University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering
Main Examination 2015

Title of Paper : **Introduction to Digital Signal Processing**

Course Number : **EE443**

Time Allowed : **3 hrs**

Instructions :

1. **Answer any four (4) questions**
2. **Each question carries 25 marks**
3. **Useful information is attached at the end of the question paper**

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

The paper consists of seven (7) pages

Question 1

(a) A processor is described by the linear difference equation

$$y[n] = x[n] + 4x[n-2]$$

Find the transfer function of the processor and hence write down the unit sample response sequence of the processor. [6]

(b) A unit-step function $u(t)$ is sampled every T seconds. Find the z-transform of the resulting sample sequence, assuming that the value of the unit step at time $t = 0$ is 1. What is the z-transform of the sampled unit step delayed by T seconds? [4]

(c) A voltage signal is modelled as the sinusoid

$$v(t) = 5\cos(3t + 0.5)$$

Express the signal in terms of the exponential frequency components and sketch the frequency-domain representation of the signal. [6]

(d) List any two (2) digital signal processing applications. [2]

(e) Find the z-transform of the following:

(i) $x(n) = 10\,u(n)$ [2]

(ii) $x(n) = (0.5)^n \sin(0.25\pi n)\,u(n)$ [5]

Question 2

(a) Given two sequences

$$x_1(n) = 5\delta(n) - 2\delta(n-2) \text{ and}$$
$$x_2(n) = 3\delta(n-3),$$

(i) Determine the z-transform of convolution of the two sequences using the convolution property of z-transform. [4]

(ii) Determine the convolution by the inverse z-transform from the result in (i). [4]

(b) Find the inverse of the following functions:

(i) $X(z) = 4 - \frac{10z}{z-1} - \frac{z}{z+0.5}$ [3]

(ii) $X(z) = \frac{-5z}{z-1} + \frac{10z}{(z-1)^2} + \frac{2z}{(z-0.8)^2}$ [3]

(c) Convert the following transfer function into its difference equation.

$$H(z) = \frac{z^2 - 1}{z^2 + 1.3z + 0.36}$$

[6]

(d) Draw the functional block of the digital signal processing scheme and state the function of the last block in the diagram. [5]

Question 3

(a) Given the following difference equation,

$$y(n) = 0.5\,x(n) + 0.5\,x(n-1)$$

(i) Find H(z) [2]

(ii) Determine the impulse response $y(n)$ if the input is $x(n) = 4\delta(n)$; [3]

(b) Given the filter

$$H(z) = \frac{1 - 0.9z^{-1} - 0.1z^{-2}}{1 + 0.3z^{-1} - 0.04z^{-2}},$$

Realize $H(z)$ and develop the difference equation using the following form

(i) Direct-form I [5]

(ii) Direct-form II [5]

(iii) Cascade (series) form via first order-sections [5]

(iv) Parallel form via first oder-sections [5]

Question 4

(a) Design a 5-tap FIR band reject filter with a lower cut-off frequency of 2 kHz, and an upper cut-off frequency of 2.4 kHz, and a sampling rate of 8 kHz using the Hamming window method. Then, determine the transfer function. [18]

(b) Design a bandpass FIR filter with the following specifications: [7]

Lower stopband = 0 - 500 Hz
Passband = 1600 - 2300 Hz
Upper stopband = 3500 - 4000Hz
Stopband attenuation = 50 dB
Passband ripple = 0.05 dB

Sampling rate = 8000 Hz

Simply show the first five (5) FIR coefficients using the Hamming window.

Question 5

(a) Design a digital lowpass Butterworth filter with the following specifications: [20]

- 3 dB attenuation at the passband frequency of 1.5 kHz
- 10 dB stopband attenuation at the frequency of 3 kHz
- Sampling frequency of 8000 Hz

(b) A digital system is described by the following difference equation:

$$y(n) = x(n) - 0.25\,x(n-2) - 1.1y(n-1) - 0.28y(n-2)$$

Find the transfer function $H(z)$, the denominator polynomial $A(z)$and the numerator polynomial $B(z)$. [5]

Table 1: Properties of z-transform

Property	Time Domain	z-Transform
Linearity	$ax_1(n)+bx_2(n)$	$aZ(x_1(n))+bZ(x_2(n))$
Shift theorem	$x(n-m)$	$z^{-m}X(z)$
Linear convolution	$x_1(n)*x_2(n)=\sum_{k=0}^{\infty}x_1(n-k)x_2(k)$	$X_1(z)X_2(z)$

Table 2: Partial fraction(s) and formulas for constant(s).

Partial fraction with the first-order real pole:

$$\frac{R}{z-p} \qquad R=(z-p)\frac{X(z)}{z}\bigg|_{z=p}$$

Partial fraction with the first-order complex poles:

$$\frac{Az}{(z-P)}+\frac{A^*z}{(z-P^*)} \qquad A=(z-P)\frac{X(z)}{z}\bigg|_{z=P}$$

P^* = complex conjugate of P

A^* = complex conjugate of A

Partial fraction with mth-order real poles:

$$\frac{R_m}{(z-p)}+\frac{R_{m-1}}{(z-p)^2}+\cdots+\frac{R_1}{(z-p)^m} \qquad R_k=\frac{1}{(k-1)!}\frac{d^{k-1}}{dz^{k-1}}\left((z-p)^m\frac{X(z)}{z}\right)\bigg|_{z=p}$$

Table 3: Summary of ideal impulse r esponses for standard FIR filters.

Filter Type	Ideal Impulse Response $h(n)$ (noncausal FIR coefficients)
Lowpass:	$h(n)=\begin{cases}\frac{\Omega_c}{\pi} & n=0\\ \frac{\sin(\Omega_c n)}{n\pi}\ \text{for } n\neq 0 & -M\le n\le M\end{cases}$
Highpass:	$h(n)=\begin{cases}\frac{\pi-\Omega_c}{\pi} & n=0\\ -\frac{\sin(\Omega_c n)}{n\pi}\ \text{for } n\neq 0 & -M\le n\le M\end{cases}$
Bandpass:	$h(n)=\begin{cases}\frac{\Omega_H-\Omega_L}{\pi} & n=0\\ \frac{\sin(\Omega_H n)}{n\pi}-\frac{\sin(\Omega_L n)}{n\pi}\ \text{for } n\neq 0 & -M\le n\le M\end{cases}$
Bandstop:	$h(n)=\begin{cases}\frac{\pi-\Omega_H+\Omega_L}{\pi} & n=0\\ -\frac{\sin(\Omega_H n)}{n\pi}+\frac{\sin(\Omega_L n)}{n\pi}\ \text{for } n\neq 0 & -M\le n\le M\end{cases}$

Causal FIR filter coefficients: shifting $h(n)$ to the right by M samples.

Transfer function:

$$H(z)=b_0+b_1z^{-1}+b_2z^{-2}+\cdots b_{2M}z^{-2M}$$

where $b_n=h(n-M)$, $n=0,1,\cdots,2M$

Table 4: The Z-transform

Line No.	$x(n),\ n\geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n)z^{-n}$	
2	$\delta(n)$	1	$\lvert z\rvert > 0$
3	$au(n)$	$\dfrac{az}{z-1}$	$\lvert z\rvert > 1$
4	$nu(n)$	$\dfrac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
5	$n^2u(n)$	$\dfrac{z(z+1)}{(z-1)^3}$	$\lvert z\rvert > 1$
6	$a^n u(n)$	$\dfrac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
7	$e^{-na}u(n)$	$\dfrac{z}{(z-e^{-a})}$	$\lvert z\rvert > e^{-a}$
8	$na^n u(n)$	$\dfrac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
9	$\sin(an)u(n)$	$\dfrac{z\sin(a)}{z^2-2z\cos(a)+1}$	$\lvert z\rvert > 1$
10	$\cos(an)u(n)$	$\dfrac{z[z-\cos(a)]}{z^2-2z\cos(a)+1}$	$\lvert z\rvert > 1$
11	$a^n\sin(bn)u(n)$	$\dfrac{[a\sin(b)]z}{z^2-[2a\cos(b)]z+a^2}$	$\lvert z\rvert > \lvert a\rvert$
12	$a^n\cos(bn)u(n)$	$\dfrac{z[z-a\cos(b)]}{z^2-[2a\cos(b)]z+a^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
13	$e^{-an}\sin(bn)u(n)$	$\dfrac{[e^{-a}\sin(b)]z}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\lvert z\rvert > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\dfrac{z[z-e^{-a}\cos(b)]}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\lvert z\rvert > e^{-a}$
15	$2\lvert A\rvert\lvert P\rvert^n\cos(n\theta+\phi)u(n)$ where P and A are complex constants defined by $P=\lvert P\rvert\angle\theta, A=\lvert A\rvert\angle\phi$	$\dfrac{Az}{z-P}+\dfrac{A^*z}{z-P^*}$	

Table 5: Conversion from analog filter specifications to lowpass prototype specifications.

Analog Filter Specifications	Lowpass Prototype Specifications
Lowpass: ω_{ap}, ω_{as}	$\nu_p = 1,\ \nu_s = \omega_{as}/\omega_{ap}$
Highpass: ω_{ap}, ω_{as}	$\nu_p = 1,\ \nu_s = \omega_{ap}/\omega_{as}$
Bandpass: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash} $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$\nu_p = 1, \nu_s = \frac{\omega_{ash}-\omega_{asl}}{\omega_{aph}-\omega_{apl}}$
Bandstop: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash} $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$\nu_p = 1, \nu_s = \frac{\omega_{aph}-\omega_{apl}}{\omega_{ash}-\omega_{asl}}$

ω_{ap}, passband frequency edge; ω_{as}, stopband frequency edge; ω_{apl}, lower cutoff frequency in passband; ω_{aph}, upper cutoff frequency in passband; ω_{asl}, lower cutoff frequency in stopband; ω_{ash}, upper cutoff frequency in stopband; ω_o, geometric center frequency.