

UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE & ENGINEERING
DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING
SIGNALS AND SYSTEMS II
COURSE CODE - EE332
SUPPLEMENTARY EXAMINATION
JULY 2016
DURATION OF THE EXAMINATION - 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. There are FIVE questions in this paper. Answer any FOUR questions.
3. Show all your steps clearly in any calculations/work.
4. State clearly any assumptions made.
5. Start each new question on a fresh page.
6. Useful Fourier Transform and Z-Transform properties are attached.
7. Make sure that this exam contains 8 pages including this one.

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QUESTION ONE (25 marks)

1. Determine the Fourier transform for each of the following signals.

(a) $x(t) = e^{(-1+j2)t}u(t)$ [6]

(b) $x(t) = \frac{1}{2\pi}e^{j\frac{\pi}{3}t} + \frac{1}{\pi}e^{j\frac{\pi}{7}t}$ [6]

(c) $x(n) = \left(\frac{1}{2}\right)^n u(n+2)$ [6]

(d) $x(t) = e^{-3t}u(t) * e^{-2t}u(t)$ [7]

QUESTION TWO (25 marks)

(a) Consider the linear constant coefficient difference equation

$$y(n) - \frac{1}{2}y(n-1) = x(n)$$

Which describes a linear, time invariant system initially at rest. What is the system

transfer function that describes $Y(e^{j\omega})$ in terms of $X(e^{j\omega})$? [5]

(b) Using Fourier transforms, find $y(n)$ if $x(n)$ is

(i) $\delta(n)$ [5]

(ii) $\delta(n - n_0)$ [7]

(iii) $\left(\frac{3}{4}\right)^n u(n)$ [8]

QUESTION THREE (25 marks)

1. State sampling theorem. Define Nyquist rate and Aliasing? [5]

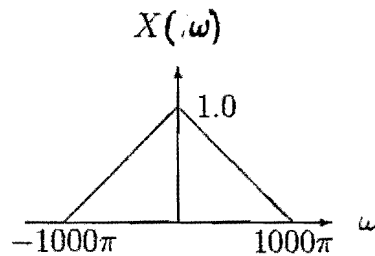
2. A signal $x(t)$ with Fourier transform $X(\omega)$ undergoes impulse-train sampling to generate $x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$, where $T = 0.5 \times 10^{-4}$. Check if the sampling theorem guarantees that $x(t)$ can be recovered from $x_p(t)$ for the following cases.

(a) $X(\omega) = 0$ for $|\omega| > 15000\pi$ [10]

(b) $X(\omega) = 0$ for $|\omega| > 25000\pi$ [10]

QUESTION FOUR (25 marks)

1. Describe how amplitude modulation and demodulation takes place for a complex exponential carrier signal. [5]
2. Let $x(t)$ be a real-valued signal for which $X(\omega) = 0$ when $|\omega| > 1000\pi$. Amplitude modulation is performed to produce the signal $g(t) = x(t) \cos(1000\pi t)$. Assume the spectrum of $x(t)$ is real valued and as given in the figure.
 - a. Find and plot the real and imaginary parts of $G(\omega)$, the spectrum of $g(t)$. [10]
 - b. Find and plot the real and imaginary parts of $W(\omega)$, the spectrum of $w(t) = g(t)\sin(1000\pi t)$. [10]

**QUESTION FIVE (25 marks)**

Consider the two discrete-time sequences.

$$x_1(n) = n \left(\frac{1}{2}\right)^n u(n) \text{ and } x_2(n) = \left(\frac{1}{3}\right)^n u(n)$$

where $u(n)$ is the unit step function. A third signal is obtained using the convolution sum, exam. $x(n) = x_1(n) * x_2(n)$

- a. Compute the z-transform of $x_1(n)$. [5]
- b. Compute the z-transform of $x_2(n)$. [5]
- c. Compute the z-transform of $x(n)$. [5]
- d. Derive $x(n)$ from the z-transform obtained in part (c). [10]

TABLE 1 Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a + j\omega}$	$a > 0$
2	$e^{at}u(-t)$	$\frac{1}{a - j\omega}$	$a > 0$
3	$e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$	$a > 0$
4	$te^{-at}u(t)$	$\frac{1}{(a + j\omega)^2}$	$a > 0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$	$a > 0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	
9	$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	
10	$\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$	
11	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	
12	$\text{sgn } t$	$\frac{2}{j\omega}$	
13	$\cos \omega_0 t u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$	
14	$\sin \omega_0 t u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$	
15	$e^{-at} \sin \omega_0 t u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
16	$e^{-at} \cos \omega_0 t u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
17	$\text{rect}\left(\frac{t}{\tau}\right)$	$\tau \text{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi} \text{sinc}(Wt)$	$\text{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2} \text{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi} \text{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$	$\omega_0 = \frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table 2 Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{ a } X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^t x(u) du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

DISCRETE-TIME FOURIER TRANSFORM

A. Properties of the discrete-time Fourier transform

Non-periodic signal	Fourier transform
$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$	$X(e^{j\omega}) \triangleq \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$
$\left. \begin{matrix} x[n] \\ y[n] \end{matrix} \right\}$	$\left. \begin{matrix} X(e^{j\omega}) \\ Y(e^{j\omega}) \end{matrix} \right\} \text{ Periodic with period } 2\pi$
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
$x[n - n_0]$	$e^{-j\omega n_0} X(e^{j\omega})$
$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$
$x^*[n]$	$X^*(e^{j(-\omega)})$
$x[-n]$	$X(e^{j(-\omega)})$
$x_{(m)}[n] = \begin{cases} x[n/m], & n \text{ multiple of } m \\ 0, & n \text{ not multiple of } m \end{cases}$	$X(e^{j(m\omega)})$
$x[n] * y[n]$	$X(e^{j\omega}) Y(e^{j\omega})$
$x[n] y[n]$	$\frac{1}{2\pi} \int_{2\pi} X(e^{j\theta}) Y(e^{j(\omega - \theta)}) d\theta$
$x[n] - x[n - 1]$	$(1 - e^{j\omega}) X(e^{j\omega})$
$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1 - e^{j\omega}} X(e^{j\omega}) + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$nx[n]$	$j \frac{d}{d\omega} X(e^{j\omega})$

If $x[n]$ is real valued then

$$x[n] \quad \left\{ \begin{array}{l} X(e^{j\omega}) = X^*(e^{j(-\omega)}) \\ \Re\{X(e^{j\omega})\} = \Re\{X(e^{j(-\omega)})\} \\ \Im\{X(e^{j\omega})\} = -\Im\{X(e^{j(-\omega)})\} \\ |X(e^{j\omega})| = |X(e^{j(-\omega)})| \\ \arg\{X(e^{j\omega})\} = -\arg\{X(e^{j(-\omega)})\} \end{array} \right.$$

$$\begin{array}{l} x_e[n] = \mathcal{E}\{x[n]\} \\ x_o[n] = \mathcal{O}\{x[n]\} \end{array} \quad \begin{array}{l} \Re\{X(e^{j\omega})\} \\ j\Im\{X(e^{j\omega})\} \end{array}$$

Parsevals relation for non-periodic signals

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} |X(e^{j\omega})|^2 d\omega$$

B. Discrete-time Fourier transform table

$x[n]$	$X(e^{j\omega})$
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\omega n_0}$
$\sum_{k=-\infty}^{\infty} \delta(n - kN)$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$
1	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$e^{j\omega_0 n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi k)$
$\cos \omega_0 n$	$\pi \sum_{k=-\infty}^{\infty} [\delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k)]$
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{k=-\infty}^{\infty} [\delta(\omega - \omega_0 - 2\pi k) - \delta(\omega + \omega_0 - 2\pi k)]$
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$a^n u(n), \quad a < 1$	$\frac{1}{1 - ae^{-j\omega}}$
$(n+1)a^n u[n], \quad a < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$
$\frac{(n+m-1)!}{n!(m-1)!} a^n u[n], \quad a < 1$	$\frac{1}{(1 - ae^{-j\omega})^m}$
$\frac{1}{1 - a^2} a^{ n }, \quad a < 1$	$\frac{1}{1 + a^2 - 2a \cos \omega}$
$\begin{cases} 1, & n \leq N_1 \\ 0, & N_1 < n \leq \frac{N}{2} \end{cases}$ period N	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$
$\begin{cases} 1, & n \leq N_1 \\ 0, & n > N_1 \end{cases}$	$\frac{\sin \omega (N_1 + \frac{1}{2})}{\sin \frac{\omega}{2}}$
$\begin{cases} \frac{\sin Wn}{\pi n} = \frac{W}{\pi} \text{sinc} \frac{Wn}{\pi} \\ 0 < W < \pi \end{cases}$	$\begin{cases} 1, & \omega \leq W \\ 0, & W < \omega \leq \pi \end{cases}$ period 2π

Table of Z-Transforms

Line No.	$x(n), n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n)z^{-n}$	
2	$\delta(n)$	1	$ z > 0$
3	$au(n)$	$\frac{az}{z-1}$	$ z > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$ z > 1$
5	$n^2u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$ z > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$ z > a $
7	$e^{-an} u(n)$	$\frac{z}{(z-e^{-a})}$	$ z > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$ z > a $
9	$\sin(an)u(n)$	$\frac{z \sin(a)}{z^2 - 2z \cos(a) + 1}$	$ z > 1$
10	$\cos(an)u(n)$	$\frac{z[z - \cos(a)]}{z^2 - 2z \cos(a) + 1}$	$ z > 1$
11	$a^n \sin(bn)u(n)$	$\frac{[a \sin(b)]z}{z^2 - [2a \cos(b)]z + a^2}$	$ z > a $
12	$a^n \cos(bn)u(n)$	$\frac{z[z - a \cos(b)]}{z^2 - [2a \cos(b)]z + a^2}$	$ z > a $
13	$e^{-an} \sin(bn)u(n)$	$\frac{[e^{-a} \sin(b)]z}{z^2 - [2e^{-a} \cos(b)]z + e^{-2a}}$	$ z > e^{-a}$
14	$e^{-an} \cos(bn)u(n)$	$\frac{[e^{-a} \cos(b)]z}{z^2 - [2e^{-a} \cos(b)]z + e^{-2a}}$	$ z > e^{-a}$

Properties of Z-Transforms

Linearity: $ax_1[k] + bx_2[k] \Leftrightarrow aX_1(z) + bX_2(z)$

Time Reversal: $x[-k] \Leftrightarrow X(1/z)$

Summation: $\sum_{n=-\infty}^k x[n] \Leftrightarrow \frac{zX(z)}{z-1}$

Initial Value: $x[0] = \lim_{z \rightarrow \infty} X(z)$

Final Value: $x[\infty] = \lim_{z \rightarrow 1} (z-1)X(z)$

Convolution: $x[k] * h[k] \Leftrightarrow X(z)H(z)$

Differencing: $x[k] - x[k-1] \Leftrightarrow (1-z^{-1})X(z)$

Differentiation: $-kx[k] \Leftrightarrow z \frac{d}{dz} X(z)$

Time Shifting: $x[n-n_0] \Leftrightarrow z^{-n_0} X(z), n_0 \geq 0$

$$x[n+n_0] \Leftrightarrow z^{n_0} \left(X(z) - \sum_{m=0}^{n_0-1} x[m]z^{-m} \right), n_0 \geq 0$$