UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

MAIN EXAMINATION 2010/2011

TITLE OF PAPER	:	NUMERICAL ANALYSIS
COURSE NUMBER	:	E472
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER <u>ANY FOUR</u> OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES <u>25</u> MARKS. MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN. STUDENTS ARE PERMITTED TO USE MAPLE TO ANSWER THE QUESTIONS.

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<u>**E472 Numerical Analysis**</u>

Question one

Given a polynomial function of x as

$f(x) = x^4 - 6x^3 - 3x^2 + 20x + 5$

(a) plot the given f(x) for $x = -2$ *to* 6. Use *fsolve* command to find its real root in the interval of $x = 0$ *to* 8. **(4 marks)**

(b) Transform $f(x) = 0$ into the form $x = g(x)$ and compute a solution of $f(x) = 0$ by fixed-point iteration method, starting from $x_0 = 4$ and doing 5 iterations. Compute the percentage difference of the root found here with the one obtained in (a). **(7 marks)**

(c) Compute a solution of $f(x) = 0$ by Newton's method, starting from $x_0 = 4$ and doing 5 iterations. Compute the percentage difference of the root found here with the one obtained in (a). **(7 marks)**

(d) Compute a solution of $f(x) = 0$ by Secant method, starting from $x_0 = 4$ and $x_1 = 4.1$ and doing 5 iterations. Compute the percentage difference of the root found here with the one obtained in (a). **(7 marks)**

Question two

Given the following data of $f(x)$ as :

$f(1)=8$, $f(2)=2$, $f(3)=-6$, $f(4)=4$, $f(5)=-7$ & $f(6)=3$

(a) use Newton's forward divided method to find its 5^{th} order Lagrange polynomial extrapolation , i.e., $P_5(x)$, of the given data of $f(x)$. Then plot $P_5(x)$ for $x=0$ *to* 6 . **(15 marks)**

(b) set $f(x)=k_1\,x^2+k_2\,x+k_3$ and use the least square error fitting to find the appropriate values of k_1 , k_2 & k_3 . Plot this extrapolated $f(x)$ as well as the given data of $f(x)$ for $x=0$ *to* 6 and show them in a single display . **(10 marks)**

Question three

(a) Given $f(1) = 10$, $f'(1) = 1$, $f(3) = -5$, $f(5) = 6$, $f'(5) = -2$, find the cubic spline interpolation of $f(x)$ and plot it for $x = 1$ *to* 5 .

(10 marks)

(b) Given the following definite integral $\int_1^5 (\sin(x))^2 \, dx$,

(i) divide the integration range into **ten** equal intervals and compute the value of the given integral by the trapezoidal rule. **(6 marks)**

(ii) divide the integration range of $(1 \le x \le 5 \;\; \& \;\; 0 \le y \le 1)$ into (10×10) equal mesh intervals and use Monte Carlo method with 1000 picks to compute the approximate value of the given integral. Compare this result with that obtained in (b)(i) to find their percentage difference. **(9 marks)**

Question four

(a) Given the following system of linear equations as

$$\begin{cases} x - 3\,y + 7\,z = 49 \\ 6\,x + 6\,y - z = -\,21 \\ x - 9\,y + 4\,z = 4 \end{cases}$$

(i) use *linsolve* command to find the solutions of x , y *and* z , **(3 marks)**

(ii) apply the Gauss-Seidel iteration (7 steps) to the given system, choosing the appropriate pivoting and starting from $x_0 = 1$, $y_0 = 1$ *and* $z_0 = 1$, and compute the iterated solutions of the system. Compare these values with the solutions obtained in (a)(i) and compute their respective percentage differences.

(8 marks)

(b) Given the differential equation $\dfrac{d^2\,y(x)}{d\,x^2} = x\,\dfrac{d\,y(x)}{d\,x} - 5\,y(x) + 2\,x$ with initial conditions of $y(0) = 4$ & $\left.\dfrac{d\,y(x)}{d\,x}\right|_{x=0} = -\,2$,

(i) use *dsolve* command to find its specific solution of $y(x)$. Also find the value of $y(1)$, **(4 marks)**

(ii) use Euler's method with $h = 0.1$ and do 10 steps to find the approximate value of $y(1)$. Compare it with that obtained in (b)(i) to find their percentage difference . **(10 marks)**

Question five

(a) Given the following function of x and y as :

$f(x,y) = x^2 - 5\,x\,y + 3\,y^2 - 10\,x + 23\,y + 9$,

(i) find the maximum value of f and the position of (x , y) that the maximum happens by solving $\dfrac{\partial f}{\partial x} = 0$ *and* $\dfrac{\partial f}{\partial y} = 0$, **(3 marks)**

(ii) use the method of steepest descent , starting with the points $x_0 = 1$ *and* $y_0 = 1$, do 5 steps to find the approximate maximum value of f and its approximate (x , y) position . Compare these values with those obtained in (a) (i) to find their respective percentage differences. **(8 marks)**

(b) Given the following function of x and y as :

$f(x,y) = 5\,x + 9\,y$ where both x and y are positive variables and are subjected to the following constrains : $-2\,x + y \le 6$ *and* $3\,x + y \le 21$,

(i) plot the constrained region for $x = 0$ *to* 7 , **(4 marks)**

(ii) use the Simplex method to find the localized maximum value of f and the position of (x , y) such that this localized maximum occurs. **(10 marks)**