UNIVERSITY OF SWAZILAND
SUPPLEMENTARY EXAMINATION, JULY 2015

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TITLE OF PAPER:	**SIGNALS AND SYSTEMS I**
COURSE NUMBER:	**EE331**
TIME ALLOWED:	**THREE HOURS**

INSTRUCTIONS:

1. There are five questions in this paper. **Answer any FOUR questions.**
2. Each question carries 25 marks.
3. Marks for different sections are shown on the right hand margin.
4. Show the steps clearly in all your work. This is because marks may be awarded for method and understanding, even if a final answer is incorrect.
5. Sheets containing useful tables are attached at the end of the question paper.

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QUESTION 1 ***(25 marks)***

(a) Find the period of each of the following signals:

(i) $x(t) = e^{j\left(\frac{\pi}{4}t-1\right)}$ *(3 marks)*

(ii) $y(t) = 10 + e^{j3t} + e^{-j3t} + e^{j6t} + e^{-j6t}$ *(4 marks)*

(b) A half-wave rectified sine wave is defined by the expression

$$x(t) = \begin{cases} A\sin(\omega_o t), & 0 \le t \le \frac{T}{2} \\ 0, & \frac{T}{2} \le t \le T \end{cases}.$$

Given that the complex Fourier Coefficients C_k of $x(t)$ are given by

$$C_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t},$$

Obtain expressions for the first two Fourier coefficients only, i.e. when $k=0$ and $k=1$. **You are not asked to get a general expression for all values of *k*, therefore simplify your integral by concentrating only on these two values of *k*.**

(12 marks)

(c) Three functions $f_1(t)$, $f_2(t)$ and $f_3(t)$, which are zero outside the interval $(0,T)$, are shown in the figure Q1c. By taking a pair of signals at a time, determine whether each pair is orthogonal or not. Note that there will be three pairs of signals. *(6 marks)*

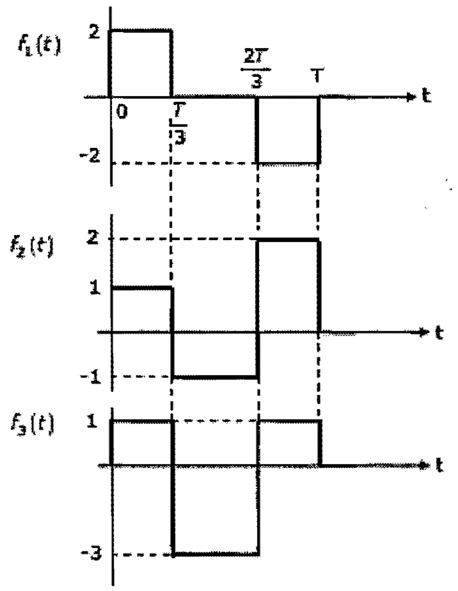


Fig. Q1c

QUESTION 2 *(25 marks)*

(a) **(i)** Find expressions for the convolution of the two signals sketched in Fig. Q2a. The convolution should be divided into clearly defined time intervals. *(12 marks)*

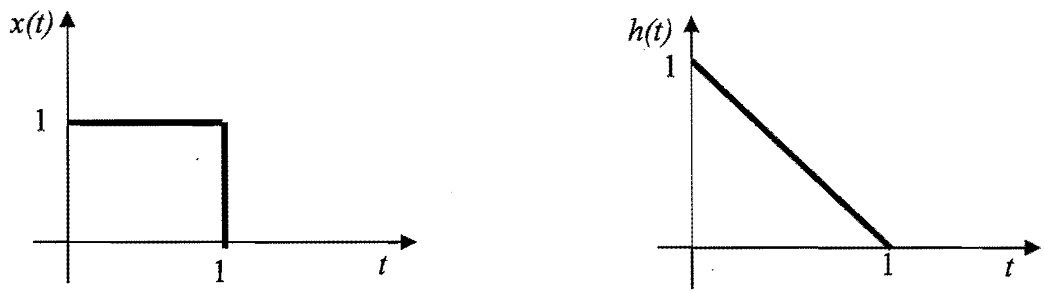


Fig. Q2a

(ii) Give a sketch of the convolution signal obtained in (i) and find the coordinates of the point where the maximum value of the convolution occurs. *(3 marks)*

(b) (i) Define the energy and the power in a signal *x(t)*. *(2 marks)*

(ii) Determine the power and energy in each of the following signals:

i. $x(t) = 5e^{-3t}, \quad t > 0$ *(4 marks)*

ii. $x(t) = 7\cos(5t + \pi/4)$ *(4 marks)*

QUESTION 3 *(25 marks)*

(a) Sketch the block diagram for the following impulse response. *(5 marks)*

$$h(t) = \left[\left[h_1(t) * h_2(t) + h_3(t)\right]\right] * h_4(t) + h_5(t)$$

(b) The classification of signals may be based on how they are represented in time and how their amplitudes are allowed to vary. Using **one cycle of a sinewave** as an example, give clear sketches of how a sine wave may be represented in the following signal sub-classes:

(i) Continuous-time and continuous-amplitude *(1 mark)*

(ii) Discrete-time and continuous-amplitude *(2 marks)*

(iii) Continuous-time and discrete-amplitude *(2 marks)*

(iv) Discrete-time and discrete-amplitude *(2 marks)*

(c) Systems can be described by the terms: linear/non-linear, time varying/time invariant, causal/non-causal.

(i) Define these three groups of terms as used in the description of systems. *(3 marks)*

(ii) By applying your definitions, derive which alternatives of the above three descriptions apply to the systems represented by the input-output signal relations below: (for example a system might be nonlinear, time invariant and causal)

i. $y(t) = x(t+1)\sin(\omega t + 1), \quad \omega > 0$ *(5 marks)*

ii. $y[n] = \left(-\frac{1}{2}\right)^n \left(x[n] + 1\right)$ *(5 marks)*

QUESTION 4 ***(25 marks)***

(a) Solve the following differential equation using Laplace Transforms. Assume all initial conditions are zero.

$$x''(t)+4x'(t)+3x(t)=4\delta(t)$$ *(10 marks)*

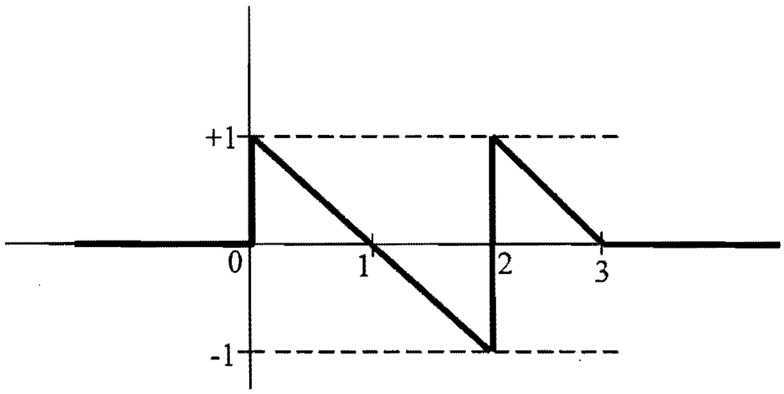
(b) Find the initial value and final value of each of the following system responses:

(i) $F(s)=\dfrac{50}{s(s^2+2s+10)}$ *(4 marks)*

(ii) $F(s)=\dfrac{(s+1)(s+2)}{(s+3)(s+4)}$ *(6 marks)*

(c) Find and sketch the even and odd components of the signal

$$x(t)=\begin{cases}-t+1, & 0<t<2\\ -t+3, & 2<t<3\\ 0, & t \text{ elsewhere}\end{cases}$$ as sketched below *(5 marks)*



QUESTION 5 *(25 marks)*

(a) A schematic diagram of a linear circuit shown in Fig.Q5a

(i) Find the Laplace domain relation $\frac{I_1(s)}{I(s)}$, for the currents $i(t)$ and $i_1(t)$, assuming that all initial current and voltage values are zero. *(8 marks)*

(ii) Hence, if $R = 6\ \Omega$, $L = 2$ H and $C = 0.25$ F, find the response $i_1(t)$ of the circuit to an impulse $i(t) = \delta(t)$. *(8 marks)*

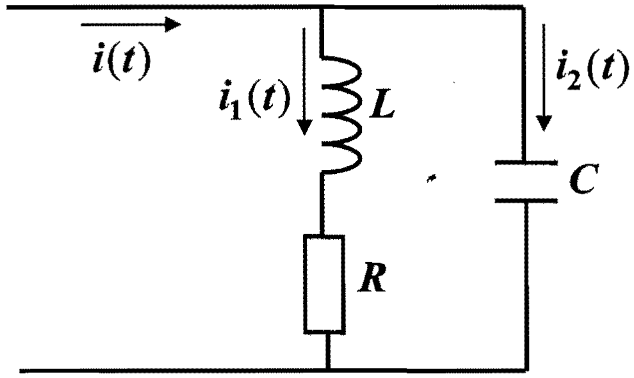


Fig.Q5a

(b) A signal $x(t)$ is sketched in Fig. Q5b. Sketch the signal given by $[x(t) + x(-t)]u(-t)$.

(9 marks)

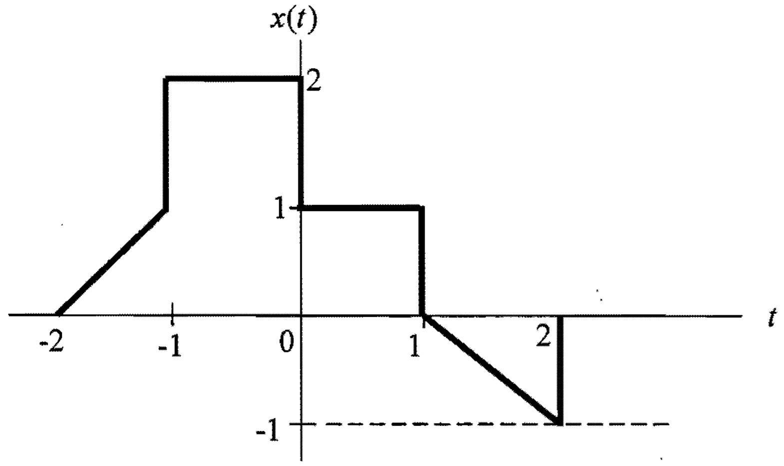


Fig. Q5b

Table of Laplace Transforms

delta function	$\delta(t)$	$\xleftrightarrow{\mathcal{L}}$	1
shifted delta function	$\delta(t-a)$	$\xleftrightarrow{\mathcal{L}}$	e^{-as}
unit step	$u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}$
ramp	$tu(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2}{s^3}$
n-th power	t^n	$\xleftrightarrow{\mathcal{L}}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\|t\|}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\xleftrightarrow{\mathcal{L}}$	$-F'(s)$
frequency n-th differentiation	$t^nf(t)$	$\xleftrightarrow{\mathcal{L}}$	$(-1)^nF^{(n)}(s)$
time differentiation	$f'(t)=\frac{d}{dt}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$sF(s)-f(0)$
time 2nd differentiation	$f''(t)=\frac{d^2}{dt^2}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^2F(s)-sf(0)-f'(0)$
time n-th differentiation	$f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau=(u*f)(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$

PROPERTIES OF LAPLACE TRANSFORMS

i) Time-shift (delay): $f(t-t_0) \xleftrightarrow{L} F(s)e^{-st_0}, \; t_0 > 0$

ii) Time differentiation: $\dfrac{df(t)}{dt} \xleftrightarrow{L} sF(s) - f(0)$

iii) Time integration: $\displaystyle\int_0^t f(t)dt \xleftrightarrow{L} \frac{F(s)}{s}$

iv) Linearity: $af(t) + bg(t) \xleftrightarrow{L} aF(s) + bF(s)$

v) Convolution Integral: $x(t) * h(t) \xleftrightarrow{L} X(s)H(s)$

vi) Frequency-shift: $e^{\alpha t} f(t) \xleftrightarrow{L} F(s-\alpha)$

vii) Multiplying by t: $tf(t) \xleftrightarrow{L} -\dfrac{dF(s)}{ds}$

viii) Scaling: $f(at) \xleftrightarrow{L} \dfrac{1}{a} F\left(\dfrac{s}{a}\right), \; a > 0$

ix) Initial Value Theorem: $\lim\limits_{s\to\infty}\{sF(s)\} = f(0)$

x) Final Value Theorem: $\lim\limits_{s\to 0}\{sF(s)\} = f(\infty)$

TABLE OF BASIC FOURIER TRANSFORM PAIRS

#	Time Domain: x(t)	Frequency Domain: X(jω)
1	$\delta(t)$	1
2	1	$2\pi\delta(\omega)$
3	$\delta(t-t_d)$	$e^{-j\omega t_d}$
4	$e^{j\omega_o t}$	$2\pi\delta(\omega-\omega_o)$
5	$e^{-at}u(t),\ (a>0)$	$\dfrac{1}{a+j\omega}$
6	$e^{bt}u(-t),\ (b>0)$	$\dfrac{1}{b-j\omega}$
7	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$
8	$u(t+\frac{1}{2}T)-u(t-\frac{1}{2}T)$	$\dfrac{\sin(\omega T/2)}{\omega/2}$
9	$\dfrac{\sin(\omega_b t)}{\pi t}$	$u(\omega+\omega_b)-u(\omega-\omega_b)$
10	$A\cos(\omega_o t+\phi)$	$\pi Ae^{j\phi}\delta(\omega-\omega_o)+\pi Ae^{-j\phi}\delta(\omega+\omega_o)$
11	$\cos(\omega_o t)$	$\pi\delta(\omega-\omega_o)+\pi\delta(\omega+\omega_o)$
12	$\sin(\omega_o t)$	$-j\pi\delta(\omega-\omega_o)+j\pi\delta(\omega+\omega_o)$
13	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$	$\sum_{k=-\infty}^{\infty} 2\pi a_k\delta(\omega-k\omega_o)$
14	$\sum_{k=-\infty}^{\infty} \delta(t-nT)$	$\dfrac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta(\omega-\dfrac{2\pi}{T}k)$

TABLE OF BASIC FOURIER TRANSFORM PROPERTIES

#	PROPERTY NAME	TIME DOMAIN: x(t)	FREQUENCY DOMAIN: X(jω)
1	Linearity	$ax_1(t)+bx_2(t)$	$aX_1(j\omega)+bX_2(j\omega)$
2	Conjugation	$x^*(t)$	$X^*(-j\omega)$
3	Time-Reversal	$x(-t)$	$X(-j\omega)$
4	Time Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X(j\frac{\omega}{a})$
5	Time Delay	$x(t-t_d)$	$e^{-j\omega t_d}X(j\omega)$
6	Modulation	$x(t)e^{j\omega_o t}$	$X[j(\omega-\omega_o)]$
7	Modulation	$x(t)\cos(\omega_o t)$	$\frac{1}{2}X[j(\omega-\omega_o)]\ +\ \frac{1}{2}X[j(\omega+\omega_o)]$
8	Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j\omega)^n X(j\omega)$
9	Convolution	$x(t)*h(t)$	$X(j\omega)H(j\omega)$
10	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega)*Y(j\omega)$