University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering
Supplementary Examination 2016

Title of Paper	:	**Introduction to Digital Signal Processing**
Course Number	:	**EE443**

Time Allowed : **3 hrs**

Instructions :

1. **Answer all four (4) questions**
2. **Each question carries 25 marks**
3. **Useful information is attached at the end of the question paper**

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The paper consists of seven (7) pages

319

Question 1

(a) Calculate the filter coefficients for a 5-tap FIR bandpass filter with a lower cut-off frequency of 2,000 Hz and an upper cut-off frequency of 2,400 Hz at a sampling rate of 8,000 Hz. Also, find the transfer function. [12]

(b) A second-order notch filter is required to satisfy the following specifications:

- Sampling rate = 8,000 Hz
- 3 dB bandwidth: BW =100 Hz
- Narrow passband centered at f_0 = 1,500 Hz

Find the transfer function using the pole-zero placement approach. [6]

(c) Draw the digital signal processor based on the Harvard architecture and briefly describe the additional units. [5]

(d) List any DSP applications [2]

Question 2

(a) Design a highpass FIR filter with the following specifications:

Stopband = 0–1,500 Hz
Passband = 2,500–4,000 Hz
Stopband attenuation = 40 dB
Passband ripple = 0.1 dB
Sampling rate = 8,000 Hz [13]

(b) Perform the FIR filter realization using the linear phase form for the designed filter in (a) [2]

(c) In one of the DSP processors, the program efficiency is greatly improved by the use of registers, explain the functions of the registers [5]

(d) Compact disc recording system is one of the DSP real world applications. Explain what happens during playback time and the purpose of oversampling in such DSP system. [5]

Question 3

(a) Design a second-order digital bandpass Butterworth filter with the following specifications:

- Upper cutoff frequency of 2.6 kHz and
- Lower cutoff frequency of 2.4 kHz,
- Sampling frequency of 8,000 Hz.

State the difference equation and the transfer function of the filter designed [15]

(b) Use direct Form I and II to realize the filter. [8]

(c) Given the following difference equation,

$$y(n) = 0.5\,x(n) + 0.5\,x(n-1)$$

Find $H(z)$ [2]

Question 4

(a) Compare the executions cycle of the two architectures, Von Neumann and Harvard architectures? [10]

(b) Find $x(n)$ if $X(z) = \frac{z^2}{(z-1)(z-0.5)^2}$ [12]

(c) Find the z-transform of the following

$$x(n) = e^{-0.1n}\cos(0.25\pi n)\,u(n)$$ [3]

Table 1: Properties of z-transform

Property	Time Domain	z-Transform
Linearity	$ax_1(n) + bx_2(n)$	$aZ(x_1(n)) + bZ(x_2(n))$
Shift theorem	$x(n-m)$	$z^{-m}X(z)$
Linear convolution	$x_1(n)*x_2(n) = \sum_{k=0}^{\infty} x_1(n-k)x_2(k)$	$X_1(z)X_2(z)$

Table 2: Partial fraction(s) and formulas for constant(s).

Partial fraction with the first-order real pole:	
$\dfrac{R}{z-p}$	$R = (z-p)\dfrac{X(z)}{z}\Big\vert_{z=p}$
Partial fraction with the first-order complex poles:	
$\dfrac{Az}{(z-P)} + \dfrac{A^*z}{(z-P^*)}$	$A = (z-P)\dfrac{X(z)}{z}\Big\vert_{z=P}$
P^* = complex conjugate of P	
A^* = complex conjugate of A	
Partial fraction with mth-order real poles:	
$\dfrac{R_m}{(z-p)} + \dfrac{R_{m-1}}{(z-p)^2} + \cdots + \dfrac{R_1}{(z-p)^m}$	$R_k = \dfrac{1}{(k-1)!}\dfrac{d^{k-1}}{dz^{k-1}}\left((z-p)^m \dfrac{X(z)}{z}\right)\Big\vert_{z=p}$

Table 3: 3 dB Butterworth lowpass prototype transfer functions ($\varepsilon = 1$)

n	$H_P(s)$
1	$\dfrac{1}{s+1}$
2	$\dfrac{1}{s^2+1.4142s+1}$
3	$\dfrac{1}{s^3+2s^2+2s+1}$
4	$\dfrac{1}{s^4+2.6131s^3+3.4142s^2+2.6131s+1}$
5	$\dfrac{1}{s^5+3.2361s^4+5.2361s^3+5.2361s^2+3.2361s+1}$
6	$\dfrac{1}{s^6+3.8637s^5+7.4641s^4+9.1416s^3+7.4641s^2+3.8637s+1}$

Table 4: Summary of ideal impulse responses for standard FIR filters.

Filter Type	Ideal Impulse Response $h(n)$ (noncausal FIR coefficients)
Lowpass:	$h(n) = \begin{cases} \frac{\Omega_c}{\pi} & n=0 \\ \frac{\sin(\Omega_c n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Highpass:	$h(n) = \begin{cases} \frac{\pi-\Omega_c}{\pi} & n=0 \\ -\frac{\sin(\Omega_c n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Bandpass:	$h(n) = \begin{cases} \frac{\Omega_H-\Omega_L}{\pi} & n=0 \\ \frac{\sin(\Omega_H n)}{n\pi} - \frac{\sin(\Omega_L n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Bandstop:	$h(n) = \begin{cases} \frac{\pi-\Omega_H+\Omega_L}{\pi} & n=0 \\ -\frac{\sin(\Omega_H n)}{n\pi} + \frac{\sin(\Omega_L n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$

Causal FIR filter coefficients: shifting $h(n)$ to the right by M samples.
Transfer function:

$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots b_{2M} z^{-2M}$
where $b_n = h(n - M)$, $n = 0, 1, \cdots, 2M$

Table 5: 3 dB Butterworth lowpass prototype transfer functions ($\varepsilon = 1$)

n	$H_P(s)$
1	$\frac{1}{s+1}$
2	$\frac{1}{s^2+1.4142s+1}$
3	$\frac{1}{s^3+2s^2+2s+1}$
4	$\frac{1}{s^4+2.6131s^3+3.4142s^2+2.6131s+1}$
5	$\frac{1}{s^5+3.2361s^4+5.2361s^3+5.2361s^2+3.2361s+1}$
6	$\frac{1}{s^6+3.8637s^5+7.4641s^4+9.1416s^3+7.4641s^2+3.8637s+1}$

Table 6: Analog lowpass prototype transformations

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$

Table 6: Conversion from analog filter specifications to lowpass prototype specifications.

Analog Filter Specifications	Lowpass Prototype Specifications
Lowpass: ω_{ap}, ω_{as}	$\nu_p = 1$, $\nu_s = \omega_{as}/\omega_{ap}$
Highpass: ω_{ap}, ω_{as}	$\nu_p = 1$, $\nu_s = \omega_{ap}/\omega_{as}$
Bandpass: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash} $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$\nu_p = 1, \nu_s = \frac{\omega_{ash}-\omega_{asl}}{\omega_{aph}-\omega_{apl}}$
Bandstop: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash} $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$\nu_p = 1, \nu_s = \frac{\omega_{aph}-\omega_{apl}}{\omega_{ash}-\omega_{asl}}$

ω_{ap}, passband frequency edge; ω_{as}, stopband frequency edge; ω_{apl}, lower cutoff frequency in passband; ω_{aph}, upper cutoff frequency in passband; ω_{asl}, lower cutoff frequency in stopband; ω_{ash}, upper cutoff frequency in stopband; ω_o, geometric center frequency.

The Z-transform

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n)z^{-n}$	
2	$\delta(n)$	1	$\lvert z\rvert > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\lvert z\rvert > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
5	$n^2u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z\rvert > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
7	$e^{-na}u(n)$	$\frac{z}{(z-e^{-a})}$	$\lvert z\rvert > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
9	$\sin(an)u(n)$	$\frac{z\sin(a)}{z^2 - 2z\cos(a) + 1}$	$\lvert z\rvert > 1$
10	$\cos(an)u(n)$	$\frac{z[z-\cos(a)]}{z^2 - 2z\cos(a) + 1}$	$\lvert z\rvert > 1$
11	$a^n \sin(bn)u(n)$	$\frac{[a\sin(b)]z}{z^2 - [2a\cos(b)]z + a^2}$	$\lvert z\rvert > \lvert a\rvert$
12	$a^n \cos(bn)u(n)$	$\frac{z[z - a\cos(b)]}{z^2 - [2a\cos(b)]z + a^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
13	$e^{-an}\sin(bn)u(n)$	$\frac{[e^{-a}\sin(b)]z}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\lvert z\rvert > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\frac{z[z - e^{-a}\cos(b)]}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\lvert z\rvert > e^{-a}$
15	$2\lvert A\rvert\lvert P\rvert^n \cos(n\theta + \phi)u(n)$ where P and A are complex constants defined by $P = \lvert P\rvert\angle\theta, A = \lvert A\rvert\angle\phi$	$\frac{Az}{z-P} + \frac{A^* z}{z-P^*}$	