UNIVERSITY OF SWAZILAND

MAIN EXAMINATION, MAY 2013

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TITLE OF PAPER:	INTRODUCTION TO DIGITAL SIGNAL PROCESSING
COURSE CODE:	EE443
TIME ALLOWED:	THREE HOURS

INSTRUCTIONS:

1. Answer any FOUR (4) of the following five questions.
2. Each question carries 25 marks.
3. Tables of selected window functions and selected Z-transform pairs are attached at the end.

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THIS PAPER CONTAINS EIGHT (8) PAGES INCLUDING THIS PAGE

QUESTION ONE *(25 marks)*

(a) State two conditions that must be satisfied for a signal to be recovered from its samples. *(2 marks*

(b) A signal *x(t)* consists of a sum of sinusoids

$$x(t) = 2\cos 1000\pi t + 3\cos 3000\pi t + 4\cos 4000\pi t$$

(i) What is should be the sampling rate for this signal? *(2 mar*

(ii) What happens to each sinusoid when sampled at half the frequency stated in (i)? *(3 mar*

(c) Given a discrete-time signal

$$x[n] = 6\sin(n\pi/100)\text{ V},\ \ n = 0,1,2,3,\ldots$$

(i) Find the peak-to-peak range of the signal? *(1 ma*

(ii) What is the quantization step (resolution) of a 10-bit ADC for this signal? *(2 mar*

(iii) If the quantization resolution is required to be below 1 mV, how many bits are required in the ADC? *(3 mar*

(iv) What is the r.m.s value of quantization noise generated if a CD quality 16-bit quantizer is used? *(2 mar*

(d) A signal has a flat uniform spectrum. A 6^{th} order Butterworth filter with cut-off frequency of 5 kHz is used to filter this signal. The filtered signal is digitized using 10-bit quantization. What should be the minimum sampling rate if the aliased signal amplitude at 2 kHz should not exceed the r.m.s value of the quantization noise?

An *n*th order Butterworth analogue filter has a magnitude response $\dfrac{1}{\sqrt{1+\left(\dfrac{f}{f_c}\right)^{2n}}}$.

(10 marks)

QUESTION TWO **(25 *marks*)**

(a) Examine and discuss the stability or otherwise of the following IIR filters:

(i) $H(z) = \dfrac{z(z-1)}{(z^2 - z + 1)(z + 0.8)}$ *(5 marks)*

(ii) $3y(n) = 3.7y(n-1) - 0.7y(n-2) + x(n-1), \quad n \geq 0$ *(7 marks)*

(b) Two first-order IIR filters are defined by the difference equations:

$$y_1(n) = x(n) - 0.5y_1(n-1), \quad n \geq 0$$

$$y_2(n) = x(n) - y_2(n-1), \quad n \geq 0$$

The filters are connected in parallel so that the combined filter has a system function $H(z) = H_1(z) + H_2(z)$. Obtain an expression for the response $y[n]$ of the filter combination to an input sequence $x[n] = (-1)^n, \quad n \geq 0$ *(13 marks)*

QUESTION THREE **(25 *marks*)**

(a) An FIR filter defined by

$$y(n) = x(n) + 2x(n-1) + 4x(n-2) + 2x(n-3) + x(n-4), \quad n \geq 0$$

(i) Obtain expressions for the magnitude and phase response of the filter. *(6 marks)*

(ii) Sketch the magnitude and phase response. *(6 marks)*

(b) An FIR has a transfer function given by $H(z) = 1 + 0.6z^{-1} + z^{-2}$. Given that the sampling rate is 7 kHz, determine the input signal frequency which will be maximally attenuated when passed through the filter. *(8 marks)*

(c) For the filter with a system function

$$H(z) = \frac{1 + 3z^{-1} + 4z^{-2}}{1 - 2z^{-1} + 5z^{-2} + -z^{-3}}$$

Sketch a realization structure for this filter. *(5 marks)*

QUESTION FOUR **(25 *marks*)**

(a) (i) How can a circular convolution of two sequences be obtained using FFTs and IFFT only?

(i) Using the above method and a radix-2 decimation-in-time FFT algorithm, find the circular convolution of the sequences:

$x_1[n] = [3, 1, 2, 5]$

$x_2[n] = [1, 2, 0, -2]$ *(15 marks)*

(b) Convert the analogue filter $H(s) = \dfrac{1}{(s+1)(s+2)}$ into a digital filter using the impulse invariant technique with a sampling interval of 0.02 s. *(10 marks)*

QUESTION FIVE **(25 *marks*)**

A linear-phase FIR filter is to be designed with the following specifications:

Filter length, $N = 9$

Normalized cut-off frequency $= \dfrac{4\pi}{9}$ rad

Window to be applied = Hanning

(a) Calculate the filter coefficients with accuracy of 4 decimal places. *(15 marks)*

(b) Explain why a window function needs to be used in this design. *(2 marks)*

(c) Calculate the magnitude and phase response of this filter at a normalized frequency of $\dfrac{\pi}{9}$ *(8 marks)*

TABLE OF Z-TRANSFORMS OF SOME COMMON SEQUENCES

Discrete-time sequence $x(n),\ n \geq 0$	**Z-transform** $H(z)$
$k\delta(n)$	k
k	$\frac{kz}{z-1}$
$ke^{-\alpha n}$	$\frac{kz}{z-e^{-\alpha}}$
$k\alpha^n$	$\frac{kz}{z-\alpha}$
kn	$\frac{kz}{(z-1)^2}$
kn^2	$\frac{kz(z+1)}{(z-1)^3}$
$kn\alpha^n$	$\frac{k\alpha z}{(z-\alpha)^2}$

QUANTIZATION

For a sine wave $SQNR = 6.02B + 1.76$ dB.

SUMMARY OF IMPORTANT FEATURES OF SELECTED WINDOW FUNCTIONS

Name of Widow	Normalized Transition Width	Passband Ripple (dB)	Main lobe relative to Sidelobe (dB)	Max. Stopband attenuation (dB)	6 dB normalized bandwidth (bins)	Window Function ω(n), |n| ≤(N-1)/2
Rectangular	0.9/N	0.7416	13	21	1.21	1
Hanning	3.1/N	0.0546	31	44	2.00	$0.5+0.5\cos\left(\frac{2\pi n}{N}\right)$
Hamming	3.3/N	0.0194	41	53	1.81	$0.54+0.46\cos\left(\frac{2\pi n}{N}\right)$
Blackman	5.5/N	0.0017	57	74	2.35	$0.42+0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{4\pi n}{N-1}\right)$
Kaiser	2.93/N (β=4.54)	0.0274		50		$\frac{I_o\left(\beta\left\{1-\left[\frac{2n}{N-1}\right]^2\right\}^{\frac{1}{2}}\right)}{I_o(\beta)}$
	4.32/N (β=6.76	0.00275		70		
	5.71/N (β=8.96)	0.000275		90		

$$\text{Bin width} = \frac{f_s}{N}\ \text{Hz}$$