University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering

Main Examination 2012

Title of Paper: Signals and Systems II

Course Number: EE332

Time Allowed: 3 hrs

Instructions:

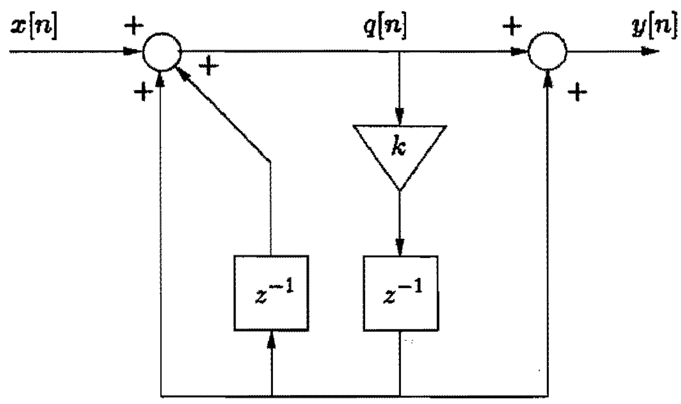
1. Answer any four (4) questions.
2. Each question carries 25 marks.
3. Useful tables are attached at the end of the question paper

This paper should not be opened until permission has been given by the invigilator.

This paper contains ten (10) pages including this page.

Question 1

(a) Calculate the transfer function, $H(z)$ of the discrete-time system shown in the figure below: [5]



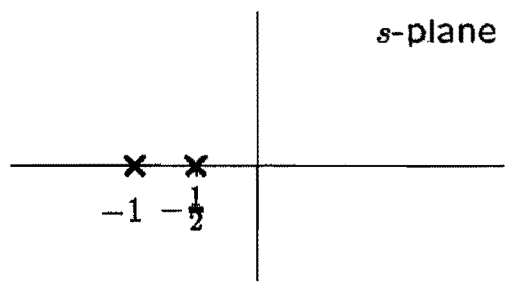
(b) Consider the following continuous-time signal:

$$x(t) = 2A\cos\left(200\pi t + \tfrac{\pi}{3}\right) + 3A\sin\left(100\pi t - \tfrac{\pi}{6}\right)$$

Determine the lowest possible sampling frequency, f_s in Hz to sample the signal without aliasing effects. [3]

(c) Compute the output, $y[n]$ for the following FIR filter with coefficients, $b_k = \{3,-1,2,1\}$ and input, $x[n]=[2,4,6]$. [5]

(d) A causal CT system has the following pole-zero diagram:



Let $y(t) = s(t)$ represent the response of this system to a unit-step signal

$$x(t) = u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & otherwise \end{cases}$$

Assume that the unit-step response $s(t)$ of this system is known to approach 1 as $t \to \infty$. Determine $y(t) = s(t)$. [12]

Question 2

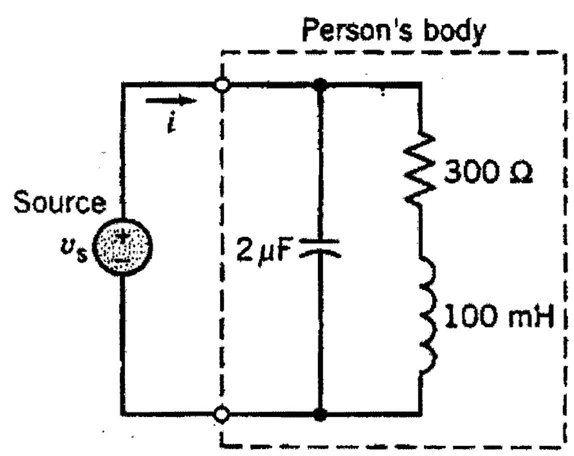
(a) Solve the following difference equation when the initial conditions are $y[0]=10$ and $y[1]=4$

$$y[n+2]-\left(\frac{3}{2}\right)y[n+1]+\left(\frac{1}{2}\right)y[n]=\left(\frac{1}{4}\right)^{n} \text{ for } ,n\geq 0.$$

[10]

(b) There are many deaths each year from electric shock. If a person makes a good contact with his hands, the circuit can be represented by the figure below, where $v_s=160\cos(\omega t)$ and $\omega=2\pi f$. Find the steady-state current $i(t)$ flowing through the body when $f=60Hz$.

[10]



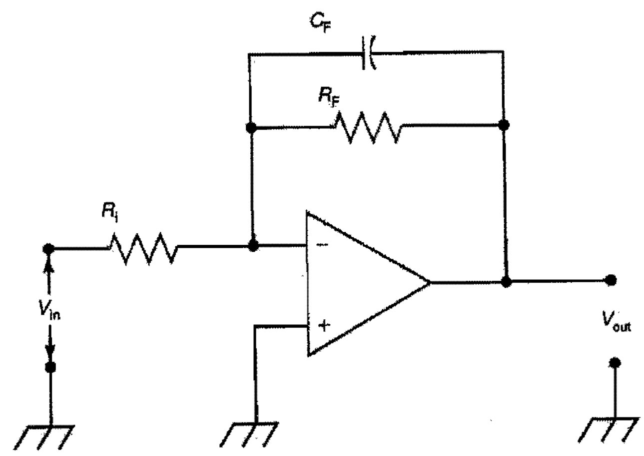
(c) Plot the spectrum of the voltage signal:

$$v(t)=2-3\cos(4t)+5\cos(6t+45°)+\cos(8t-75°).$$

[5]

Question 3

Consider the circuit below:



where $R_i = 2k\Omega$, $R_F = 100k\Omega$, $C_F = 0.1\mu F$

(a) What kind of filter is this? [1]

(b) Find the transfer function, $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$ [9]

(c) Find $V_{out}(t)$ when the input, $V_{in}(t)$ is a step of 0.1V whose Laplace transform is $\frac{0.1}{s}$. [10]

(d) Given the following differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 5y(t) = \frac{dx(t)}{dt}$$

(i) Write its characteristic equation. [2]
(ii) Determine its general solution. [3]

Question 4

(a) Given the following transfer function:

$$H(s) = 10\frac{s+10}{s^2+3s}$$

Construct the Bode plot for the magnitude and phase with respect to the angular frequency, ω, showing all the elements involved in your construction. [20]

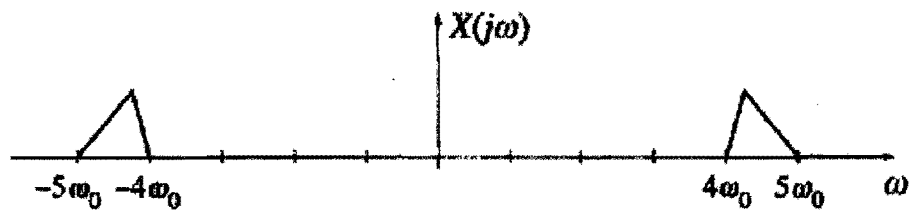
(b) Identify the region of convergenc, ROC associated with the z-transform for $w[n] = \left(\frac{-1}{2}\right)^n u[-n] + 2\left(\frac{1}{4}\right)^n u[-n]$.

[5]

Question 5

(a) If a channel has a bandwidth of $100kHz$, how many baseband signals each with a bandwidth of $15kHz$ could be multiplexed into a single composite signal and transmitted through the channel? [2]

(b) A signal with the sketched spectrum is sampled ideally with intervals T.



i) Is the signal a high-pass, a low-pass or a band-pass signal? [1]
ii) Is it complex or real? [1]
iii) Is critical sampling possible? [1]

(c) For a linear system with a unit feedback represented by:

$$G(s) = \frac{4}{s^2 + 2s + 4}$$

Calculate :

(i) Settling time. [2]
(ii) Maximum overshoot. [2]
(iii) Peak time. [2]
(iv) Is this system over-damped, under-damped or critically damped? [1]

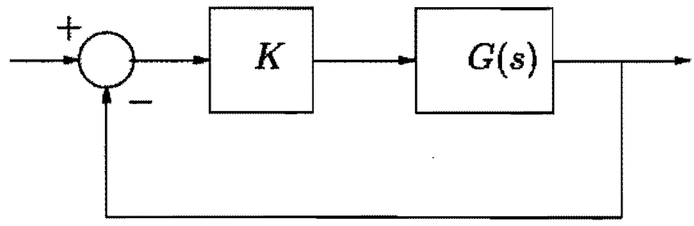
(d) A system is governed by the difference equation:

$$y[n] = -\frac{1}{2}x[n+2] - y[n+1]$$

where x is the input and y is the output. Is the system

(i) Linear? [1]
(ii) Time invariant? [2]
(iii) Bounded input bounded output (BIBO) stable? [2]
(iv) Memoryless? [2]
(v) Causal? [2]

(e) For the following closed loop system, calculate the steady-state error to a unit step input. [4]



where $G(s) = \dfrac{0.5(s+11)}{(s+1)^3}$

Table of Z-Transforms

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n)z^{-n}$	
2	$\delta(n)$	1	$\|z\| > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\|z\| > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\|z\| > 1$
5	$n^2u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\|z\| > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$\|z\| > \|a\|$
7	$e^{-na}u(n)$	$\frac{z}{(z-e^{-a})}$	$\|z\| > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\|z\| > \|a\|$
9	$\sin(an)u(n)$	$\frac{z\sin(a)}{z^2-2z\cos(a)+1}$	$\|z\| > 1$
10	$\cos(an)u(n)$	$\frac{z[z-\cos(a)]}{z^2-2z\cos(a)+1}$	$\|z\| > 1$
11	$a^n \sin(bn)u(n)$	$\frac{[a\sin(b)]z}{z^2-[2a\cos(b)]z+a^2}$	$\|z\| > \|a\|$
12	$a^n \cos(bn)u(n)$	$\frac{z[z-a\cos(b)]}{z^2-[2a\cos(b)]z+a^{-2}}$	$\|z\| > \|a\|$
13	$e^{-an}\sin(bn)u(n)$	$\frac{[e^{-a}\sin(b)]z}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\|z\| > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\frac{z[z-e^{-a}\cos(b)]}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\|z\| > e^{-a}$

<u>Tables of Laplace Transforms</u>

$f(t) = {}^{-1}\{F(s)\} = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(s)e^{st}ds$	$\xleftrightarrow{\mathcal{L}}$	$F(s) = \mathcal{L}\{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-st}dt$
$f(t)$	$\xleftrightarrow{\mathcal{L}}$	$F(s)$
$f(t-a) \quad t \geqslant a > 0$	$\xleftrightarrow{\mathcal{L}}$	$a^{-as}F(s)$
$e^{-at}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$F(s+a)$
$f(at) \quad a > 0$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{a}F(\frac{s}{a})$
$af_1(t) + bf_2(t)$	$\xleftrightarrow{\mathcal{L}}$	$aF_1(s) + bF_2(s)$
$f_1(t)f_2(t)$	$\xleftrightarrow{\mathcal{L}}$	$F_1(s) * F_2(s)$
$f_1(t) * f_2(t)$	$\xleftrightarrow{\mathcal{L}}$	$F_1(s)F_2(s)$
$\delta(t)$	$\xleftrightarrow{\mathcal{L}}$	1
1	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}$
t	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s^2}$
e^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$
te^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{(s+a)^2}$
$1 - e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{a}{s(s+a)}$
$\frac{1}{a}e^{-\frac{t}{a}}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{1+as}$
$\frac{1}{a}\left(1 - e^{-at}\right)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$
$\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2+\omega^2}$
$\sinh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2-\omega^2}$
$\cosh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2-\omega^2}$
$e^{-at}\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s+a}{(s+a)^2+\omega^2}$
t^n	$\xleftrightarrow{\mathcal{L}}$	$\frac{n!}{s+n+1}$
$t^n f(t)$	$\xleftrightarrow{\mathcal{L}}$	$(-1)^n F^{(n)}(s)$
$f'(t) = \frac{d}{dt}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$sF(s) - f(0)$
$f''(t) = \frac{d^2}{dt^2}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^2F(s) - sf(0) - f'(0)$
In general : $f^{(n)}(t) = \frac{d^n}{dt^n}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^nF(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$
$\int_0^t f(u)du$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}F(s)$
$\frac{1}{t}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$\int_s^\infty F(u)du$
$f^{-1}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)-f^{-1}}{s}$
$f^{-n}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)}{s^n} + \frac{f^{-1}(0)}{s^n} + \frac{f^{-2}(0)}{s^{n-1}} + \ldots + \frac{f^{-n}(0)}{s}$

$F(s)$	$f(t) \quad 0 \le t$	
1. 1	$\delta(t)$	unit impulse at $t = 0$
2. $\frac{1}{s}$	1 or $u(t)$	unit step starting at $t = 0$
3. $\frac{1}{s^2}$	$t \cdot u(t)$ or t	ramp function
4. $\frac{1}{s^n}$	$\frac{1}{(n-1)!} t^{n-1}$	n = positive integer
5. $\frac{1}{s} e^{-as}$	$u(t - a)$	unit step starting at $t = a$
6. $\frac{1}{s}(1 - e^{-as})$	$u(t) - u(t-a)$	rectangular pulse
7. $\frac{1}{s+a}$	e^{-at}	exponential decay
8. $\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!} t^{n-1} e^{-at}$	n = positive integer
9. $\frac{1}{s(s+a)}$	$\frac{1}{a}(1 - e^{-at})$	

Properties of Z-Transforms

Linearity: $x_1[k] + x_2[k] \Leftrightarrow X_1(z) + X_2(z)$

Real Scaling: $ax[k] \Leftrightarrow aX(z)$

Complex Scaling: $a^k x[k] \Leftrightarrow X(z/a)$

Time Reversal: $x[-k] \Leftrightarrow X(1/z)$

Summation: $\sum_{n=-\infty}^{k} x[n] \Leftrightarrow \frac{zX(z)}{z-1}$

Initial Value: $x[0] = \lim_{z\to\infty} X(z)$

Final Value: $x[\infty] = \lim_{z\to 1} (z-1)X(z)$

Convolution: $x[k] * h[k] \Leftrightarrow X(z)H(z)$

Differencing:
$x[k] - x[k-1] \Leftrightarrow (1 - z^{-1})X(z)$

Differentiation: $-kx[k] \Leftrightarrow z\frac{d}{dz}X(z)$

Time Shifting:
$x[n - n_o] \Leftrightarrow z^{-n_o}X(z), n_o \ge 0$

$x[n + n_o] \Leftrightarrow z^{n_o}\left(X(z) - \sum_{m=0}^{n_0-1} x[m]z^{-m}\right), n_o \ge 0$