University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering
Main Examination 2015

Title of Paper	:	**Control Engineering I**
Course Number	:	**EE431**
Time Allowed	:	**3 hrs**
Instructions	:	

1. **Answer any four (4) questions**
2. **Each question carries 25 marks**
3. **Useful information is attached at the end of the question paper**

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

The paper consists of nine (9) pages

Question 1

(a) How do closed loop systems compensate for disturbances and state one of their drawbacks. [3]

(b) Based on the natural response definition of stability, explain based on linear time invariant system as to when a system is said to be stable, unstable and marginally stable. [3]

(c) The imaginary part of a pole generates what part of a response. Sketch their system response when the complex values are $\pm j3$ and the input is a step function. [3]

(d) The physical realization of a system to control room temperature is shown in Figure 1.1. Here the output signal from a temperature sensing device is compared with the desired temperature. Any difference or error causes the controller to send a control signal to the gas solenoid valve which produces a linear movement of the value stem, thus adjusting the flow of a gas to the burner of the gas fire. The desired temperature is usually obtained from manual adjustment of a potentiometer. Draw the block diagram of the room temperature control system. [7]

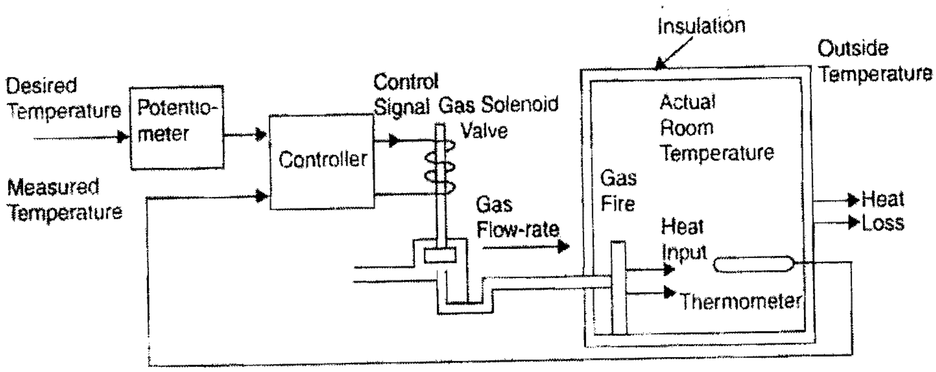


Figure 1.1

(e) Find the number of poles in the left half-plane, right half-plane, and at the $j\omega$ –axis for the system of Figure 1.2. Draw conclusions about the stability of the closed-loop system. [9]

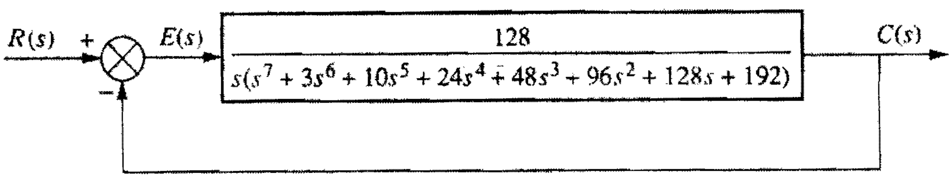


Figure 1.2

Question 2

(a) Find the transfer function, $T(s) = Y(s)/R(s)$, for the following system represented in state space. [10]

$$\dot{x} = \begin{bmatrix} 2 & 3 & -8 \\ 0 & 5 & 3 \\ -3 & -5 & -4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} r$$

$$y = [1 \;\; 3 \;\; 6]\,\mathbf{x}$$

(b) Represent the following transfer function in state space, also show the decomposed transfer function and the equivalent block diagram. Give your answer in vector-matrix form. [15]

$$T(s) = \frac{s^2+3s+7}{(s+1)(s^2+5s+4)}$$

Question 3

(a) Find the overall closed-loop transfer function of the system shown in figure 3.1 [6]

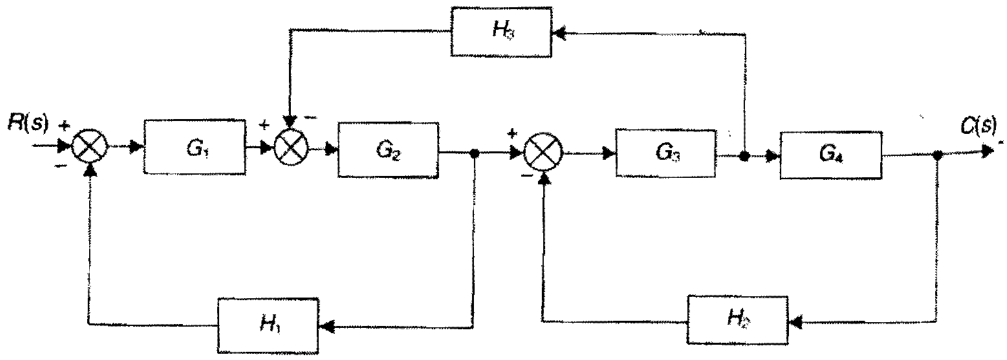


Figure 3.1

(b) With reference to Figure 3.2

(i) Sketch the root locus for the system [4]

(ii) Find the frequency and gain, K, for which the root locus crosses the imaginary axis. For what range of K is the system stable? [15]

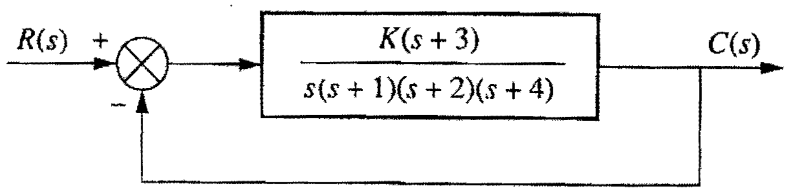


Figure 3.2

Question 4

(a) For the system shown in Figure 4.1, find the system type, the appropriate error constant associated with the system type, and the steady-state error for a unit step input. [5]

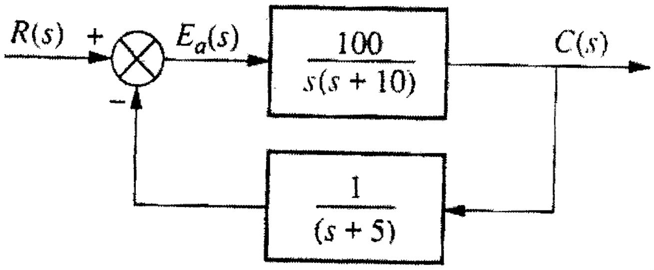


Figure 4.1

(b) Represent the system below in state space in phase variable and input feed-forward forms. Draw the signal-flow graphs. [8]

$$T(s) = \frac{s^3+2s^2+7s+1}{s^4+3s^3+5s^2+6s+4}$$

(c) Find the value of the proportional controller gain K_1 to make the controller system shown in Figure 4.2 just unstable. Also, find the roots of the characteristics equation and the transient response $c(t)$. [12]

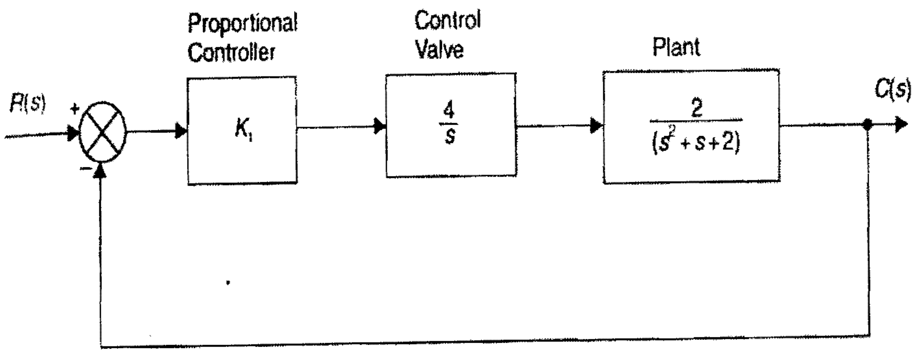


Figure 4.2

Question 5

(a) For the system in Figure 5.1, evaluate the static error constants and find the expected error for the standard step, ramp, and parabolic inputs. [5]

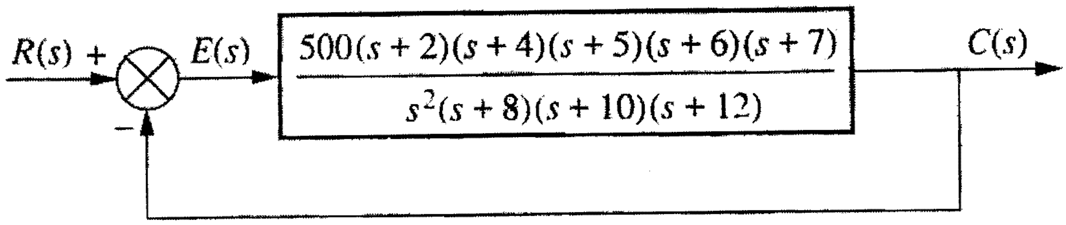


Figure 5.1

(b) Using Mason's rule, find the transfer function, T(s) = C(s)/R(s), for the system represented in Figure 5.2 below.[10]

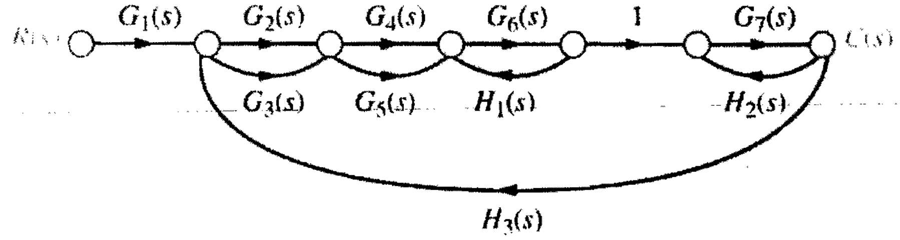


Figure 5.2

(c) A DC motor is connected to an op-amp circuit in cascade as shown in the figure 5.3 (a). The op-amp circuit subsystem is shown in figure 5.3 (b); the input to the op-amp is a voltage source $v_i\ (t)$, the output is the voltage $v_s\ (t)$, and the transfer function of this subsystem is $G_1\ (s)$. The DC motor subsystem is shown in figure 5.3 (c); the input to the DC motor is the op-amp's output $v_s\ (t)$, the output is the angular velocity $\omega\ (t)$ of a shaft connected to the motor, and the transfer function of this subsystem is $G_2\ (s)$. The DC motor subsystem is not loading the op-amp circuit subsystem.

(i) Derive the transfer function $G_1(s)$ of the op–amp circuit subsystem. Locate the poles and zeros of $G_1(s)$ on the s–plane. [5]

(ii) Derive the time–domain response $\omega\ (t)$ when the input $v_i(t)$ is a step function of amplitude 1V (i.e., the unit–step response.) Given that the transfer function of the DC motor subsystem $G_2(s) = \frac{1}{s+2}$. [5]

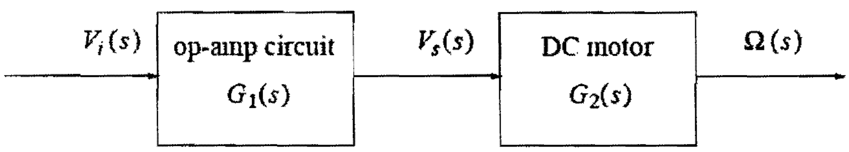


Figure 5.3 (a)

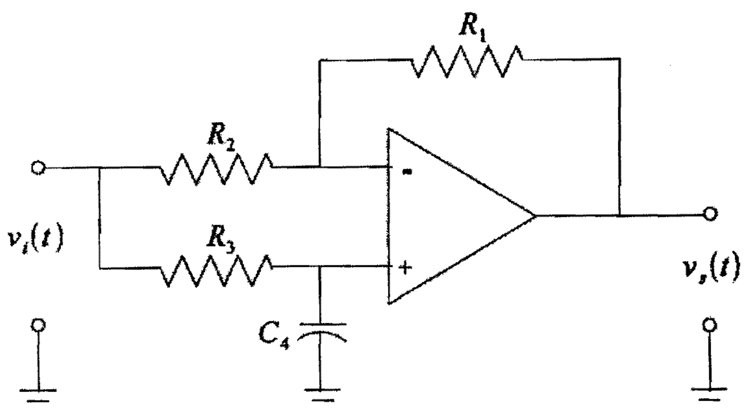


Figure 5.3 (b)

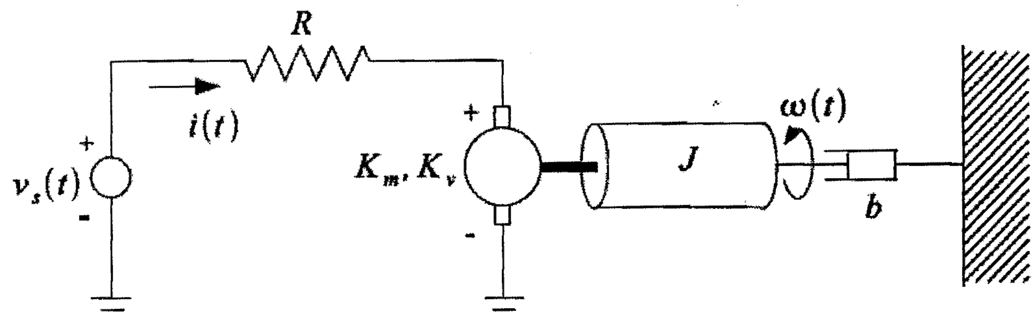


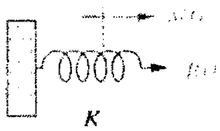
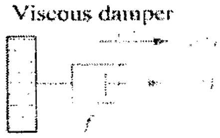
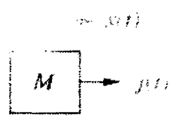
Figure 5.3 (c)

Table 1

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ = V (volts), $i(t)$ = A (amps), $q(t)$ = Q (coulombs), C = F (farads), R = Ω (ohms), G = ℧ (mhos), L = H (henries).

Table 2

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v\frac{dx(t)}{dt}$	$f_v s$
M 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds²/meter).

Table 3

Input	Steady-state error formula	Type 0 Static error constant	Type 0 Error	Type 1 Static error constant	Type 1 Error	Type 2 Static error constant	Type 2 Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

Static Error Constants

For a step input, $u(t)$,

$$e(\infty) = e_{step}(\infty) = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

For a ramp input, $tu(t)$,

$$e(\infty) = e_{ramp}(\infty) = \frac{1}{\lim_{s \to 0} sG(s)}$$

For a parabolic input, $\frac{1}{2}t^2u(t)$,

$$e(\infty) = e_{parabola}(\infty) = \frac{1}{\lim_{s \to 0} s^2G(s)}$$

Position constant, K_p, where

$$K_p = \lim_{s \to 0} G(s)$$

Velocity constant, K_v, where

$$K_v = \lim_{s \to 0} sG(s)$$

Acceleration constant, K_a, where

$$K_a = \lim_{s \to 0} s^2G(s)$$