UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

Department of Electronic and Electrical Engineering

MAIN EXAMINATION 2014

Title of the Paper:

Electromagnetic Fields I

Course Number: **EE341**
Time Allowed: **Three Hours.**

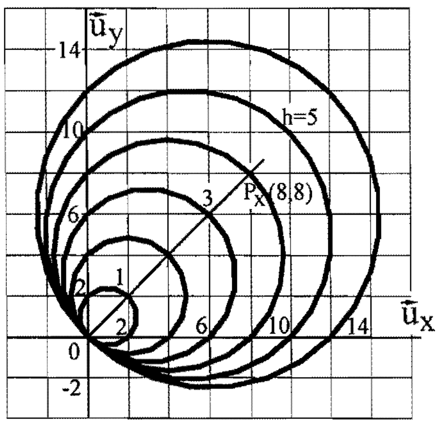
Instructions:

1. To answer, pick any to sum a total of 100% from 12 questions in the following pages.
2. The answer is better written in the space provided in the question book. Use the answer book as a scratch pad.
3. Mark big X for not picked questions; otherwise, it is up to the grading person to pick valid questions.
4. This paper has 9 pages, including this page.

DO NOT OPEN THE PAPER
UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Q1, 10 pts: Given a scalar function $f(x,y,z) = x^2 \cdot y$, find (i) $\int f \cdot d\vec{l}$ and (ii) $\int f \cdot dl$ along a straight line from (1,1,0) to (0,0,0).

Q2, 10 pts: Given a scalar function, $h(x,y) = (x^2 + y^2)/2(x+y)$, the height of a slanted cone shown in Fig. Q2-1, (i) calculate graphically the maximum change (gradient) of the height at the location $P_x(8,8)$ and the direction of the change; (ii) calculate the same but analytically. Check if the two answers are close. (5 pts each. In the 5, 3 pts for the direction part)



h(x,y)=(x²+y²)/2(x+y)
h-axis out of the paper
contour (constant height, "h")
of a slant cone.

Fig. Q2-1

Q3, 10 pts: Given the field patterns shown in Fig. Q3-1, which are in xy-plane only and no contribution in z-axis, by inspection determine and mark the area which has curl≠0 or div≠0 or both≠0 of the pattern. Then analytically calculate the non-zero curl or divergence to prove. Take closed surface anywhere in the pattern but must be marked or specified. The closed surface may be a square or a circle.

$\mathbf{A} = \hat{\mathbf{x}}\, xy^2 + \hat{\mathbf{y}}\, x^2 y,$

for $-10 \leq x, y \leq 10$

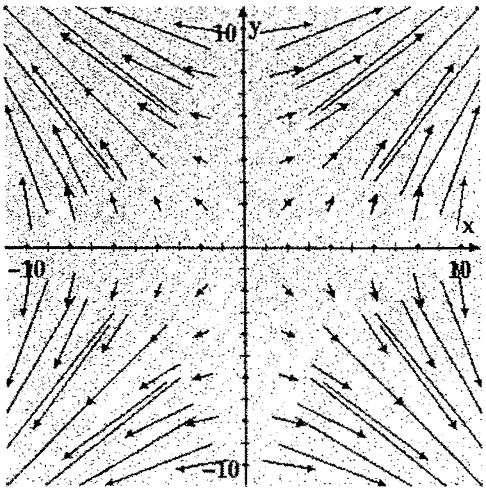


Fig. Q3-1

Q4, 10 pts: List any five pairs of dual equation in electromagnetic fields.

term	Electric Fields	Magnetic Fields

Q5, 10 pts: A coaxial cable has an inner radius r_i and outer radius r_o with insulation material ε/μ_o. Consider no end fringing effects. (i) Find the total electric energy stored in this 1 meter long cable, energized by a source charge q_l Coul/Mtr. (ii) Find the total magnetic energy stored in this 1 meter long cable, energized by a source current I_s.

Q6, 10 pts: An infinitely long line charge with a line density $+q_l$ Coul/Mtr is located d Mtr above an infinitive perfect conducting plane. Find the charge density on the plane. Use the image method. Is there any dual method in static magnetic fields and give the reason behind? (6 pts for the first question, 4 pts for the second).

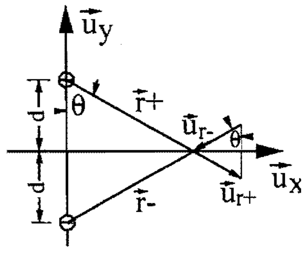


Fig. Q6-1

Q7, 10 pts: A coaxial cable has an inner radius r_i and outer radius r_o with insulation material ε/μ_o. Consider no end fringing effects. (i) Calculate the cable per unit inductance and capacitance. (ii) the Characteristic impedance z_o. (4 pts for each answer in (i) and 2 pts for (ii))

Q8, 10 pts: An electric dipole has a dipole moment $\vec{p}$ and its direction in z-axis as shown in Fig. Q8-1. The potential produced by the dipole is given: $V = \dfrac{\vec{p} \circ \vec{u}_r}{4\pi\varepsilon_0 \cdot r^2}$, where p=qd. (i). Find the electric field "E" of the dipole. (ii). Through the dual principle, give directly the magnetic field "B" equation and depict the geometry of the magnetic dipole "m" and definition of "m".

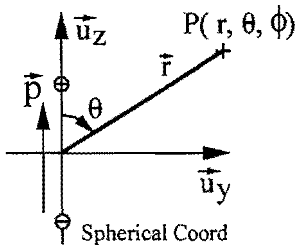


Fig. Q8-1

Q9, 10 pts: Prove (i) which equation or law in electric fields will degenerate into Kirchhoff's Voltage Law, specifying the necessary conditions; (ii) which will degenerate into Kirchhoff's Current Law likewise.

Q10, 10 pts (5 pts for each): (i) Show that if no surface current densities exist at the parallel interfaces shown in Fig. Q10-1, the relationship between θ_4 and θ_1 is independent of μ_2. (ii) Show the same for independent of ε_2 for electric fields if no surface charge densities exist like- wise.

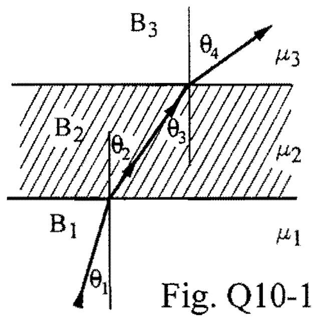


Fig. Q10-1

Q11, 20pts: A square current coil of sides 2-Mtr. carries a current I. Determine the vector potential of this coil at the point on its axis and z_0 meters away from the coil plane.

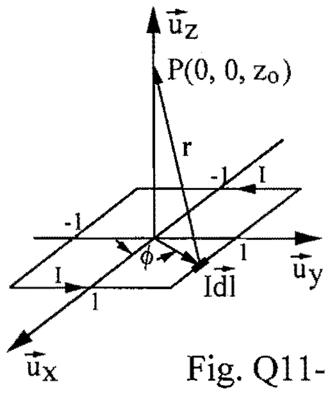


Fig. Q11-1

Q12, 20 pts: In Fig. Q12-1 is shown a magnetic circuit has 2 windings and 2 air gaps with a core permeability $\mu \rightarrow \infty$. The two air gaps size is shown in the figure and the cross sectional areas of the two gaps are respectively A_1 and A_2. The current through N_1 is i_1, through N_2 is i_2. Find (i), the self-inductances of coil 1 and 2, and (ii), the two mutual inductances between the two coils.

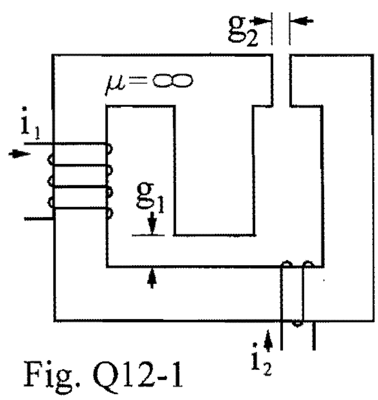


Fig. Q12-1