UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

MAIN EXAMINATION DECEMBER 2010

TITLE OF PAPER: **SIGNALS AND SYSTEMS**

COURSE CODE: **EE331**

TIME ALLOWED: **THREE (3)** HOURS

INSTRUCTIONS:

1. Answer question **one** and any other **three** questions.
2. All question carry 25 marks each.
3. Marks for different sections are shown in the right-hand margin

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This paper contains three (4) pages including this page

QUESTION 1

a) Sketch the following signals :

(i) $x_1(t) = r(t) - r(t-2) - 4u(t-5) \quad t \succ 0$ (5 marks)

(ii) $x_2(t) = 2.5u(t)u(2-t) \quad t \succ 0$ (2 marks)

b) Obtain the Laplace transform of $x(t) = (1 - e^{-5t})u(t)$ (3 marks)

c) Obtain the inverse Laplace transform of $Y(s) = \dfrac{-19s^2 + 64}{s(s+8)(s+2)^2}$ (15 marks)

QUESTION 2

Find the trigonometric Fourier series for the full rectified sine wave defined by

$$x(t) = A\sin(\omega_0 t) \quad 0 \le t \le \frac{T_0}{2}$$

with $x(t) = x(t + \frac{T_0}{2})$ (25 marks)

QUESTION 3

a) Consider the circuit shown in Figure 1, with the initial conditions assumed to be zero,

(i) obtain a differential and integral equation, and

(ii) obtain the voltage across the resistor as a function of time for t .> 0. (17 marks)

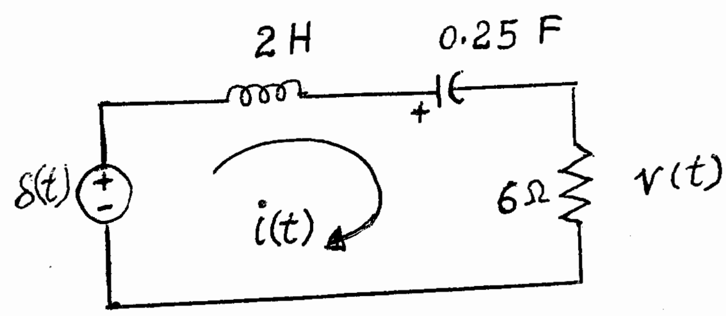

Figure 1

b) Show that the product of two even signals or two odd signals is an even signal and that the product of an even signal and odd signal is an odd signal. (8 marks)

QUESTION 4

Given the following signals

$$x_1(t) = \cos 10\pi t + \sin 5\pi t$$
$$x_2(t) = e^{-2t}u(t)$$

a) Show whether or not each one of the signals is periodic. If period then give the period. (17 marks)

b) Compute the energy and average power for $x_2(t)$. (8 marks)

QUESTION 5

a) i) What are Walsh functions? (2 marks)

ii) Sketch the function $\phi_2^1(t) = \begin{cases} 1, & 0 \le t \prec \frac{1}{4} \; and \; \frac{3}{4} \prec t \le 1 \\ -1, & \frac{1}{4} \prec t \prec \frac{3}{4} \end{cases}$ (5 marks)

b) Given two rectangular pulse signals $x(t) = \prod(\frac{t-5}{2})$ and $h(t) = 2\prod(\frac{t-2}{2})$

i) write x(t) and h(t) in terms of singularity functions (4 marks)

ii) use the convolution integral to determine $y(t) = x(t) * h(t)$. (14 marks)

Partial Tables of z-and s-Transforms

	f(t)	F(s)	F(z)	f(kt)
A)	u(t)	$\frac{1}{s}$	$\frac{z}{z-1}$	u(kT)
B)	t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$	kT
C)	t^n	$\frac{n!}{s^{n+1}}$	$\lim_{a\to 0}(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n$
D)	e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$	e^{-akT}
E)	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n e^{-akT}$
F)	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z\sin\omega T}{z^2-2z\cos\omega T+1}$	$\sin(\omega kT)$
G)	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$\frac{z(z-\cos\omega T)}{z^2-2z\cos\omega T+1}$	$\cos(\omega kT)$
H)	$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{ze^{-aT}\sin\omega T}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$	$e^{-akT}\sin(\omega kT)$
I)	$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{ze^{-aT}\sin\omega T}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$	$e^{-akT}\cos(\omega kT)$
10			$\frac{z}{z+a}$	$a^k\cos(k\pi)$

z-Transform Theorems

	Name	Theorem
1.	Linearity theorem	$z\{af(t)\} = aF(z)$
2.	Linearity theorem	$z\{f_1(t)+f_2(t)\}=F_1(z) + F_2(z)$
3.	Complex differentiation	$z\{e^{-at}f(t)\}=F(e^{aT}z)$
4.	Real translation	$z\{f(t-nT)\}=z^{-n}F(z)$
5.	Complex differentiation	$z\{tf(t)\} = -Tz\frac{dF(z)}{dz}$
6.	Initial value theorem	$f(0) = \lim_{z\to\infty}F(z)$ If the limit exists
7.	Final value theorem	$f(\infty) = \lim_{z\to 1}(1-z^{-1})F(z)$ if the limit exists and the system is stable