University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering

Main Examination 2011

Title of Paper: Signals and Systems II

Course Number: EE332

Time Allowed: 3 hrs

Instructions:

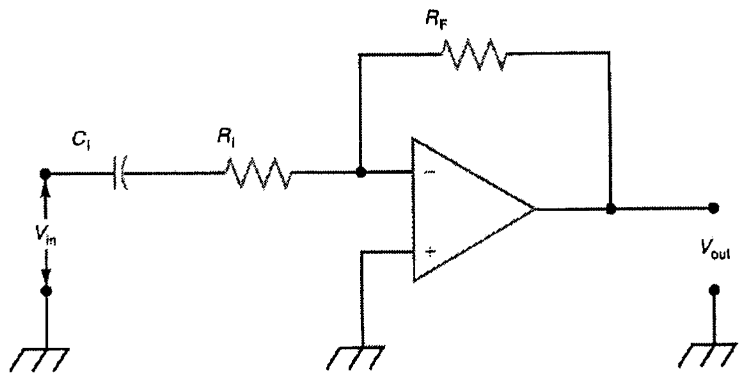
1. Answer any four (4) questions.
2. Each questions carries 25 marks.
3. Useful tables are attached at the end of the question paper

This paper should not be opened until permission has been given by the invigilator.

This paper contains eight (8) pages including this page.

Question 1

Consider the circuit below:



where $R_i = 2k\Omega$, $C_i = 0.1\mu F$, $R_F = 100k\Omega$.

a) What kind of filter is this? (1)

b) Find the transfer function, $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$. (5)

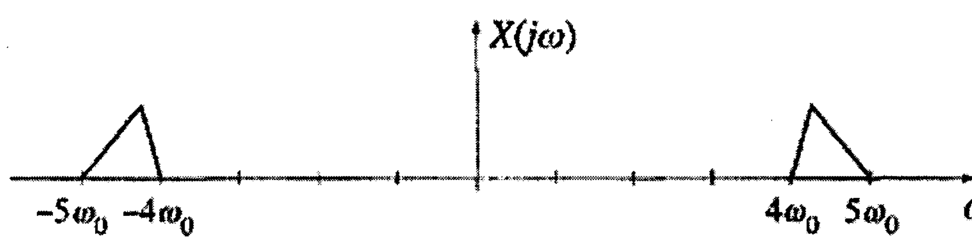
c) Given the transfer function, $H(s) = \frac{-5000}{s+100}$. Find $V_{out}(t)$ when the input, $V_{in}(t)$ is a step of 0.1. (10)

d) State the sampling theorem. (2)

e) If the signal $x(t) = \cos(2000\pi t)$ is sampled at 5000Hz such that $x[n] = x(nT_s)$. What is the fundamental frequency of $x[n]$ in radians/sample? (2)

f) If a channel has a bandwidth of $100kHz$, how many baseband signals each with a bandwidth of $15kHz$ could be multiplexed into a single composite signal and transmitted through the channel? (2)

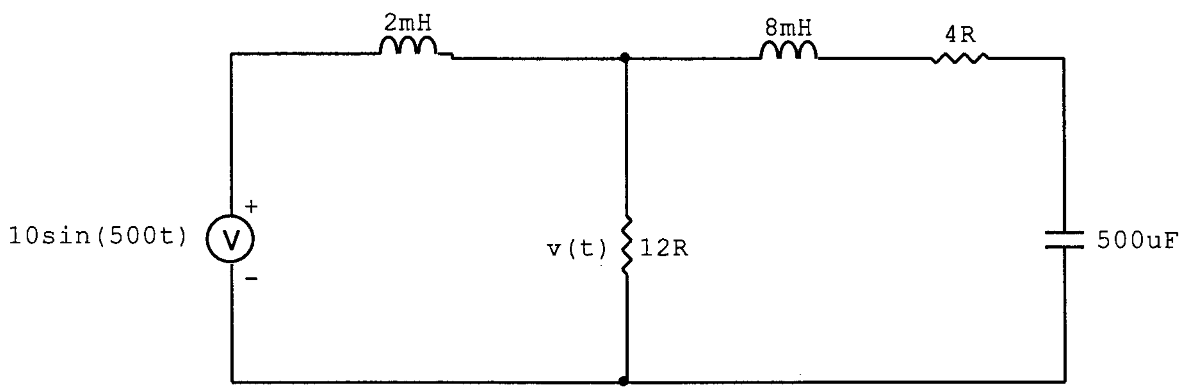
g) A signal with the sketched spectrum is sampled ideally with intervals T.



i) Is the signal a high-pass, a low-pass or a band-pass signal? (1)
ii) Is it complex or real? (1)
iii) Is critical sampling possible? (1)

Question 2

For the following circuit



a) Find the equivalent impedance in phasor form. (5)

b) Determine the steady-state voltage, v(t), using the phasor form. (5)

c) Given this transfer function

$$H(s) = 100\frac{(s+1)}{s^2 + 110s + 1000}.$$

Construct the Bode plot for the magnitude and phase response with respect to the angular frequency, ω. Show all the elements involved in your construction.

(15)

Question 3

a) A second-order system has the following transfer function

$$\frac{B(s)}{A(s)} = \frac{4}{s^2 + 2s + 4}$$

i) Calculate the peak time, t_p. (3)

ii) Find the settling time, t_s. (3)

iii) Calculate the maximum percentage overshoot, $MPOS$. (3)

iv) Is this system critically damped, over-damped or under-damped? (1)

b) Given the following differential equation

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = \frac{dx(t)}{dt}$$

i) Write its characteristic equation. (2)

ii) Determine its general solution. (4)

c) Consider the system whose open-loop transfer function is given by

$$H(s)G(s) = \frac{s+3}{(s+1)(s+2)}$$

Find the steady state error when the desired output is a unit step function. (4)

d) Identify the ROC associated with the z-transform for

$$y[n] = \left(\frac{-1}{2}\right)^n u[n] + 2\left(\frac{1}{4}\right)^n u[n].$$

(5)

Question 4

a) Draw a system block diagram for the following transfer function

$$H(z) = \frac{z}{z^2 + z + 1}$$

showing all necessary working. (4)

b) Find the difference equation for this transfer function

$$H(z) = \frac{(z+1)^2}{\left(z - \frac{1}{2}\right)\left(z + \frac{3}{4}\right)}.$$

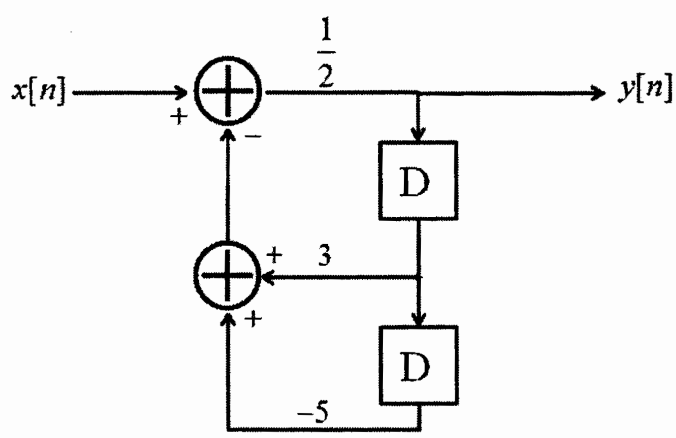
(7)

c) Solve the difference equation when the initial condition is $y(-1) = 1$

$$y[n] - 0.5y[n-1] = 5(0.2)^n u[n].$$

(10)

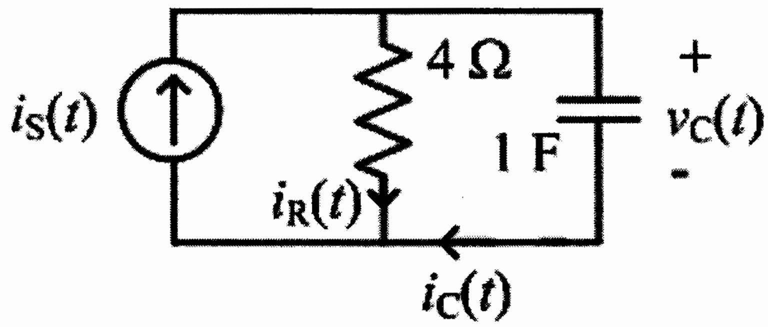
d) Write the difference equation for this block diagram.



(4)

Question 5

a)



For the circuit shown above, use Laplace transforms to find the steady-state response at $v_C(t)$ when $i_S(t) = \cos(t)u(t)$. (10)

b) Construct the Bode plot for the magnitude and phase response with respect to the angular frequency, ω for:

$$H(s) = 10\frac{s+10}{s^2+3s}$$

Show all the elements involved in your construction.

(15)

Table of Z-Transforms

$f[k], k \geq 0$	$F(z) = \mathscr{Z}[f[k]]$
Unit pulses, $\delta[k]$, $\delta[k - k_0]$,	1, z^{-k_0}
Unit step function, $u[k]$	$\dfrac{z}{z-1}$
$a^k \cdot u[k]$	$\dfrac{z}{z-a}$
$ka^k \cdot u[k]$	$\dfrac{az}{(z-a)^2}$
$(k+1)a^k \cdot u[k]$	$\dfrac{z^2}{(z-a)^2}$
$\sin(\alpha k) \cdot u[k]$	$\dfrac{z \sin \alpha}{z^2 - 2z \cos \alpha + 1}$
$\cos(\alpha k) \cdot u[k]$	$\dfrac{z(z - \cos \alpha)}{z^2 - 2z \cos \alpha + 1}$
$a^k f[k] \cdot u[k]$	$F(z/a)$

Table of Laplace Transforms

$F(s)$	$f(t) \quad 0 \le t$	
1. 1	$\delta(t)$	unit impulse at $t = 0$
2. $\frac{1}{s}$	1 or $u(t)$	unit step starting at $t = 0$
3. $\frac{1}{s^2}$	$t \cdot u(t)$ or t	ramp function
4. $\frac{1}{s^n}$	$\frac{1}{(n-1)!} t^{n-1}$	n = positive integer
5. $\frac{1}{s} e^{-as}$	$u(t - a)$	unit step starting at $t = a$
6. $\frac{1}{s}(1-e^{-as})$	$u(t) - u(t-a)$	rectangular pulse
7. $\frac{1}{s+a}$	e^{-at}	exponential decay
8. $\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!} t^{n-1} e^{-at}$	n = positive integer
9. $\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$	

Properties of Z-Transforms

Linearity: $x_1[k] + x_2[k] \Leftrightarrow X_1(z) + X_2(z)$

Real Scaling: $ax[k] \Leftrightarrow aX(z)$

Complex Scaling: $a^k x[k] \Leftrightarrow X(z/a)$

Time Reversal: $x[-k] \Leftrightarrow X(1/z)$

Summation: $\sum_{n=-\infty}^{k} x[n] \Leftrightarrow \frac{zX(z)}{z-1}$

Initial Value: $x[0] = \lim_{z \to \infty} X(z)$

Final Value: $x[\infty] = \lim_{z \to 1} (z-1)X(z)$

Convolution: $x[k] * h[k] \Leftrightarrow X(z)H(z)$

Differencing: $x[k] - x[k-1] \Leftrightarrow \left(1 - z^{-1}\right) X(z)$

Differentiation: $-kx[k] \Leftrightarrow z\frac{d}{dz}X(z)$

Time Shifting: $x[n - n_o] \Leftrightarrow z^{-n_o} X(z), n_o \ge 0$

$$x[n + n_o] \Leftrightarrow z^{n_o} \left(X(z) - \sum_{m=0}^{n_0 - 1} x[m] z^{-m} \right), n_o \ge 0$$