# Faculty of Science
# Department of Electrical and Electronic Engineering
# Main Examination 2015

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**Title of Paper** : **Signals and Systems II**

**University of Swaziland**

**Course Number** : **EE332**

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**Time Allowed** : **3 hrs**

**Instructions** :

1. **Answer any four (4) questions**
2. **Each question carries 25 marks**
3. **Useful information is attached at the end of the question paper**
4. **Semi-log paper is attached to the question paper**

**THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR**

**The paper consists of nine (9) pages**

## Question 1 [25]

a) Define the following terms: [5]

i) Transient response
ii) Rise time
iii) Asymptotic stability
iv) Steady – state error
v) High pass filter

b) Find the Bode log magnitude and phase angle plot for the transfer function: [12]

$$TF = \frac{200(s+20)}{s(2s+1)(s+40)}$$

c) Given: [8]

$$x(t) = 30\cos\left(2\pi \cdot f_o \cdot t - \frac{\pi}{2}\right) + B\cos(2\pi \cdot f_0 \cdot t + \varphi) + 60\sin(2\pi \cdot f_0 . t + \frac{\pi}{4})$$

**And**

$$x(t) = 50\cos(2\pi \cdot f_0 \cdot t + \frac{\pi}{4})$$

Find $X_B = Be^{j\varphi}$.

## Question 2 [25]

a) If the input voltage, $e_{in}(t)$, of the following system ***(figure 2.1)*** is a unit step, and $e_{out}(t)$, is the output voltage

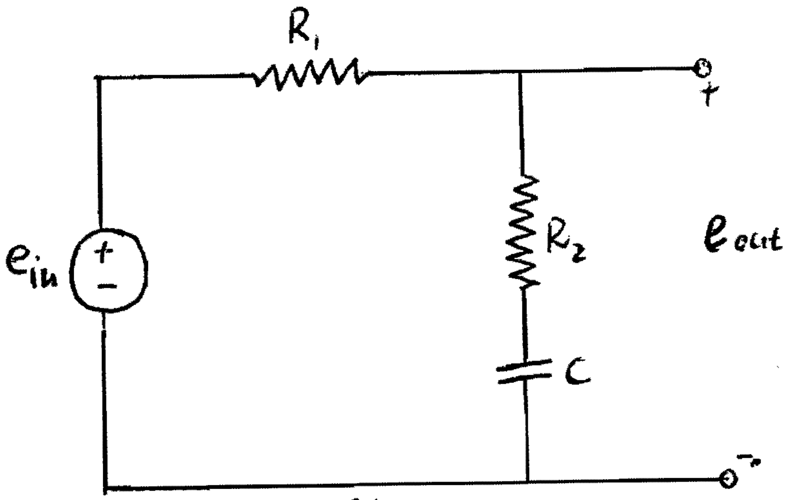


figure 2.1

i) Find the transfer function of the system. The transfer function should be in terms of $R_1, R_2, and\ C.$ [6]

ii) Find the unit step response. [6]

iii) Given that $R_1, = R_2 = 1K\Omega$ and $C = 1\mu F$, find $e_{out}(t)$ [2]

b) The closed – loop transfer function, T(s), is given by:

$$\boldsymbol{T(s) = \frac{C(s)}{R(s)} = \frac{5000}{s(s+75)+5000}}$$

i) Calculate the damping ratio, $\zeta$. [4]

ii) Calculate the undamped natural frequency, $\omega_n$ [3]

iii) Calculate the maximum overshoot percentage [2]

iv) Calculate 5% setting time [2]

## Question 3 [25]

a) Determine the fundamental frequency of the following signal, and plot its sinusoidal spectra (both magnitude and phase) and it exponential spectra. [10]

$$x(t) = 2 + 3\cos(0.2t) + \cos\left(0.25t + \frac{\pi}{2}\right) + 4\cos(0.3t - \pi)$$

b) Using ***figure 3.1***, find an expression for *x(t)* and plot the spectrum. [6]

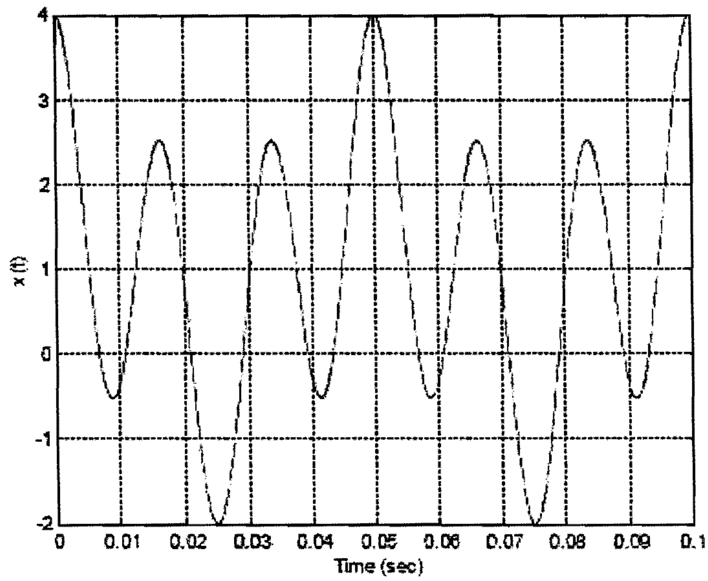


***figure 3.1***

c) A ***1.5 MHz*** carrier is amplitude modulated by 2 sinusoidal signals of frequency: ***500Hz*** and ***800Hz***. What are the frequencies in the AM spectrum? [2]

d) A transmitter puts out a total power of ***25*** Watts of ***30%*** AM signal :

i) How much power is contained in the carrier? [4]

ii) How much power is contained in each of the sideband? [3]

**Question 4 [25]**

a) Calculate the inverse Fourier transform of the following signals: [12]

$$X(j\omega) = \frac{i}{j}[\delta(\omega - 1) - \delta(\omega + 1)] + 3[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$$

b) Obtain the *z-transform* of the transfer function *F(z)*: [13]

i) $F(z) = \frac{z+1}{z^2+0.2z+0.1}$

ii) $F(z) = \frac{z+1}{z^2+0.3z+0.02}$

## Question 5 [25]

a) Draw the block diagram of the discrete-time system represented by the recurrence equation:

i) $y(n) = x(n-1) - 4y(n-1) + 7y(n-2) + 3y(n-3)$ [3]

b) Find the z-transform of the following sequence:

i) $x(n) = 10\sin(0.25\pi n)\,u(n)$ [3]

c) A digital signal processing (DSP) system is described by the following differential equation with zero initial condition:

$$y(n) + 0.1y(n-1) - 0.2y(n-2) = x(n) + x(n-1)$$

i) Determine system response $y(n)$ due to the unit step function excitation, where $u(n) = 1$ for $n \geq 0$. [12]

d) Consider the analog signal

$$x(t) = 3\cos(2000\pi t) + 5\sin(6000\pi t) + 10\cos(12000\pi t)$$

i) What is the Nyquist rate for this signal? [1]

ii) Suppose that the signal is sampled at the rate of $f_s = 5000$ samples per second, what is the discrete-time signal obtained after sampling? [3]

iii) What is the analog signal we can reconstruct from the samples? [3]

## Properties of Laplace Transforms

Time-shift (delay): $f(t-t_0) \xleftrightarrow{L} F(s)e^{-st_0}$, $t_0 > 0$

Time differentiation: $\frac{df(t)}{dt} \xleftrightarrow{L} sF(s) - f(0)$

Time integration: $\int_0^t f(t)dt \xleftrightarrow{L} \frac{F(s)}{s}$

Linearity: $af(t) + bg(t) \xleftrightarrow{L} aF(s) + bF(s)$

Convolution Integral: $x(t) * h(t) \xleftrightarrow{L} X(s)H(s)$

Frequency-shift: $e^{\alpha t} f(t) \xleftrightarrow{L} F(s-\alpha)$

Multiplying by $t$: $tf(t) \xleftrightarrow{L} -\frac{dF(s)}{ds}$

Scaling: $f(at) \xleftrightarrow{L} \frac{1}{a} F\left(\frac{s)}{a}\right)$, $a > 0$

Initial Value Theorem: $\lim_{s \to \infty} \{sF(s)\} = f(0)$

Final Value Theorem: $\lim_{s \to 0} \{sF(s)\} = \lim_{t \to \infty} f(t)$

<u>Table of Z-Transforms</u>

| Line No. | $x(n),\ n \geq 0$ | z-Transform $X(z)$ | Region of Convergence |
|---|---|---|---|
| 1 | $x(n)$ | $\sum_{n=0}^{\infty} x(n)z^{-n}$ | |
| 2 | $\delta(n)$ | $1$ | $\lvert z\rvert > 0$ |
| 3 | $au(n)$ | $\frac{az}{z-1}$ | $\lvert z\rvert > 1$ |
| 4 | $nu(n)$ | $\frac{z}{(z-1)^2}$ | $\lvert z\rvert > 1$ |
| 5 | $n^2u(n)$ | $\frac{z(z+1)}{(z-1)^3}$ | $\lvert z\rvert > 1$ |
| 6 | $a^n u(n)$ | $\frac{z}{z-a}$ | $\lvert z\rvert > \lvert a\rvert$ |
| 7 | $e^{-an}u(n)$ | $\frac{z}{(z-e^{-a})}$ | $\lvert z\rvert > e^{-a}$ |
| 8 | $na^n u(n)$ | $\frac{az}{(z-a)^2}$ | $\lvert z\rvert > \lvert a\rvert$ |
| 9 | $\sin(an)u(n)$ | $\frac{z\sin(a)}{z^2-2z\cos(a)+1}$ | $\lvert z\rvert > 1$ |
| 10 | $\cos(an)u(n)$ | $\frac{z[z-\cos(a)]}{z^2-2z\cos(a)+1}$ | $\lvert z\rvert > 1$ |
| 11 | $a^n\sin(bn)u(n)$ | $\frac{[a\sin(b)]z}{z^2-[2a\cos(b)]z+a^2}$ | $\lvert z\rvert > \lvert a\rvert$ |
| 12 | $a^n\cos(bn)u(n)$ | $\frac{z[z-a\cos(b)]}{z^2-[2a\cos(b)]z+a^{-2}}$ | $\lvert z\rvert > \lvert a\rvert$ |
| 13 | $e^{-an}\sin(bn)u(n)$ | $\frac{[e^{-a}\sin(b)]z}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$ | $\lvert z\rvert > e^{-a}$ |
| 14 | $e^{-an}\cos(bn)u(n)$ | $\frac{z[z-e^{-a}\cos(b)]}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$ | $\lvert z\rvert > e^{-a}$ |

<u>Properties of Z-Transforms</u>

Linearity: $ax_1[k]+bx_2[k] \Leftrightarrow aX_1(z)+bX_2(z)$

Time Reversal: $x[-k] \Leftrightarrow X(1/z)$

Summation: $\sum_{n=-\infty}^{k} x[n] \Leftrightarrow \frac{zX(z)}{z-1}$

Initial Value: $x[0] = \lim_{z\to\infty} X(z)$

Final Value: $x[\infty] = \lim_{z\to 1}(z-1)X(z)$

Convolution: $x[k]*h[k] \Leftrightarrow X(z)H(z)$

Differencing: $x[k]-x[k-1] \Leftrightarrow \left(1-z^{-1}\right)X(z)$

Differentiation: $-kx[k] \Leftrightarrow z\frac{d}{dz}X(z)$

Time Shifting: $x[n-n_o] \Leftrightarrow z^{-n_o}X(z), n_o \geq 0$

$$x[n+n_o] \Leftrightarrow z^{n_o}\left(X(z)-\sum_{m=0}^{n_o-1} x[m]z^{-m}\right), n_o \geq 0$$

# Table of Laplace Transforms

| | | | |
|---|---|---|---|
| delta function | $\delta(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $1$ |
| shifted delta function | $\delta(t-a)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $e^{-as}$ |
| unit step | $u(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{1}{s}$ |
| ramp | $tu(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{1}{s^2}$ |
| parabola | $t^2u(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{2}{s^3}$ |
| $n$-th power | $t^n$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{n!}{s^{n+1}}$ |
| exponential decay | $e^{-at}$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{1}{s+a}$ |
| two-sided exponential decay | $e^{-a\|t\|}$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{2a}{a^2-s^2}$ |
| | $te^{-at}$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{1}{(s+a)^2}$ |
| | $(1-at)e^{-at}$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{s}{(s+a)^2}$ |
| exponential approach | $1-e^{-at}$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{a}{s(s+a)}$ |
| sine | $\sin(\omega t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{\omega}{s^2+\omega^2}$ |
| cosine | $\cos(\omega t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{s}{s^2+\omega^2}$ |
| hyperbolic sine | $\sinh(\omega t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{\omega}{s^2-\omega^2}$ |
| hyperbolic cosine | $\cosh(\omega t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{s}{s^2-\omega^2}$ |
| exponentially decaying sine | $e^{-at}\sin(\omega t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{\omega}{(s+a)^2+\omega^2}$ |
| exponentially decaying cosine | $e^{-at}\cos(\omega t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{s+a}{(s+a)^2+\omega^2}$ |
| frequency differentiation | $tf(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $-F'(s)$ |
| frequency $n$-th differentiation | $t^nf(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $(-1)^nF^{(n)}(s)$ |
| time differentiation | $f'(t)=\frac{d}{dt}f(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $sF(s)-f(0)$ |
| time 2nd differentiation | $f''(t)=\frac{d^2}{dt^2}f(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $s^2F(s)-sf(0)-f'(0)$ |
| time $n$-th differentiation | $f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$ |
| time integration | $\int_0^t f(\tau)d\tau=(u*f)(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{1}{s}F(s)$ |
| frequency integration | $\frac{1}{t}f(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\int_s^\infty F(u)du$ |
| time inverse | $f^{-1}(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{F(s)-f^{-1}}{s}$ |
| time differentiation | $f^{-n}(t)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$ |

TITLE

| NAME | DATE |
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