UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE
DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

MAIN EXAMINATION DECEMBER 2011

TITLE OF PAPER: **CONTROL ENGINEERING I**

COURSE CODE: **EE 431**

TIME ALLOWED: **THREE HOURS**

Student Name:	
Student Number:	

INSTRUCTIONS:

1. Answer all questions.
2. Give your answers on the question paper, and if more space is required, complete your answer on the back of the paper or in your answer book and mention about the place of your answer completion.
3. Put the question sheet inside the answer book upon submission of your exam paper.
   (DON'T FORGET TO SUBMIT BOTH OF THE ANSWER BOOK AND QUESTION PAPER)
4. Marks for different questions are indicated on the beginning of the question.
5. Rough work maybe done in the examination answer book and crossed through.

DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

This paper starts at page 1 and ends at page 21

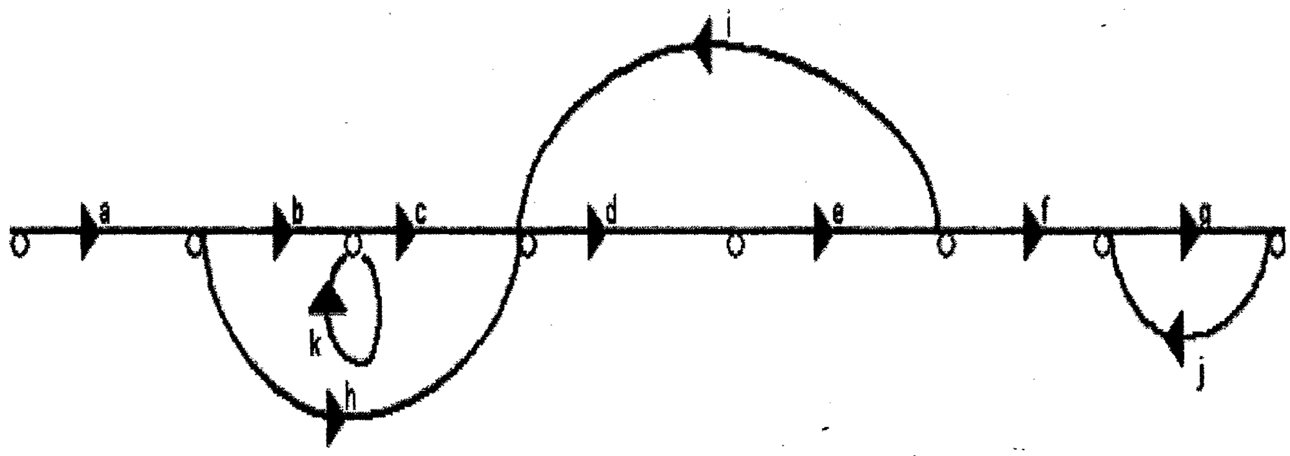
Question 1: Solve the following questions (17 marks)

a) **If the transfer function of a system** $\frac{C(s)}{R(s)} = \frac{5(s-10)}{s^5 + 2s^4 - 14s^3 + 12s^2 + 5s - 150}$, **using Routh's stability criterion, obtain the number of unstable poles in the system if they exist.**

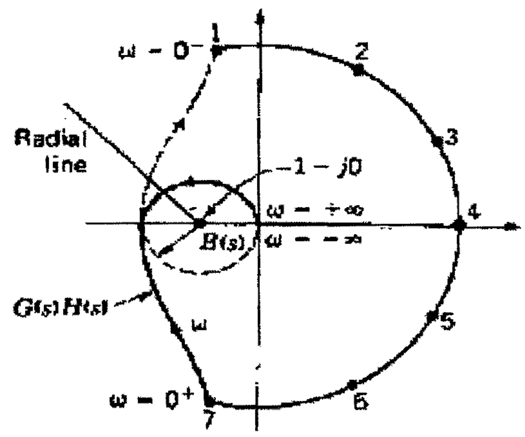
s^5			
s^4			
s^3			
s^2			
s^1			
s^0			

The number of unstable poles =

b) Using mason theorem, find the overall gain of the control system represented by the following signal flow diagram.



c) **If the open loop transfer function** $G(s)H(s)=\dfrac{K}{s(1+sT_1)(1+sT_2)}$ **has the following direct polar plot. Determine for the closed loop system** $\dfrac{C(s)}{R(s)}=\dfrac{G(s)}{1+G(s)H(s)}=\dfrac{A(s)}{B(s)}$, **the number of zeros of B(s) on the right half of s-plane** Z_R**, the number of poles of B(s) on the right half of s-plane** P_R **and the number of unstable poles in the closed loop system if they exist.**



Question 2: Solve the following questions (7 marks)

a) Mention on the following table if any of the elements in the rows of the table will correspond to any of the elements in the columns of the table using a right mark (√).

	Proportional integral controller	**Proportional derivative controller**	**Phase lead compensator**	**Phase lag compensator**	**Lead lag compensator**	**Optim state feedba contro with trackii**
Improves the transient response and reduces settling time.						
Reduces the steady state error						
The control law will be function of system states and desired value for output and system distaurbances. $u = F_x x + F_r r + F_d d$						
Has transfer function $C = \frac{A}{\alpha}\frac{s+1/T}{s+1/(\alpha T)}$ **The zero** $s=-1/T$ **and the pole** $s=-1/(\alpha T)$ **are selected to the left and close together near the origin with** α **are chosen large value such as 10.**						
Has transfer function $G_c = A\frac{(s+\frac{1}{T_1})(s+\frac{1}{T_2})}{(s+\frac{1}{\alpha T_1})(s+\frac{\alpha}{T_2})}$						

b) Mention on the following table what are the ranges of the damping coefficient that causes the following system responses.

The transient response	**The range of damping coefficient ζ**
The system is critically damped.	
The system is damped with maximum overshoot =0.	
System responses are pure sinusoidal.	
The system is damped with maximum overshoot >0.	
System is unstable and response increases without bound.	

Question 3: Solve the following questions (18 marks)

In a DC motor system, the resistance of the armature winding is denoted by R (ohm) and the self-inductance of the armature by L (H). The torque (N.m) seen at the shaft of the motor is proportional to the current i (A) induced by the applied voltage (V), $T_{ind} = K_m i$. The back (induced) electromotive force, E_A (V), is a voltage proportional to the angular rate seen at the shaft, $E_A = K_b \omega$

The state-space model of the DC motor when $R = 3\,\text{ohm}$, $L = 0.6$ H, $K_m = 0.025$, $K_b = 0.025$, $K_f = 0.3$, $J = 0.03$ and $T_L = 0$ will be

$$\frac{d}{dt}x(t) = Ax(t) + Bu(t) \qquad y(t) = Cx(t) + Bu(t)$$

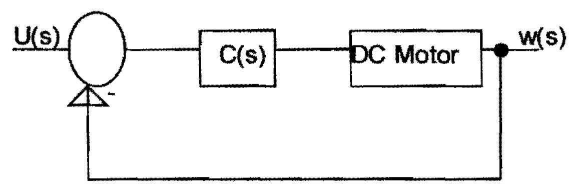
Where $x(t) = \begin{bmatrix} i(t) \\ \omega(t) \end{bmatrix}$ $\qquad u(t) = [V]$

Where

$$A = \begin{bmatrix} -5 & -0.04167 \\ 0.8333 & -10 \end{bmatrix}, \quad B = \begin{bmatrix} 1.6667 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad D = [0]$$

a) By taking the Lap lace transform of the state space equations, find the transfer function of motor angular speed $\dfrac{\omega(s)}{V(s)}$. Assume the initial values for the speed and current equal to zero.

b) Consider a closed loop DC motor system with negative unity feedback, and a forward compensator $C(s)$ added in series with the DC motor to control the applied voltage to DC motor. The compensator is assumed to proportional integral compensator with the following transfer function $C(s) = K\ (1+\frac{1}{s})$. Sketch roughly the root locus of the system.



d) Sketch roughly the root locus of the system assuming the controller is a proportional integral derivative compensator with the following transfer function

$$C(s) = K\ (1 + s + \frac{1}{s}) = K\frac{(s^2 + s + 1)}{s} = K\frac{(s + 0.5 + j0.8666)(s + 0.5 - j0.8666)}{s}.$$

e) If the gain of the proportional integral controller is chosen to be K=50, so $C(s) = 50 * (1 + \frac{1}{s})$**, the overall system gain will be**

$$\frac{\omega(s)}{U(s)} = \frac{C(s) * G(s)}{1 + C(s) * G(s)H(s)} = \frac{69.444 * (s + 1)}{(s + 0.6288)(s^2 + 14.37s + 110.4)}$$

Where feedback gain H=1, and G(s) is the transfer function for the DC motor $\frac{\omega(s)}{V(s)}$**.**

Calculate the steady state value for motor speed when the control input $U(s)$ **is unit step function.**

Question 4: Solve the following questions (34 marks)

Given that a closed loop negative feedback system with the following transfer functions for the forward and feedback gains:

$$G(s) = \frac{K}{(s^2 + 7s + 6)} = \frac{K}{(s+1)*(s+6)}, \quad H(s) = \frac{1}{(s+3)}$$

The objective is to plot the root locus of the open loop transfer function $G(s)H(s)$ **and to obtain the system transient response when** $\zeta = 0.5$.

a) Calculate the real axis intercept of the asymptotes and the asymptotes of the root locus as s approaches infinity.

b) Sketch roughly the initial estimate for the root locus of $G(s)H(s)$**.**

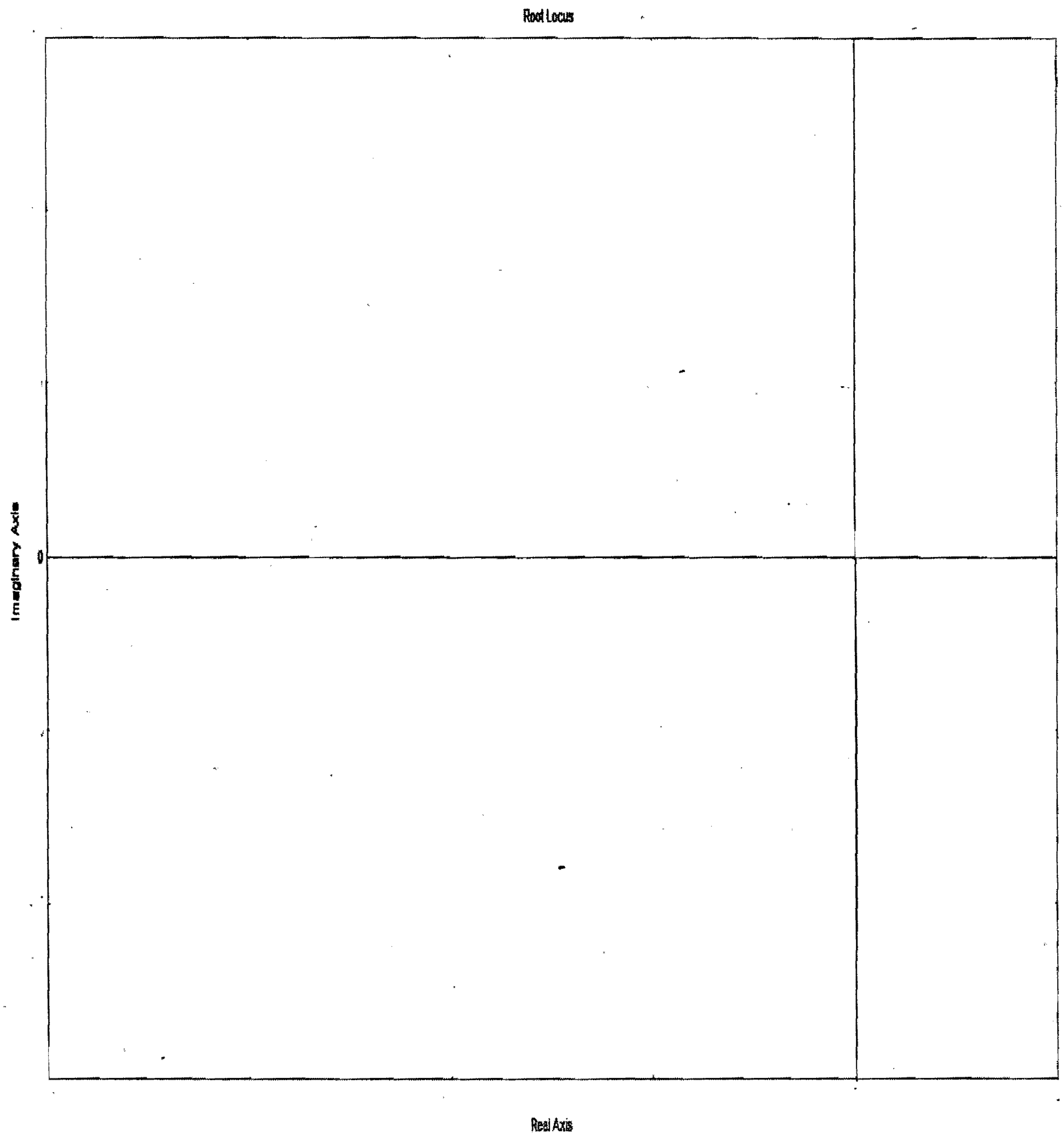
c) Calculate the breakaway point on the real axis and the value of loop sensitivity K at the breakaway point if exist.

d) Write the overall system gain $\frac{C(s)}{R(s)}$ of the negative feedback system.

e) Calculate the imaginary axis crossover points and the value of loop sensitivity K at the crossover points if exist.

s^3		
s^2		
s^1		
s^0		

f) Draw the root locus of $G(s)H(s)$ based on the results obtained in the previous parts.



g) If a radial line corresponding to $\zeta = 0.5$ is drawn on the graph, determine the dominant poles and the value of loop sensitivity K at these points.

h) Calculate any additional roots using the grants rule.

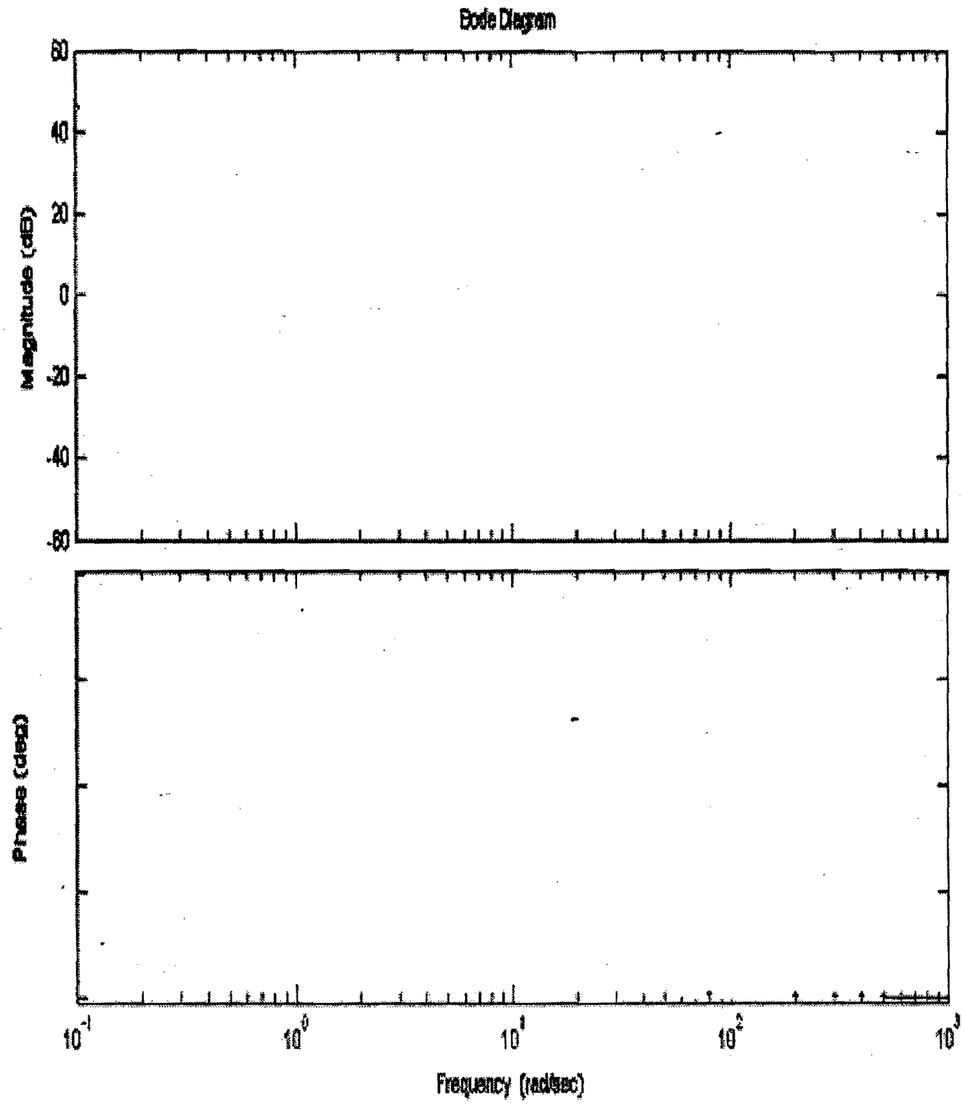
i) Write the overall system transfer function $\frac{C(s)}{R(s)}$ **with the value of gain and roots calculated previously corresponding to** $\zeta = 0.5$ **and determine the transient response of** $c(t)$ **for a unit step input** $r(t) = u(t)$**.**

Question 5: Solve the following questions (24 marks)

The objective is to draw the bode plot and the polar plot for the following open loop transfer function

$$G(j\omega)H(j\omega) = \frac{0.8}{j\omega(1+0.15j\omega+0.25^2(j\omega)^2)}$$

a) Draw the log magnitude and phase plots for the factor $\frac{1}{(1+0.15j\omega+0.25^2(j\omega)^2)}$



b) Complete the following table:

Term	Cutoff frequency If exist	Log magnitude equation	Angle equation
0.8			
$\frac{1}{j\omega}$			
$\frac{1}{(1+0.15j\omega+0.25^2(j\omega)^2)}$			

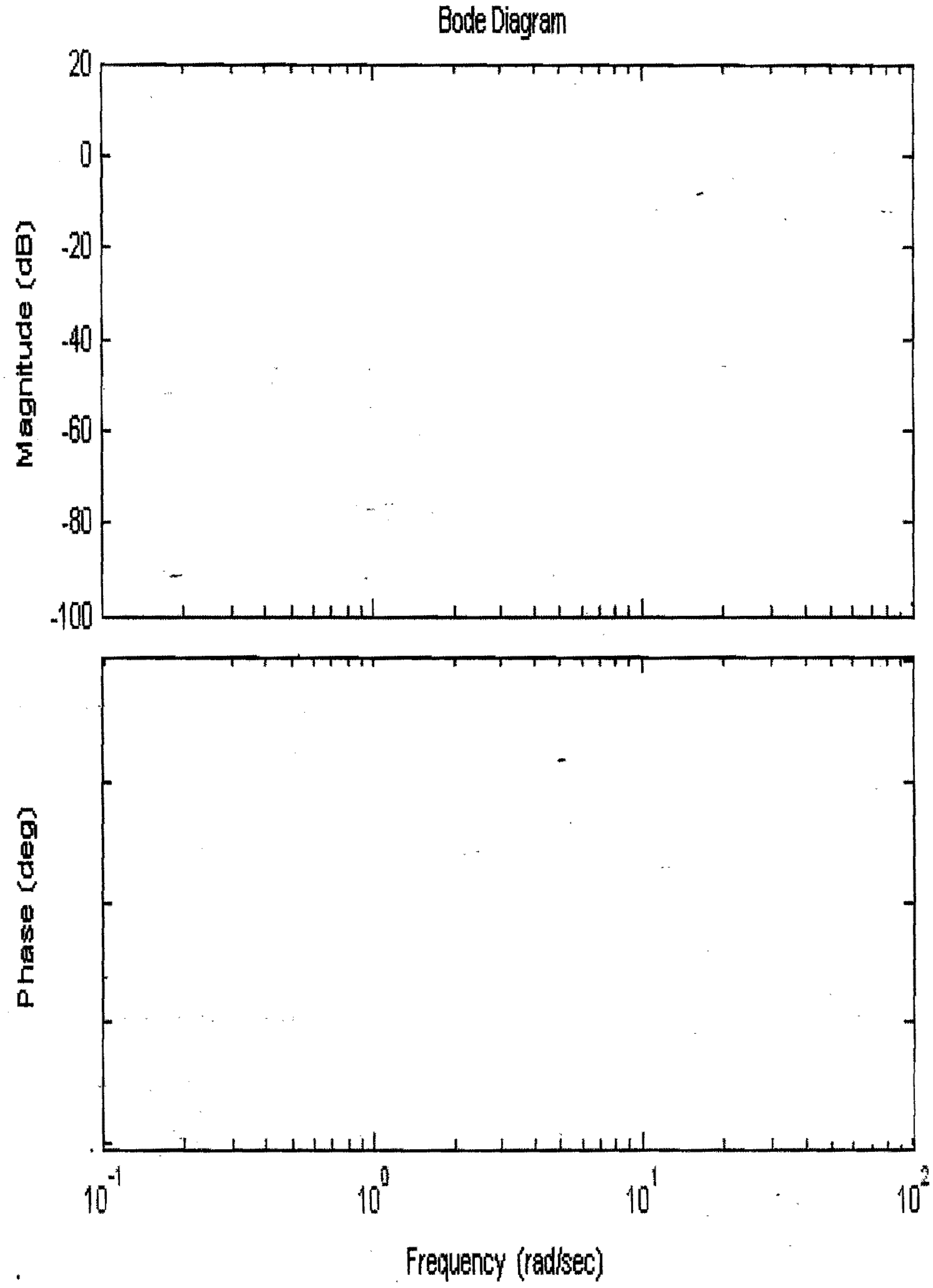
c) **Complete the following table:**

Frequency	Log Magnitude at that frequency	Angle at that frequency
0	**(Not required)**	
Decade before first cutoff frequency $0.1\omega_1$		
First cutoff frequency ω_1		
Decade after cutoff frequency $10\omega_1$		
INFINITY	**(Not Required)**	

d) Draw the composite log magnitude and phase plots for overall open loop transfer function

$$G(j\omega)H(j\omega) = \frac{0.8}{j\omega(1+0.15j\omega+0.25^2(j\omega)^2)}$$

Note: In the following bode diagram, put the scale that is suitable with your analysis and results.



e) From the composite log magnitude and phase angles curves, estimate

Gain crossover frequency:	
Phase margin:	
Phase crossover frequency:	
Gain margin:	

f) Indicate with reasoning if the closed loop system stable or not. If the system stable calculate the gain that will cause the system to be unstable.

g) Draw roughly the polar (Nyquist) plot for the previous open loop transfer function $G(j\omega)H(j\omega)=\dfrac{0.8}{j\omega(1+0.15j\omega+0.25^2(j\omega)^2)}$. **Mention on the plot the location of phase crossover frequency and gain crossover frequency. What should be phase crossover frequency and how much the gain margin?**

Note: You can utilize some of the results obtained in the bode plot section in order to draw the polar plot.