Faculty of Science
Department of Electrical and Electronic Engineering
Main Examination 2015

Title of Paper : **Signals and Systems I**

University of Swaziland

Course Number : **EE331**

Time Allowed : **3 hrs**

Instructions :

1. **Answer all four (4) questions**
2. **Each question carries 25 marks**

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The paper consists of five (7) pages including the cover page

Question 1 [25]

a) Define the following terms:

i.	Signal	[2]
ii.	System	[2]
iii.	Deterministic signal	[2]
iv.	Random signal	[2]

b) For any arbitrary signal $x(t)$, which is an even signal, show that: [8]

$$\int_{-\infty}^{\infty} x(t)dt = 2\int_{0}^{\infty} x(t)dt$$

c) Using figure 1.1, write the mathematical expression of:

i.	$x(t)$	[3]
ii.	$\dot{x}(t)$	[3]
iii.	And plot $\dot{x}(t)$	[3]

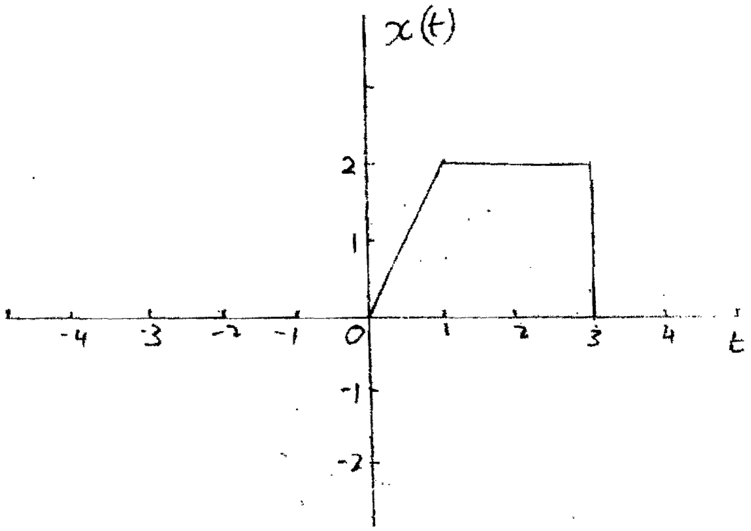

Figure 1.1

Question 2 [25]

a) A continuous- time signal $x(t)$ is shown in figure 2.1, sketch and label each of the following signals.

i. $x\left(4-\frac{t}{2}\right)$ [3]

ii. $[x(t)+x(-t)]u(t)$ [4]

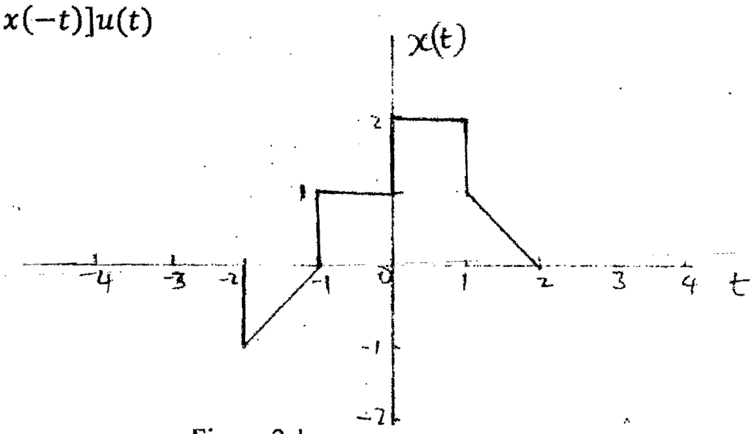

Figure 2,1

b) Determine if the following signals are periodic; if periodic, give the period.

i. $x(t)=\cos(4t)+2\sin(8t)$ [3]

ii. $x(t)=\cos(3\pi t)+2\cos(4\pi t)$ [3]

iii. $x[n]=10\cos(16\pi n)$ [2]

c) Determine if the following systems are: (i) time-invariant, (ii) linear, (iii) causal, (iv) and (v) memoryless

i. $y[n+1]+4y[n]=3x[n+1]-x[n]$ [5]

ii. $y[n]=nx[2n]$ [5]

Question 3 [25]

a) Find the convolution integral of $x(t)$ and $h(t)$ and sketch the convolved signal. [12]
$x(t) = (t-1)\{u(t-1) - u(t-3)\}$ and $h(t) = [u(t+1) - 2u(t-2)]$

b) Consider the Resistor-Inductance (RL) circuit in figure 3.1. Find the differential relating the output voltage $y(t)$ across R and the input voltage $x(t)$ [5]

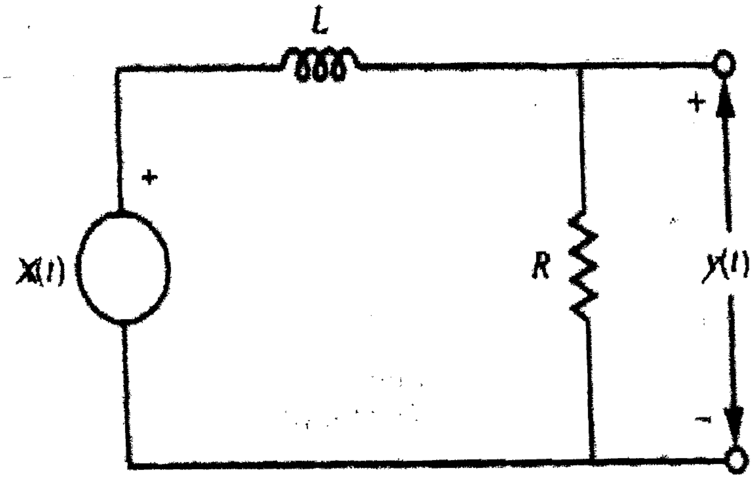

Figure 3.1

c) Determine the discrete-time convolution sum of the given sequences: [8]

$$x[n] = \{1, \underset{\uparrow}{2}, 3, 4\} \text{ and } h[n] = \{1, \underset{\uparrow}{5}, 1\}$$

Question 4 [25]

a) Write the input-output equation for the system shown in figure 4.1. [5]

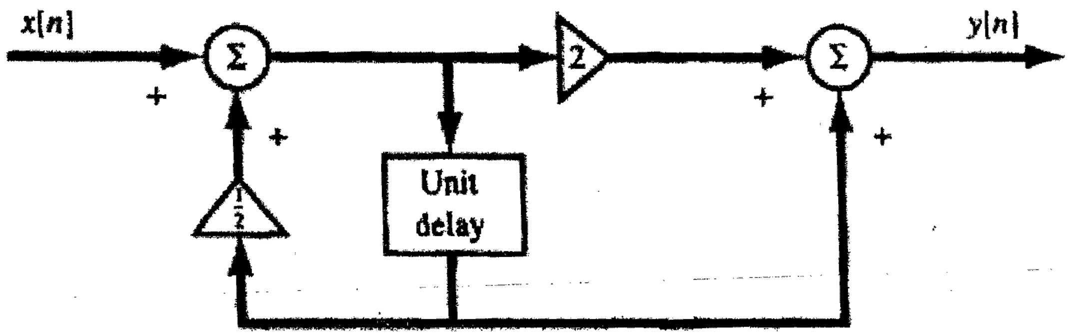

Figure 4.1

b) Find the total response of the system given by: [10]

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2x(t),$$

$$with\; y(0) = -1;\; \left.\frac{dy(t)}{dt}\right|_{t=0} = 1 \quad and\; x(t) = \cos(t)\, u(t)$$

c) Compute the inverse Laplace Transforms of the following functions:

i. $X(s) = \frac{10(s+1)}{(s^2+4s+8)s}$ [3]

ii. $X(s) = \frac{10(s+1)}{s^2+4s+3}e^{-2s}$ [4]

d) Compute the Laplace Transforms of the following function:

i. $x(t) = u(t) - e^{-2t}\cos(10t)\, u(t)$ [3]

<u>Table of Laplace Transforms</u>

delta function	$\delta(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	1
shifted delta function	$\delta(t-a)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	e^{-as}
unit step	$u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}$
ramp	$tu(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2}{s^3}$
n-th power	t^n	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\|t\|}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$-F'(s)$
frequency n-th differentiation	$t^nf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$(-1)^nF^{(n)}(s)$
time differentiation	$f'(t)=\frac{d}{dt}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$sF(s)-f(0)$
time 2nd differentiation	$f''(t)=\frac{d^2}{dt^2}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^2F(s)-sf(0)-f'(0)$
time n-th differentiation	$f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau=(u*f)(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$

Properties of Laplace Transforms

i) Time-shift (delay): $f(t-t_0)\overset{L}{\longleftrightarrow}F(s)e^{-st_0},\ t_0>0$

ii) Time differentiation: $\frac{df(t)}{dt}\overset{L}{\longleftrightarrow}sF(s)-f(0)$

iii) Time integration: $\int_0^t f(t)dt\overset{L}{\longleftrightarrow}\frac{F(s)}{s}$

iv) Linearity: $af(t)+bg(t)\overset{L}{\longleftrightarrow}aF(s)+bF(s)$

v) Convolution Integral: $x(t)*h(t)\overset{L}{\longleftrightarrow}X(s)H(s)$

vi) Frequency-shift: $e^{\alpha t}f(t)\overset{L}{\longleftrightarrow}F(s-\alpha)$

vii) Multiplying by t: $tf(t)\overset{L}{\longleftrightarrow}-\frac{dF(s)}{ds}$

viii) Scaling: $f(at)\overset{L}{\longleftrightarrow}\frac{1}{a}F\left(\frac{s}{a}\right),\ a>0$

ix) Initial Value Theorem: $\lim_{s\to\infty}\{sF(s)\}=f(0)$

x) Final Value Theorem: $\lim_{s\to 0}\{sF(s)\}=f(\infty)$

Standard Table of Forced Response or Particular Solutions

	Input	Particular Solution
1	$cx^m(t)$	$a_0+a_1x(t)+\ldots+a_mx^m(t)$
2	$cx^m(t)e^{ax(t)}$	$\left(a_0+a_1x(t)+\ldots+a_mx^m(t)\right)e^{ax(t)}$
3	$cx^m(t)\cos\left(bx(t)\right)$	$\left(a_0+a_1x(t)+\ldots+a_mx^m(t)\right)\cos\left(bx(t)\right)+\left(c_0+c_1x(t)+\ldots+c_mx^m(t)\right)\sin\left(bx(t)\right)$
4	$cx^m(t)\sin\left(bx(t)\right)$	$\left(a_0+a_1x(t)+\ldots+a_mx^m(t)\right)\sin\left(bx(t)\right)+\left(c_0+c_1x(t)+\ldots+c_mx^m(t)\right)\cos\left(bx(t)\right)$

where $c, a_0, a_1, a_m, c_0, c_1, c_m$ are constants.