University of Swaziland
Faculty of Science and Engineering
Department of Electrical and Electronic Engineering

Main Examination 2014

Title of Paper: Signals and Systems II

Course Number: EE332

Time Allowed: 3 hrs

Instructions:

1. Answer any four (4) questions
2. Each question carries 25 marks
3. Useful tables are attached at the end of the question paper
4. Linear-log paper is attached to the question paper

This paper should not be opened until permission has been given by the invigilator.

This paper contains nine (9) pages including this page.

Question 1

a) Based on the system $G(s)$ shown below:

$$G(s) = \frac{10(s+1)}{s(s^2+60s+800)}$$

Sketch the magnitude and phase Bode plots. Showing all the elements involved in your construction. [10]

b) Plot the magnitude and phase spectrum of the voltage signal: [6]

$$v(t) = 2 - 3\cos(4t) + 5\cos(6t + 45°) - \sin(8t - 75°).$$

c) What are the limitations of amplitude modulation? [4]

d) Find the steady state error for a unit feedback control system represented by:

$$G(s)H(s) = \frac{5}{s^2(s+1)(2s+1)}$$

Assuming that the input signal is given by: $2t^2 - 3t - 2$ [5]

Question 2

a) Draw a simple block diagram showing how amplitude demodulation occurs. Sketch all the input signals and the overall output signal. [5]

b) Using phasors find the steady-state current, $i(t)$ in Fig.2
[10]

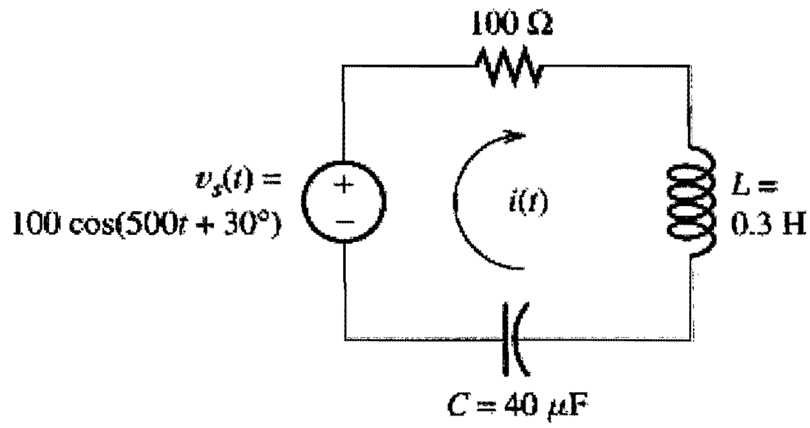

Fig. 2

c) For the following signal, $x[n] = \left(\frac{1}{2}\right)^n u[n] + (2)^n u[-n-1]$:

i) Find the z-transform [3]
ii) Find the region of convergence. [2]
iii) Sketch the pole zero diagram and the region of convergence. [5]

Question 3

a) Obtain the z-transform of: [3]

$$X(s) = \frac{1}{s(s+1)}$$

b) Let $x(t)$ be a signal with spectrum depicted in Fig. 3(b):

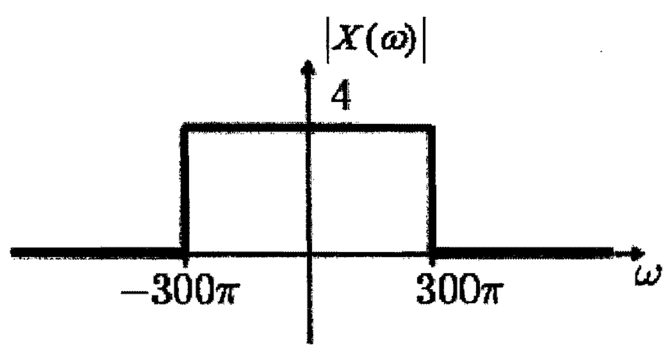

Fig. 3(b)

i) What is the Nyquist sampling frequency in Hz of signal $x(t)$? [2]

ii) Sketch the spectrum of the sampled signal, $|X_s(\omega)|$, assuming we sample at $F_s = 400Hz$. Show all your working. [5]

c) Let $y(t) = [A + x(t)]\cos(\omega_c t)$ be an AM large carrier signal, where $x(t)$ is the message waveform. Here we only consider a sinusoidal message, i.e. $x(t) = K\cos(\omega_m t + \theta)$ where $\omega_m \ll \omega_c$.

i) Show that the AM large carrier wave can be written as: [2]

$$y(t) = A[1 + b\cos(\omega_m t + \theta)]\cos(\omega_c t)$$

ii) Find an expression for parameter b in terms of K and A. [2]

iii) What constraint must be imposed upon b to ensure that overmodulation does not occur? [1]

d) Consider the circuit in Fig. 3(d):

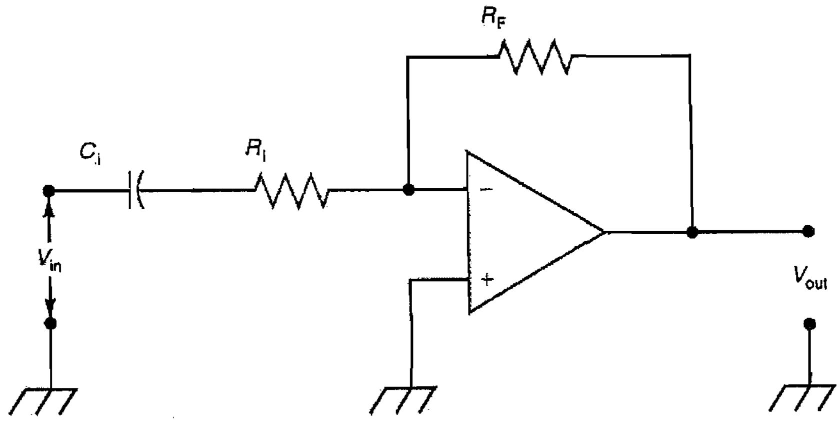

Fig. 3(d)

where $R_i = 2k\Omega$, $C_i = 0.1\mu F$, $R_F = 100k\Omega$.

i) What kind of filter is this? [1]

ii) Find the transfer function, $H(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$. [5]

e) Find the transfer function, $H(z)$ of the discrete-time system shown in the Fig. 3(e): [4]

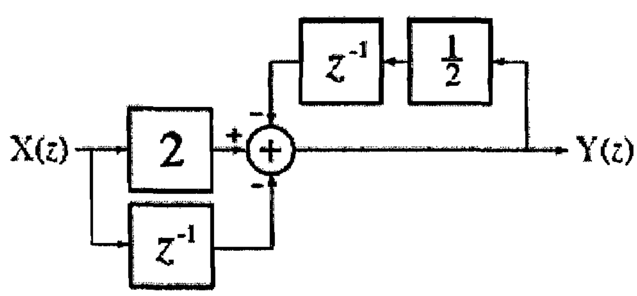

Fig. 3(e)

Question 4

A unit step is applied to a system with the transfer function, $H(s) = \frac{12}{s^2 + 2s + 9}$. Find the following:

a) The roots of the characteristic equation and general solution [3]
b) The undamped natural frequency, ω_n [2]
c) The damping ratio, ζ [2]
d) The damped natural frequency, ω_d [3]
e) The stability ratio, σ [3]
f) The percentage overshoot [3]
g) The settling time, t_s [3]
h) The peak time, t_p [3]
i) The DC gain, K [1]
j) Is this system underdamped, overdamped or critically damped? Explain your answer? [2]

Question 5

a) Define the following terms as used in the analysis of linear signals and systems:

i) Zero-order hold sampling [2]
ii) Single sideband transmission [2]
iii) Aliasing [2]
iv) Modulating factor [2]

b) Solve the following difference equation using z-transforms: [15]

$$x[n+2] + 0.4x[n+1] - 0.32x[n] = u[n], \quad x[0] = 0, \quad x[1] = 1.$$

c) Draw a simple envelope diode detector that can be used to demodulate an AM signal. [2]

Table of Laplace Transforms

delta function	$\delta(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	1
shifted delta function	$\delta(t-a)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	e^{-as}
unit step	$u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}$
ramp	$tu(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2}{s^3}$
n-th power	t^n	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\|t\|}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$-F'(s)$
frequency n-th differentiation	$t^n f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$(-1)^n F^{(n)}(s)$
time differentiation	$f'(t)=\frac{d}{dt}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$sF(s)-f(0)$
time 2nd differentiation	$f''(t)=\frac{d^2}{dt^2}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^2F(s)-sf(0)-f'(0)$
time n-th differentiation	$f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau=(u*f)(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$

<u>Properties of Laplace Transforms</u>

Time-shift (delay): $f(t-t_0) \xleftrightarrow{L} F(s)e^{-st_0}, \quad t_0 > 0$

Time differentiation: $\dfrac{df(t)}{dt} \xleftrightarrow{L} sF(s) - f(0)$

Time integration: $\displaystyle\int_0^t f(t)dt \xleftrightarrow{L} \frac{F(s)}{s}$

Linearity: $af(t) + bg(t) \xleftrightarrow{L} aF(s) + bF(s)$

Convolution Integral: $x(t) * h(t) \xleftrightarrow{L} X(s)H(s)$

Frequency-shift: $e^{\alpha t} f(t) \xleftrightarrow{L} F(s-\alpha)$

Multiplying by t: $tf(t) \xleftrightarrow{L} -\dfrac{dF(s)}{ds}$

Scaling: $f(at) \xleftrightarrow{L} \dfrac{1}{a} F\left(\dfrac{s}{a}\right), \quad a > 0$

Initial Value Theorem: $\lim_{s \to \infty} \{sF(s)\} = f(0)$

Final Value Theorem: $\lim_{s \to 0} \{sF(s)\} = \lim_{t \to \infty} f(t)$

Table of Z-Transforms

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n) z^{-n}$	
2	$\delta(n)$	1	$\|z\| > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\|z\| > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\|z\| > 1$
5	$n^2 u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\|z\| > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$\|z\| > \|a\|$
7	$e^{-na} u(n)$	$\frac{z}{(z-e^{-a})}$	$\|z\| > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\|z\| > \|a\|$
9	$\sin(an)u(n)$	$\frac{z \sin(a)}{z^2 - 2z\cos(a) + 1}$	$\|z\| > 1$
10	$\cos(an)u(n)$	$\frac{z[z - \cos(a)]}{z^2 - 2z\cos(a) + 1}$	$\|z\| > 1$
11	$a^n \sin(bn)u(n)$	$\frac{[a \sin(b)]z}{z^2 - [2a\cos(b)]z + a^2}$	$\|z\| > \|a\|$
12	$a^n \cos(bn)u(n)$	$\frac{z[z - a\cos(b)]}{z^2 - [2a\cos(b)]z + a^{-2}}$	$\|z\| > \|a\|$
13	$e^{-an} \sin(bn)u(n)$	$\frac{[e^{-a}\sin(b)]z}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\|z\| > e^{-a}$
14	$e^{-an} \cos(bn)u(n)$	$\frac{z[z - e^{-a}\cos(b)]}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\|z\| > e^{-a}$

Properties of Z-Transforms

Linearity: $ax_1[k] + bx_2[k] \Leftrightarrow aX_1(z) + bX_2(z)$

Time Reversal: $x[-k] \Leftrightarrow X(1/z)$

Summation: $\sum_{n=-\infty}^{k} x[n] \Leftrightarrow \frac{zX(z)}{z-1}$

Initial Value: $x[0] = \lim_{z\to\infty} X(z)$

Final Value: $x[\infty] = \lim_{z\to 1} (z-1)X(z)$

Convolution: $x[k] * h[k] \Leftrightarrow X(z)H(z)$

Differencing: $x[k] - x[k-1] \Leftrightarrow \left(1 - z^{-1}\right)X(z)$

Differentiation: $-kx[k] \Leftrightarrow z\frac{d}{dz}X(z)$

Time Shifting: $x[n-n_o] \Leftrightarrow z^{-n_o}X(z), n_o \geq 0$

$$x[n+n_o] \Leftrightarrow z^{n_o}\left(X(z) - \sum_{m=0}^{n_o-1} x[m]z^{-m}\right), n_o \geq 0$$