University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering
Supplementary Examination 2018

Title of Paper	:	Control Engineering I
Course Number	:	EE431
Time Allowed	:	3 hrs

Instructions :

1. This paper has five questions.
2. Answer any four questions
3. Each question carries 25 marks
4. Useful information is attached at the end of the question paper

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The paper consists of Seven (7) pages including this page

Question 1 (25 Marks)

(a) Given that the state and output equations are

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Show that the transfer function $T(s) = C(sI - A)^{-1}B + D$ [5]

(b) Determine the statespace model of the following electrical circuit, given that the output is the voltage across C_1 [10]

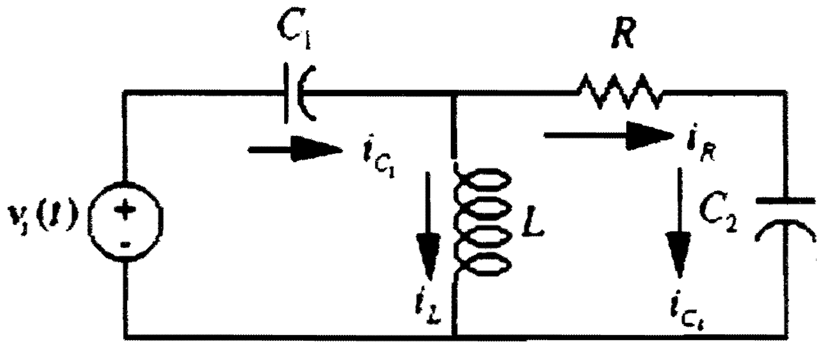


Fig Q. 1

(c) Given $C_1 = 0.1\ F$, $C_2 = 10\ F$, $R = 1\Omega$,
Find the transfer function of the circuit in Fig. Q.1. [10]

Question 2 (25 Marks)

(a) Draw the block diagram of a closed loop control system for a disk drive. [5]

(b) Given the block diagram shown in Fig. Q.2. Determine the transfer function using block diagram reduction techniques. [10]

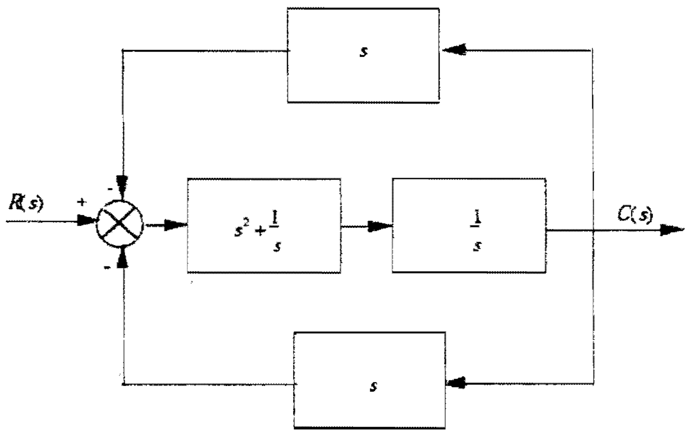


Fig Q. 2

(c) Verify your answer in (b) by using Mason's rule [10]

Question 3 (25 Marks)

(a) For the system shown in Fig Q.3, Find the system type, the appropriate error constant associated with the system type, and the steady-state error for a unit step input [5]

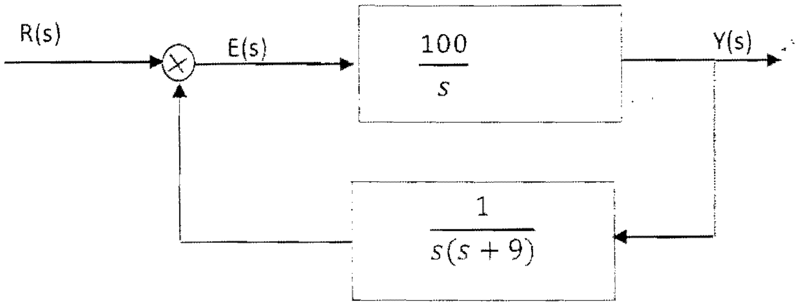


Fig Q.3

(b) Given the control system in the figure below, find the value of K so that there is 5 % error in the steady-state [5]

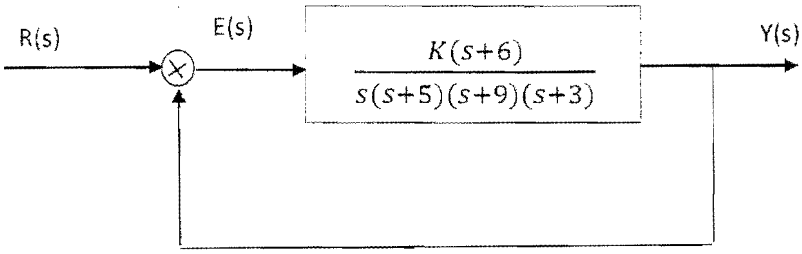


Fig Q.4

(c) Given the following system in state space. How many poles are on the right-hand half of the system. [5]

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 5 \\ -5 & 3 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 0 & 3 & 7 \end{bmatrix} x$$

(d) Determine the number of poles on the right-half plane, left-half plane and on $j\omega$ axis for the following system. Comment about system stability. [10]

$$T(s) = \frac{s^2 + 1}{s^8 + 3s^7 + 10s^6 + 24s^5 + 48s^4 + 96s^3 + 128s^2 + 192s + 128}$$

Question 4 (25 Marks)

(a) Answer the following question

  (i) Discuss stability in digital systems [3]

  (ii) What causes an entire row of zeros to show up in a Routh table [2]

(b) Show that for a unit ramp function where $f(kT) = kT$, the Z-transform is $\frac{Tz}{(z-1)^2}$ [10]

(c) Given the system shown in Fig. Q. 2 below.

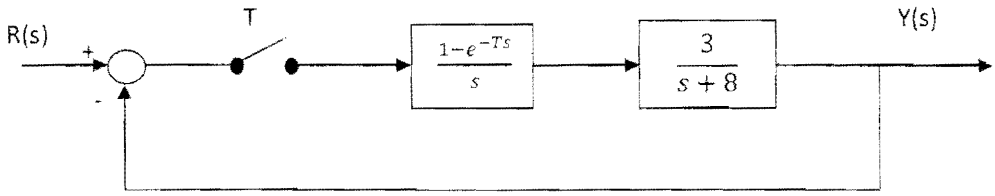


Fig. Q.2

Determine the range of the sampling interval T that will make the system stable [10]

Question 5 (25 Marks)

(a) Given the system shown below

$$G(s) = \frac{361}{s^2 + 16s + 361}$$

Find the Natural frequency of the system ω_n. Damping ratio ζ, Settling time T_s, Peak time T_p and Percentage overshoot, $\% \, OS$ [5]

(b) Given a feedback system whose open-loop transfer function is

$$G(s) = \frac{K(s+3)}{(S+5)(S+8)(s+12)}$$

Where K is the feedback gain. Evaluate the system's close-loop behaviour using the root locus technique.

(i) How many asymptotes are there in this system's root locus? What are the asymptotes angles? [2]

(ii) Where is the asymptotes real-axis intercept? [2]

(iii) Sketch the root locus based on the information from the previous questions. [6]
NB: No need to annotate break-in/away points and imaginary axis intercepts, if there are any.

(iv) If you had to recommend this system to a customer, what would you advise with respect to increasing the feedback gain K indefinitely? [3]

(c) Study the diagram below and answer the questions that follow.

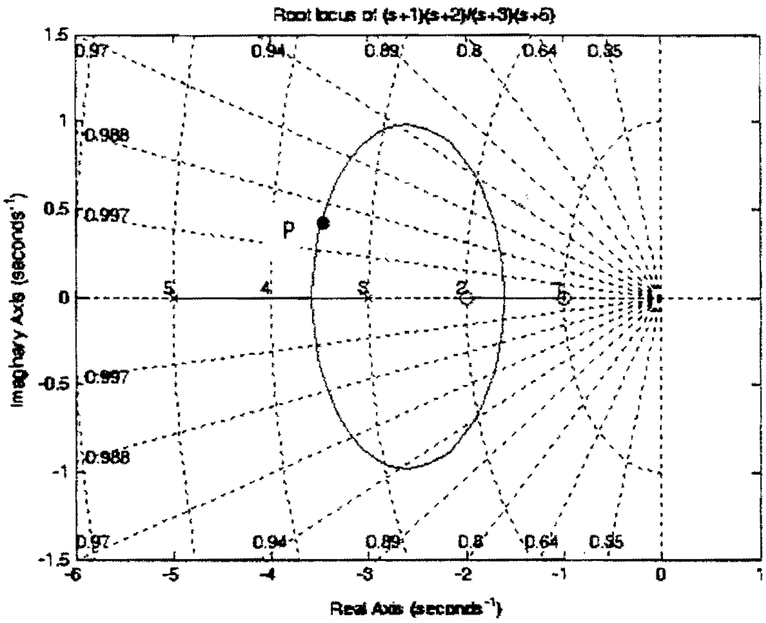


Fig. Q.50

i. Is it possible to tune this system to achieve a damping ratio of 0.707 Explain your answer? [3]

ii. Is it possible to achieve the following settling time. Explain your answer

Ts = 1 sec [2]

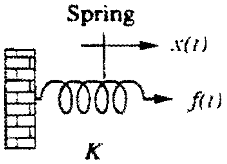
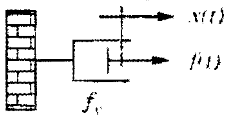
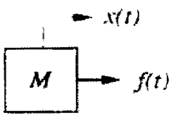
Ts = 4 sec [2]

Table 1

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ = V (volts), $i(t)$ = A (amps), $q(t)$ = Q (coulombs), C = F (farads), R = Ω (ohms), G = ℧ (mhos), L = H (henries).

Table 2

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)\,d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v\frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds²/meter).

Table 3

Input	Steady-state error formula	Type 0 Static error constant	Type 0 Error	Type 1 Static error constant	Type 1 Error	Type 2 Static error constant	Type 2 Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

Static Error Constants

For a step input, $u(t)$,

$$e(\infty) = e_{\text{step}}(\infty) = \frac{1}{1 + \lim\limits_{s\to 0} G(s)}$$

For a ramp input, $tu(t)$,

$$e(\infty) = e_{\text{ramp}}(\infty) = \frac{1}{\lim\limits_{s\to 0} sG(s)}$$

For a parabolic input, $\frac{1}{2}t^2u(t)$,

$$e(\infty) = e_{\text{parabola}}(\infty) = \frac{1}{\lim\limits_{s\to 0} s^2G(s)}$$

Position constant, K_p, where

$$K_p = \lim_{s\to 0} G(s)$$

Velocity constant, K_v, where

$$K_v = \lim_{s\to 0} sG(s)$$

Acceleration constant, K_a, where

$$K_a = \lim_{s\to 0} s^2G(s)$$

$$f^*(t) = \sum_{k=0}^{\infty} kT\delta(t - kT)$$

$$F^*(s) = \sum_{k=0}^{\infty} kTe^{-kTs}$$

$$e^{-kTs} = Z^{-k}$$