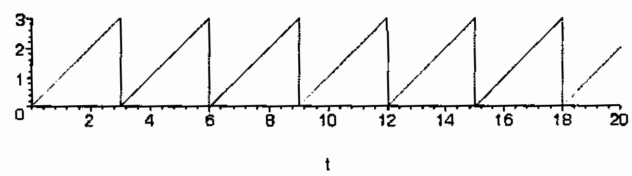
(iii) if the initial conditions are given as $y(0) = 4$ & $\left.\frac{d\,y(t)}{d\,t}\right|_{t=0} = -1$, then find its specific solution and plot it for x = 0 to 5 . **(5 marks)**

Question two

Given the following differential equation

$$\frac{d^2\,y(t)}{dt^2} + 6\,\frac{d\,y(t)}{dt} + 13\,y(t) = h(t)$$

with the initial conditions of $y(0) = -3$ & $\left.\frac{d\,y(t)}{dt}\right|_{t=0} = 5$, and h(t) is the following jigsaw periodic function with the period of 3 as below



(a) find the Laplace transform of h(t), i.e., H(s),

(Hint : $H(s) = \dfrac{R(s)}{1 - e^{-3s}}$ where R(s) is the laplace transform of r(t)

where $r(t) = \begin{cases} t & if \quad 0 \le t \le 3 \\ 0 & if \quad t > 3 \end{cases}$) **(6 marks)**

(b) find the Laplace transform of y(t), i.e., Y(s), and show that Y(s) = F(s) + G(s) H(s) where

$F(s) = -\dfrac{3\,s + 13}{s^2 + 6\,s + 13}$ & $G(s) = \dfrac{1}{s^2 + 6\,s + 13}$ **(8 marks)**

(c) find the inverse Laplace transforms of $F(s)$, $G(s)$ & $H(s)$ and name them $f(t)$, $g(t)$ & $h(t)$ respectively, **(3 marks)**

(d) thus the inverse Laplace transform of Y(s), i.e., y(t), can be found by $y(t) = f(t) + (convolution\ of\ \ g(t)\ \&\ h(t))$. Find y(t) and then plot it for t = 0 to 20 and make a brief comment. **(8 marks)**

Question three

Given the following Bessel's equation as

$$x^2 \frac{d^2\, y(x)}{d\,x^2} + x\frac{d\,y(x)}{d\,x} + \left(x^2 - 4\right)y(x) = 0 \quad ,$$

(a) (i) use *dsolve* command to find its general solution , **(2 marks)**

(ii) use *series* command to express BesselJ(2,x) & BesselY(2,x) into their power series up to x^{11} (i.e., would appear with $0\left(x^{12}\right)$). Then convert them into polynomials. **(4 marks)**

(b) (i) set $y(x) = \sum_{n=0}^{\infty} a_n\, x^{n+s}$ *and* $a_0 \neq 0$, utilize the power series method to find its indicial equations and thus find the values of s & a_1 , **(6 marks)**

(ii) for $s = 2$, set $a_0 = 1$, use the recurrence relation to find the values of a_n up to $n = 8$. Show that this polynomial solution is linearly dependent to the independent solutions in (a)(ii). **(9 marks)**

(iii) for $s = -2$, set $a_0 = 1$, use the recurrence relation to find the values of a_n up to $n = 8$. Show that this polynomial solution can not be found directly by power series method. **(4 marks)**

Question four

(a) Given a discrete probability function $f(1) = 0.05$, $f(2) = 0.15$, $f(3) = 0.27$, $f(4) = 0.21$, $f(5) = 0.18$ and $f(6) = 0.14$, find its cumulative probability distribution function G(x) , i.e., find the values of G(1) , G(2) , G(3) , G(4) , G(5) and G(6). . Plot a bar chart of G(x) for $x = 0$ *to* 6 . **(7 marks)**

(b) If on the average there are 3 defects out of every 400 products in a factory,

(i) write down the probability of finding a defected and non-defected product, denote them as p and q respectively , **(2 marks)**

(ii) randomly picking up 300 of such products , find the probability of finding at least 2 but not more than 5 defected products among them. Find the answer using both Binomial and Poisson's cumulative distribution functions and compare them to find their percentage difference. **(10 marks)**

(c) Use the random number generator to generate an ensemble of 30 data of x with its values ranging from 5 to 200 , then find its mean value , variance and standard deviation . **(6 marks)**

Question five

Given the following data of x as
$X = [\,4, 3.99, 4, 4.01, 4, 3.99, 4, 4.01, 4, 3.99, 4, 4.01, 4, 3.99, 4, 4.01, 4, 3.98, 4, 4.02, 4, 3.98, 4, 4.02, 4, 3.97, 4, 4.03\,]$
which represent on the average the distribution of 4 inch nail products

(a) plot the histogram of X starting from $x = 3.965$ with the increment of 0.01 until it reaches $x = 4.035$ **(7 marks)**

(b) find the mean value and standard deviation of the given X, **(2 marks)**

(c) (i) if the confidence level is 0.999, what would be the corresponding confidence range for the given X, **(6 marks)**

(ii) if the confidence range is $3.98 < x < 4.02$, what would be the corresponding confidence level for the given X, **(4 marks)**

(iii) if customer's requested confidence level and confidence range are 0.9999 and $3.999 < x < 4.001$ respectively, what should be the value of the standard deviation in order to meet the customer's demand ?

(6 marks)