UNIVERSITY OF SWAZILAND
Faculty of Science and Engineering
Department of Electrical and Electronic Engineering
Supplementary Examination 2015

Title of Paper	**:**	**Signals and Systems II**
Course Code	**:**	**EE332**
Time Allowed	**:**	**3 hrs**

Instructions:

1. Answer **all** four (4) questions
2. Each question carries 25 marks
3. Useful information and special graph paper are attached at the end of the question paper

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The paper consists of eight (8) pages including the cover page

Question 1 [25]

a) Find the Bode log magnitude plot for the transfer function: [6]

$$TF = \frac{200(s+20)}{s(2s+1)(s+40)}$$

b) Determine the fundamental frequency of the following signal, and plot its sinusoidal spectra (both magnitude and phase) and it exponential spectra. [10]

$$x(t) = 2 + 3\cos(0.2t) + \cos\left(0.25t + \frac{\pi}{2}\right) + 4\cos(0.3t - \pi)$$

c) A *1.5 MHz* carrier is amplitude modulated by 2 sinusoidal signals of frequency: ***500Hz*** and ***800Hz***. What are the frequencies in the AM spectrum? [2]

d) A transmitter puts out a total power of ***25*** Watts of ***30%*** AM signal :

i) How much power is contained in the carrier? [4]

ii) How much power is contained in each of the sideband? [3]

Question 2 [25]

a) Draw the block diagram of the discrete-time system represented by the recurrence equation: [3]

i) $y(n) = x(n-1) - 4y(n-1) + 7y(n-2) + 3y(n-3)$

b) Find the z-transform of the following sequence: [3]

i) $x(n) = 10\sin(0.25\pi n)\,u(n)$

c) A digital signal processing (DSP) system is described by the following differential equation with zero initial condition:

$$y(n) + 0.1y(n-1) - 0.2y(n-2) = x(n) + x(n-1)$$

i) Determine system response $y(n)$ due to the unit step function excitation, where $u(n) = 1$ for $n \geq 0$. [12]

d) Consider the analog signal

$$x(t) = 3\cos(2000\pi t) + 5\sin(6000\pi t) + 10\cos(12000\pi t)$$

i) What is the Nyquist rate for this signal? [1]

ii) Suppose that the signal is sampled at the rate of $f_s = 5000$ samples per second, what is the discrete-time signal obtained after sampling? [3]

iii) What is the analog signal we can reconstruct from the samples? [3]

Question 3 [25]

a) If the input voltage, $e_{in}(t)$, of the following system (***figure3.1***) is a unit step, and $e_{out}(t)$, is the output voltage

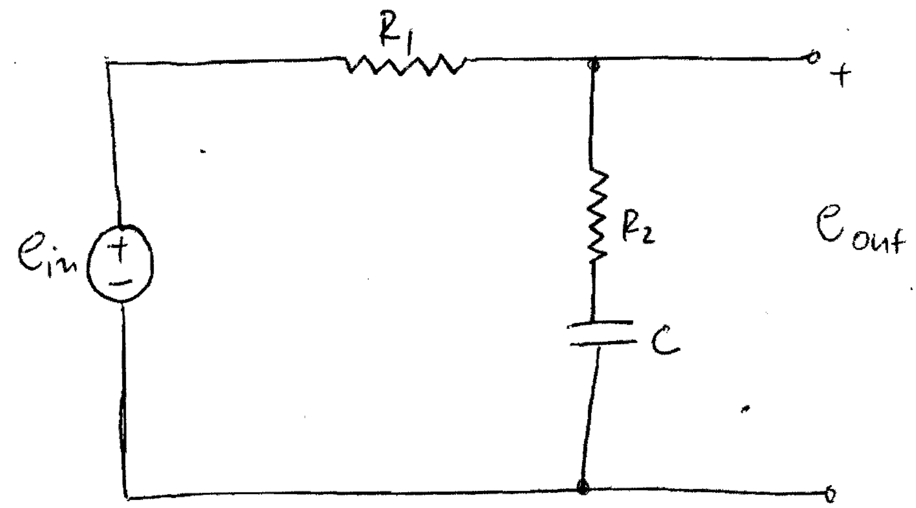

figure 3.1

i) Find the transfer function of the system. The transfer function should be in terms of $R_1, R_2, and\ C$. [6]

ii) Find the unit step response. [6]

iii) Given that $R_1, = R_2 = 1K\Omega$ and $C = 1\mu F$, find $e_{out}(t)$ [2]

b) The closed – loop transfer function, T(s), is given by:

$$\boldsymbol{T(s) = \frac{C(s)}{R(s)} = \frac{5000}{s(s+75)+5000}}$$

i) Calculate the damping ratio, ζ. [4]

ii) Calculate the undamped natural frequency, ω_n [3]

iii) Calculate the maximum overshoot percentage [2]

iv) Calculate 5% setting time [2]

Question 4 [25]

a) Determine if the following systems are time-invariant, linear, causal and/or memoryless? [6]

i) $\frac{dy}{dt} + 6y(t) = 4x(t)$

ii) $y[n] + 2y[n-1] = x[n+1]$

b) Solve the following differential equation using the Laplace Transform method: [5]

i) $\frac{dy}{dt} + 4y(t) = 3x(t): x(t) = \sin(2t), y(0) = 1$

c) Compute the inverse Laplace Transform of the following functions: [3]

i) $X(s) = \frac{2s+100}{(s+1)(s+8)(s+10)}$

d) Find the inverse Z-transforms of the following signal: [4]

i) $X(z) = \frac{(z-1)(z+0.8)}{(z-0.5)(z+0.5)}$

e) Determine the stability of the following systems: [2]

i) $H(s) = \frac{s+2}{(s+3)(s+2)}$

f) Sketch the response of the system below to a step input. [5]

i) $H(s) = \frac{10}{s+2}$

Table of Laplace Transforms

delta function	$\delta(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	1
shifted delta function	$\delta(t-a)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	e^{-as}
unit step	$u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}$
ramp	$tu(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2}{s^3}$
n-th power	t^n	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\|t\|}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$-F'(s)$
frequency n-th differentiation	$t^nf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$(-1)^nF^{(n)}(s)$
time differentiation	$f'(t)=\frac{d}{dt}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$sF(s)-f(0)$
time 2nd differentiation	$f''(t)=\frac{d^2}{dt^2}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^2F(s)-sf(0)-f'(0)$
time n-th differentiation	$f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau=(u*f)(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$

Properties of Laplace Transforms

Time-shift (delay): $f(t-t_0) \xleftrightarrow{L} F(s)e^{-st_0}, \quad t_0 > 0$

Time differentiation: $\frac{df(t)}{dt} \xleftrightarrow{L} sF(s) - f(0)$

Time integration: $\int_0^t f(t)dt \xleftrightarrow{L} \frac{F(s)}{s}$

Linearity: $af(t) + bg(t) \xleftrightarrow{L} aF(s) + bF(s)$

Convolution Integral: $x(t) * h(t) \xleftrightarrow{L} X(s)H(s)$

Frequency-shift: $e^{\alpha t} f(t) \xleftrightarrow{L} F(s-\alpha)$

Multiplying by t: $tf(t) \xleftrightarrow{L} -\frac{dF(s)}{ds}$

Scaling: $f(at) \xleftrightarrow{L} \frac{1}{a}F\left(\frac{s}{a}\right), \quad a > 0$

Initial Value Theorem: $\lim_{s\to\infty}\{sF(s)\} = f(0)$

Final Value Theorem: $\lim_{s\to 0}\{sF(s)\} = \lim_{t\to\infty} f(t)$

Table of Z-Transforms

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n)z^{-n}$	
2	$\delta(n)$	1	$\|z\| > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\|z\| > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\|z\| > 1$
5	$n^2u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\|z\| > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$\|z\| > \|a\|$
7	$e^{-an}u(n)$	$\frac{z}{(z-e^{-a})}$	$\|z\| > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\|z\| > \|a\|$
9	$\sin(an)u(n)$	$\frac{z\sin(a)}{z^2 - 2z\cos(a) + 1}$	$\|z\| > 1$
10	$\cos(an)u(n)$	$\frac{z[z-\cos(a)]}{z^2 - 2z\cos(a) + 1}$	$\|z\| > 1$
11	$a^n \sin(bn)u(n)$	$\frac{[a\sin(b)]z}{z^2 - [2a\cos(b)]z + a^2}$	$\|z\| > \|a\|$
12	$a^n \cos(bn)u(n)$	$\frac{z[z - a\cos(b)]}{z^2 - [2a\cos(b)]z + a^{-2}}$	$\|z\| > \|a\|$
13	$e^{-an}\sin(bn)u(n)$	$\frac{[e^{-a}\sin(b)]z}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\|z\| > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\frac{z[z - e^{-a}\cos(b)]}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\|z\| > e^{-a}$

Properties of Z-Transforms

Linearity: $ax_1[k] + bx_2[k] \Leftrightarrow aX_1(z) + bX_2(z)$

Time Reversal: $x[-k] \Leftrightarrow X(1/z)$

Summation: $\sum_{n=-\infty}^{k} x[n] \Leftrightarrow \frac{zX(z)}{z-1}$

Initial Value: $x[0] = \lim_{z\to\infty} X(z)$

Final Value: $x[\infty] = \lim_{z\to 1}(z-1)X(z)$

Convolution: $x[k] * h[k] \Leftrightarrow X(z)H(z)$

Differencing: $x[k] - x[k-1] \Leftrightarrow \left(1 - z^{-1}\right)X(z)$

Differentiation: $-kx[k] \Leftrightarrow z\frac{d}{dz}X(z)$

Time Shifting: $x[n-n_o] \Leftrightarrow z^{-n_o}X(z), n_o \geq 0$

$$x[n+n_o] \Leftrightarrow z^{n_o}\left(X(z) - \sum_{m=0}^{n_o-1} x[m]z^{-m}\right), n_o \geq 0$$

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