University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering
Main Examination 2017

Title of Paper	:	Control Engineering I
Course Number	:	EE431
Time Allowed	:	3 hrs

Instructions :

1. Answer any four (4) questions
2. Each question carries 25 marks
3. Useful information is attached at the end of the question paper

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The paper consists of Six (6) pages including this page

Question 1 (25 Marks)

(a) For the electric circuit given in Figure Q.1

(i) Determine the state space representation, [10]

(ii) The transfer function $G(s) = \frac{V_c(s)}{V_i(s)}$,hint $G(s) = C(sI - A)^{-1}B + D$ [10]

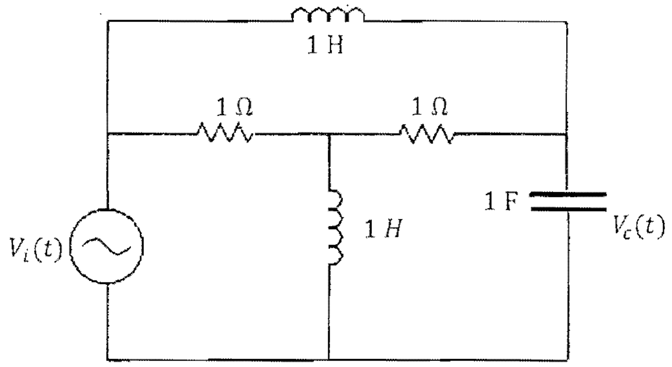


Figure Q.1

(b) Given the following transfer function $\frac{Y(s)}{R(s)} = \frac{s+2}{(s+3)(s+5)(s+7)}$, determine its step response. [5]

Question 2 (25 Marks)

(a) Answer the following question

(i) Discuss stability in digital systems [3]

(ii) What causes an entire row of zeros to show up in a Routh table [2]

(b) Given the following transfer function $G(s) = \frac{1}{2s+100}$,Sketch the Bode diagram of the system [10]

(c) Show that for a unit ramp function where $f(kT) = kT$, the z-transform is $\frac{Tz}{(z-1)^2}$ [10]

Question 3 (25 Marks)

(a) Define the following terms , *Rise Time, Peak time, Percentage overshoot* for 2nd order system. [3]

(b) Given the following transfer function $G(s) = \frac{225}{s^2+30s+225}$, determine the following

- (i) Natural frequency [1]
- (ii) Damping ratio [1]
- (iii) State the step response in relation with the damping ratio, ξ. [1]
- (iv) Settling Time [1]

(c) Given a feedback system whose open-loop transfer function is

$$G(s) = \frac{K(s+3)}{(S+5)(S+8)(s+12)}$$

Where K is the feedback gain. Evaluate the system's close-loop behaviour using the root locus technique.

- (i) How many asymptotes are there in this system's root locus? What are the asymptotes angles? [2]
- (ii) Where is the asymptotes real-axis intercept? [2]
- (iii) Sketch the root locus based on the information from the previous questions. [6]

NB: No need to annotate break-in/away points and imaginary axis intercepts, if there are any.

- (iv) If you had to recommend this system to a customer, what would you advise with respect to increasing the feedback gain K indefinitely? [3]

(d) Study the diagram below and answer the questions that follow.

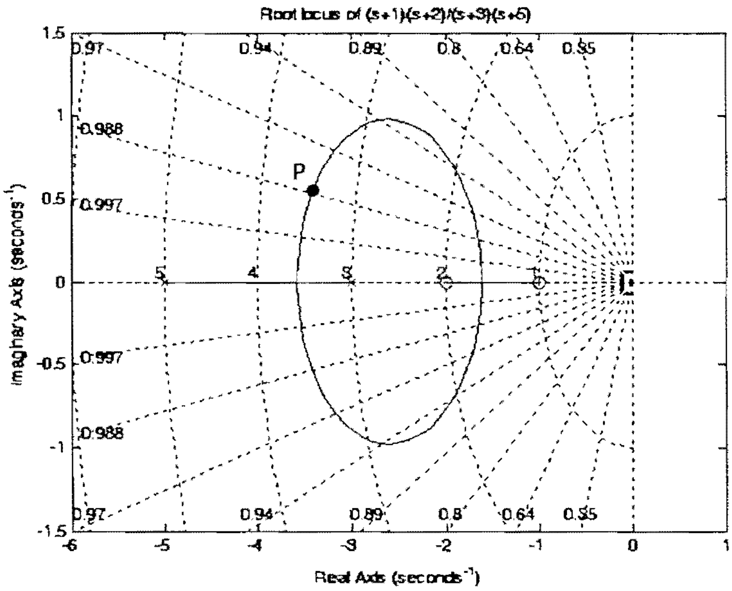


Figure Q.2

Is it possible to tune this system to achieve a damping ratio of $\frac{\sqrt{2}}{2}$ Explain your answer? [3]

Is it possible to achieve the following settling time. Explain your answer

- (i) Ts = 1 sec [1]
- (ii) Ts = 4 sec [1]

Question 4 (25 Marks)

(a) Given the following transfer function

$$T(s) = \frac{5}{s^2 + 7s + 10}$$

Find the steady state error for the following input functions

(i) For Unit step [3]
(ii) Unit Ramp [2]

(b) Determine the range of K to make the following system stable and is it possible to get a steady state error of 5% with this design of K ?. Determine the expected minimum steady state error for this system [15]

$$G(s) = \frac{K(s+20)}{s(s+2)(s+3)}$$

(c) Determine the magnitude and phase angle expressions and hence sketch polar plot the following transfer function: [5]

$$G(j\omega) = \frac{e^{-j\omega L}}{(1 + j\omega T)}$$

Question 5 (25 Marks)

(a) Given the following system draw the signal flow diagram [5]

$$\dot{x} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = [0 \quad 1 \quad 0]x$$

(b) Reduce the system shown in Figure **Q.3** to a single transfer function using Mason's rule [10]

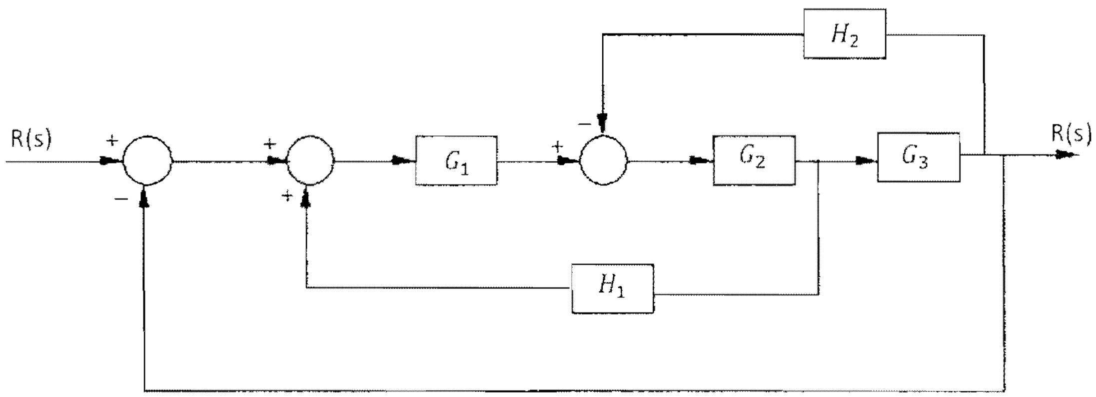
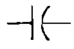


Figure Q.3

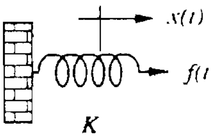
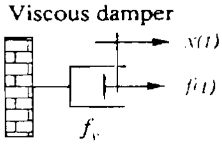
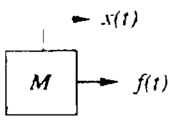
(c) Verify your answer in (b) using the block reduction method. [10]

Table 1

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ = V (volts), $i(t)$ = A (amps), $q(t)$ = Q (coulombs), C = F (farads), R = Ω (ohms), G = ℧ (mhos), L = H (henries).

Table 2

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring  K	$f(t) = K\int_0^t v(\tau)\,d\tau$	$f(t) = Kx(t)$	K
Viscous damper  f_v	$f(t) = f_v v(t)$	$f(t) = f_v\frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).

Table 3

Input	Steady-state error formula	Type 0		Type 1		Type 2	
		Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

Static Error Constants

For a step input, $u(t)$,

$$e(\infty) = e_{\text{step}}(\infty) = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

For a ramp input, $tu(t)$,

$$e(\infty) = e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s \to 0} sG(s)}$$

For a parabolic input, $\frac{1}{2}t^2u(t)$,

$$e(\infty) = e_{\text{parabola}}(\infty) = \frac{1}{\lim_{s \to 0} s^2G(s)}$$

Position constant, K_p, where

$$K_p = \lim_{s \to 0} G(s)$$

Velocity constant, K_v, where

$$K_v = \lim_{s \to 0} sG(s)$$

Acceleration constant, K_a, where

$$K_a = \lim_{s \to 0} s^2G(s)$$

$$f^*(t) = \sum_{k=0}^{\infty} kT\delta(t - kT)$$

$$F^*(s) = \sum_{k=0}^{\infty} kTe^{-kTs}$$

$$e^{-kTs} = Z^{-k}$$