

UNIVERSITY OF ESWATINI



MAIN EXAMINATION PAPER 2018

TITLE OF PAPER : PROBABILITY AND STATISTICS

COURSE CODE : EEE301

TIME ALLOWED : 3 HOURS

INSTRUCTIONS : ANSWER ANY FIVE QUESTIONS.

REQUIREMENTS : SCIENTIFIC CALCULATOR AND
STATISTICAL TABLES.

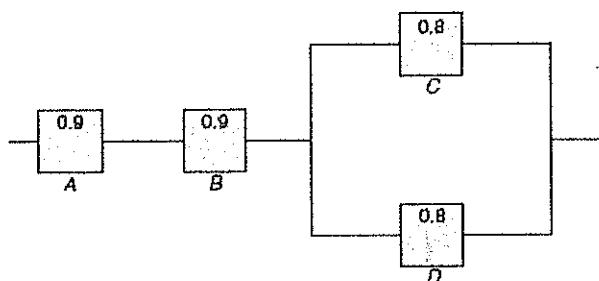
Question 1

- a) A cell-phone tower has a circular coverage area of radius 10 Km. If a call is initiated from a random point in the coverage area, find the probability that the call comes from within 2 Km of the tower.
- b) Consider the duration of a cell-phone call. For the sample space $\Omega = [0, \infty)$, find the probability that the duration is between 5 and 7 minutes.
- c) Three bits are transmitted across a noisy channel and the number of correct receptions is noted. Find the probability that the number of correctly received bits is two, assuming bit errors are mutually independent and that on each transmission the probability of correct reception is λ for some fixed $0 \leq \lambda \leq 1$.
- d) The number of hits to a popular website during a 1-minute interval is given by X_i random variable. Find the probability that there is at least one hit between 3:00 a.m and 3:01 a.m if rate parameter is 2 hits per minute. Then find the probability that there are at least 2 hits during this time interval.

(5+5+5+5 Marks)

Question 2

- a) Due to an Internet configuration error, packets sent from Mbabane to University of Eswatini are routed through Ntontozi with probability $3/4$. Given that a packet is routed through Ntontozi, suppose it has probability $1/3$ of being dropped. Given that a packet is not routed through Ntontozi, suppose it has probability $1/4$ of being dropped.
 - i. Find the probability that a packet is dropped.
 - ii. Find the probability that a packet is routed through Ntontozi when it is not dropped.
- b) An electrical system consists of four components.



The system works if components A and B work and either of the components C or D works. The reliability (probability of working) of each component is also shown in the graph. Find the probability that

- i. The entire system works

- ii. component C does not work, given that the entire system works. Assume that the four components work independently.

(5+5 Marks)

Question 3

- a) Let X denote the life time (in hundreds of hours) of a certain type of electronic component. These components frequently fail immediately upon insertion into the system. It has been observed that the probability of immediate failure is $1/4$. If a component does not fail immediately, the life-length distribution has the exponential density:

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{Otherwise} \end{cases}$$

Find the distribution function for X and evaluate $P(X > 10)$.

(10 Marks)

- b) If X is a random variable characterized by the pdf $f(x) = \lambda e^{-\lambda x}, x > 0$ variable with parameter $\lambda = 1$, Show that k^{th} moments of X is $k!$.

(10 Marks)

Question 4

- a) A company prices its hurricane insurance using the following assumptions:
- In any calendar year, there can be at most one hurricane.
 - In any calendar year, the probability of a hurricane is 0.05.
 - The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

(10 Marks)

- b) An insurance policy on an electrical device pays a benefit of 4000 if the device fails during the first year. The amount of the benefit decreases by 1000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4. What is the expected benefit under this policy?

(10 Marks)

Question 5

- a) The probability function for random variable X is

$$f(x) = \begin{cases} 2^{-x}, & x = 1, 2, \dots \\ 0, & \text{otherwise} \end{cases}$$

Find the probability function of a random variable $U = X^4 + 1$.

(10 Marks)

- b) The probability function of a random variable X is given by

$$f(x) = \begin{cases} x^2/18, & -3 < x < 6 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability function for the random variable $U = \frac{1}{3}(12 - X)$.

(10 Marks)

Question 6

A company manufacturing pacemakers is testing a new electrode. The electrodes must adhere to a silicone substrate for at least 20 years. The company is going to test the hypothesis that the mean adherence time is 20 years vs. the alternative that it is less than 20 years at the significance level $\alpha = 0.05$. The experiment will be conducted with a sample of 25 volunteers. Assume that the population distribution for the adherence time is approximately normally distributed.

The average adherence time for the pacemakers in the 25 volunteers is found to be 18.8 years and the standard deviation of the sample is found to be 3 years.

- a) Is the null hypothesis rejected?

(10 Marks)

- b) If the company wants to decrease the probability of making a type I error without increasing the sample size, should the critical value be increased or decreased? Justify your answer.

(5 Marks)

- c) Find the 95% confidence interval for the population variance σ^2 .

(5 Marks)

Question 7

A company manufacturing light bulbs is testing a new model. The company is going to test the hypothesis that the mean life time is 1000 hours vs. the alternative hypothesis that it is less than 1000 hours at the significance level = 0.02. Assume that the population distribution for life time is approximately normal. A sample of 16 light bulbs are found to have sample mean $\bar{x} = 987.5$ hours and sample variance $S^2 = 400$.

- a) State the critical region and answer whether the null hypothesis H_0 is rejected.

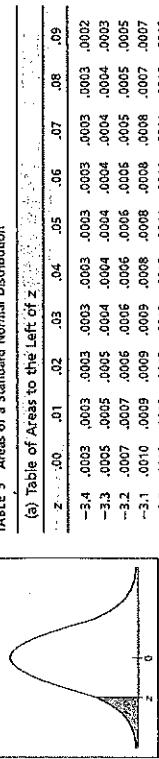
(12 Marks)

- b) Find a 90% confidence interval for the population variance σ^2 .

(8 Marks)

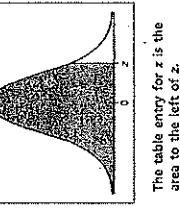
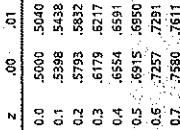
A22 Appendix II Tables

TABLE 5 Areas of Standard Normal Distribution

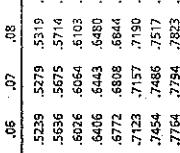


(a) Table of Areas to the Left of z

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0005	.0005	.0005	.0005
-3.1	.0010	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0010	.0010
-2.9	.0016	.0016	.0016	.0016	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0020	.0020	.0020	.0020	.0020	.0020	.0019	.0019	.0019	.0019
-2.7	.0025	.0025	.0024	.0023	.0023	.0022	.0021	.0020	.0019	.0019
-2.6	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.5	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.4	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.3	.0082	.0080	.0078	.0078	.0075	.0073	.0071	.0069	.0068	.0066
-2.2	.0107	.0104	.0102	.0102	.0099	.0096	.0094	.0091	.0089	.0084
-2.1	.0139	.0136	.0132	.0130	.0125	.0122	.0119	.0116	.0113	.0110
-2.0	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-1.9	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.8	.0287	.0281	.0277	.0288	.0282	.0262	.0256	.0250	.0244	.0239
-1.7	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.6	.0446	.0436	.0422	.0418	.0405	.0392	.0384	.0375	.0367	.0356
-1.5	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.4	.0668	.0655	.0643	.0630	.0618	.0605	.0594	.0582	.0571	.0559
-1.3	.0798	.0783	.0778	.0764	.0755	.0749	.0735	.0721	.0708	.0694
-1.2	.1068	.1051	.1034	.1018	.1001	.0985	.0969	.0953	.0938	.0923
-1.1	.1351	.1311	.1293	.1263	.1233	.1203	.1175	.1156	.1138	.1103
-1.0	.1887	.1862	.1839	.1815	.1792	.1768	.1745	.1711	.1685	.1650
-0.9	.2419	.2399	.2361	.2333	.2305	.2277	.2249	.2217	.2187	.2155
-0.8	.3070	.2959	.2839	.2727	.2605	.2483	.2361	.2236	.2115	.2085
-0.7	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.6	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.5	.5406	.5305	.5195	.5095	.4995	.4895	.4795	.4695	.4595	.4495
-0.4	.6446	.6349	.6249	.6149	.6049	.5949	.5849	.5749	.5649	.5549
-0.3	.7621	.7383	.7145	.6907	.6669	.6429	.6189	.5949	.5699	.5449
-0.2	.897	.8207	.7427	.6647	.5867	.5087	.4307	.3527	.2747	.1967
-0.1	.997	.997	.997	.997	.997	.997	.997	.997	.997	.997
0.0	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997

For values of *z* less than -3.49, use 0.000 to approximate the area.The table entry for *z* is the area to the left of *z*.The table entry for *z* is the area to the left of *z*.

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0005	.0005	.0005	.0005
-3.1	.0010	.0010	.0009	.0009	.0008	.0008	.0008	.0007	.0007	.0007
-3.0	.0013	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0010	.0010
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-2.7	.0025	.0025	.0024	.0023	.0023	.0022	.0021	.0020	.0019	.0019
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-2.5	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
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-2.3	.0082	.0080	.0078	.0078	.0075	.0073	.0071	.0069	.0068	.0066
-2.2	.0107	.0104	.0102	.0102	.0099	.0096	.0094	.0091	.0089	.0084
-2.1	.0139	.0136	.0132	.0130	.0125	.0122	.0119	.0116	.0113	.0110
-2.0	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-1.9	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
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-0.1	.997	.997	.997	.997	.997	.997	.997	.997	.997	.997

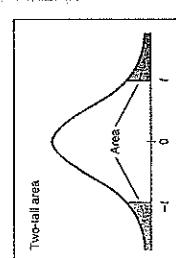
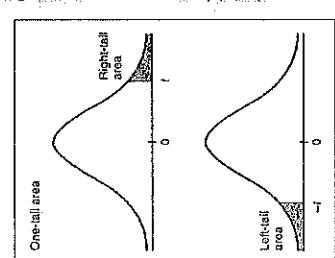
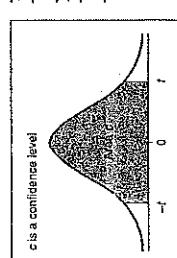
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-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0005	.0005	.0005	.0005
-3.1	.0010	.0010	.0009	.0009	.0008	.0008	.0008	.0007	.0007	.0007
-3.0	.0013	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0010	.0010
-2.9	.0016	.0016	.0016	.0016	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0020	.0020	.0020	.0020	.0020	.0020	.0019	.0019	.0019	.0019
-2.7	.0025	.0025	.0024	.0023	.0023	.0022	.0021	.0020	.0019	.0019
-2.6	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.5	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.4	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.3	.0082	.0080	.0078	.0076	.0073	.0071	.0069	.0067	.0065	.0064
-2.2	.0107	.0104	.0102	.0102	.0099	.0096	.0093	.0090	.0087	.0084
-2.1	.0139	.0136	.0132	.0130	.0125	.0122	.0119	.0116	.0113	.0110
-2.0	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
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-1.7	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301</	

A24 Appendix II Tables

TABLE 6 Critical Values for Student's t Distribution

d.f.	One-tail area		0.050		0.025		0.010		0.005	
	Two-tail area	c	0.050	0.045	0.030	0.020	0.010	0.005	0.0010	0.0005
1	1.000	2.414	3.078	4.165	6.314	12.706	31.821	63.857	686.619	5,699
2	0.816	1.604	1.885	2.282	2.920	4.203	6.965	9.325	31.999	31.999
3	0.675	1.423	1.638	1.924	2.353	3.182	4.541	5.641	21.224	21.224
4	0.541	1.344	1.533	1.778	2.132	2.776	3.747	4.604	8.610	8.610
5	0.427	1.301	1.476	1.859	2.015	2.571	3.365	4.032	6.659	6.659
6	0.338	1.273	1.440	1.850	1.943	2.445	3.143	3.707	5.659	5.659
7	0.271	1.256	1.415	1.617	1.895	2.365	2.998	3.489	5.408	5.408
8	0.226	1.240	1.397	1.592	1.865	2.306	2.895	3.355	5.041	5.041
9	0.195	1.230	1.389	1.575	1.883	2.262	2.821	3.250	4.781	4.781
10	0.170	1.222	1.372	1.543	2.028	2.764	3.168	3.689	4.587	4.587
11	0.147	1.214	1.363	1.548	1.795	2.201	2.718	3.108	4.437	4.437
12	0.130	1.209	1.356	1.538	1.782	2.173	2.681	3.055	4.318	4.318
13	0.116	1.204	1.350	1.530	1.771	2.160	2.650	3.012	4.221	4.221
14	0.102	1.200	1.345	1.523	1.751	2.145	2.624	2.977	4.140	4.140
15	0.091	1.197	1.341	1.517	1.753	2.131	2.602	2.947	4.073	4.073
16	0.080	1.194	1.337	1.512	1.746	2.120	2.583	2.921	4.015	4.015
17	0.070	1.191	1.335	1.508	1.740	2.110	2.567	2.891	3.965	3.965
18	0.060	1.188	1.330	1.504	1.736	2.073	2.552	2.878	3.822	3.822
19	0.052	1.187	1.328	1.500	1.728	2.093	2.539	2.861	3.883	3.883
20	0.046	1.185	1.325	1.497	1.725	2.088	2.528	2.845	3.850	3.850
21	0.040	1.183	1.323	1.494	1.721	2.080	2.518	2.831	3.848	3.848
22	0.035	1.182	1.321	1.492	1.717	2.074	2.508	2.819	3.822	3.822
23	0.030	1.180	1.319	1.489	1.714	2.059	2.500	2.807	3.768	3.768
24	0.026	1.179	1.318	1.487	1.711	2.064	2.492	2.797	3.745	3.745
25	0.023	1.178	1.316	1.485	1.708	2.060	2.485	2.787	3.725	3.725
26	0.020	1.177	1.315	1.483	1.705	2.055	2.479	2.779	3.707	3.707
27	0.017	1.176	1.314	1.482	1.703	2.054	2.473	2.771	3.690	3.690
28	0.015	1.175	1.313	1.480	1.701	2.048	2.465	2.763	3.674	3.674
29	0.013	1.176	1.311	1.479	1.691	2.045	2.467	2.755	3.659	3.659
30	0.011	1.172	1.310	1.477	1.697	2.042	2.457	2.750	3.646	3.646
35	0.008	1.162	1.306	1.472	1.690	2.039	2.438	2.724	3.591	3.591
40	0.006	1.167	1.303	1.468	1.684	2.024	2.423	2.704	3.551	3.551
45	0.005	1.165	1.301	1.465	1.679	2.014	2.412	2.690	3.520	3.520
50	0.004	1.164	1.299	1.462	1.676	2.009	2.403	2.678	3.496	3.496
60	0.003	1.162	1.296	1.461	1.671	2.000	2.390	2.660	3.460	3.460
70	0.002	1.160	1.294	1.459	1.667	1.997	2.386	2.648	3.435	3.435
80	0.001	1.159	1.292	1.453	1.664	1.996	2.374	2.639	3.416	3.416
90	0.001	1.157	1.290	1.451	1.660	1.984	2.364	2.626	3.390	3.390
100	0.001	1.156	1.288	1.442	1.658	1.985	2.354	2.616	3.369	3.369
1000	0.001	1.151	1.282	1.441	1.646	1.982	2.330	2.581	3.300	3.300

TABLE 6 Critical Values for Student's t DistributionTABLE 7 The χ^2 Distribution

d.f.	Right-tail Area		Right-tail Area	
	.995	.990	.975	.950
1	0.0593	0.0157	0.0092	0.0038
2	0.0106	0.0201	0.0516	0.131
3	0.027	0.15	0.216	0.584
4	0.207	0.39	0.864	1.011
5	0.476	0.854	1.145	1.61
6	0.575	0.972	1.24	1.64
7	0.989	1.34	1.69	2.33
8	1.34	1.65	2.18	2.73
9	1.73	2.09	2.70	3.47
10	2.16	2.56	3.25	4.87
11	2.60	3.05	3.82	5.58
12	3.07	3.57	4.20	6.53
13	3.57	4.11	5.01	7.98
14	4.07	4.66	5.63	8.57
15	4.60	5.33	6.26	9.26
16	5.14	5.81	6.91	10.56
17	5.70	6.41	7.56	10.99
18	6.26	7.01	8.23	11.45
19	6.84	7.57	8.63	11.65
20	7.43	8.26	9.39	12.44
21	8.03	8.90	10.28	11.99
22	8.64	9.54	11.96	13.24
23	9.26	10.20	11.69	13.99
24	9.89	10.86	12.40	13.85
25	10.52	11.52	13.12	14.04
26	11.16	12.20	13.84	15.38
27	11.81	12.88	14.57	16.15
28	12.46	13.56	15.31	16.94
29	13.21	14.26	15.95	17.77
30	13.79	14.85	16.79	18.49
40	20.71	22.16	24.43	26.51
50	27.99	29.71	32.35	34.76
60	35.53	37.48	40.48	43.19
70	43.26	45.44	48.75	51.74
80	51.17	53.84	57.15	60.39
90	59.20	61.75	65.65	68.13
100	67.33	70.06	74.22	77.93

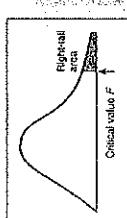
Source: From H. L. Hettler, Biometrika, June 1954. Printed by permission of the Biometrika Trustees.

For degrees of freedom $d.f.$ not in the table, see the closest $d.f.$ that is smaller.

0.674 1.150 1.282 1.440 1.645 1.850 2.226 2.576 3.251

TABLE 8 Critical Values For F Distribution

Degrees of freedom numerator, d.f.N										
Right-tail area		1	2	3	4	5	6	7	8	9
0.100	36.86	45.50	53.59	55.83	57.24	58.20	58.51	58.84	59.26	
0.050	161.45	195.50	215.71	224.58	230.16	233.99	238.88	240.54		
1	647.79	798.59	864.16	893.58	921.95	927.11	946.22	956.56	963.28	
0.010	4052.2	4995.5	5403.4	5624.6	5783.6	5828.4	5981.1	6022.5		
0.001	405284	500000	50379	562500	57605	585937	592873	598346	602284	
0.100	1.53	3.00	3.16	3.24	3.29	3.33	3.37	3.41	3.45	
0.050	18.51	19.00	19.16	19.22	19.30	19.33	19.35	19.37	19.38	
2	0.025	36.51	39.00	39.31	39.50	39.71	39.86	39.95	39.99	
0.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	
0.001	998.50	998.00	998.17	998.25	998.30	999.33	999.35	999.37	999.39	
0.100	5.54	5.59	5.64	5.71	5.76	5.82	5.87	5.92	5.98	
0.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.81		
3	0.025	17.44	16.04	15.10	14.88	14.73	14.62	14.51	14.47	
0.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	
0.001	157.03	184.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86	
0.100	0.54	4.32	4.69	5.11	4.65	4.61	4.58	4.55	4.54	
0.050	1.71	6.94	6.59	6.39	6.26	6.16	6.04	6.00		
4	0.025	11.22	10.65	9.98	9.60	9.35	9.07	8.81		
0.010	22.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.68	
0.001	74.14	51.25	56.18	53.44	51.71	50.53	49.65	49.00	48.47	
0.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	
0.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82		
5	0.025	10.01	8.43	7.76	7.39	7.15	6.88	6.85	6.76	
0.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	
0.001	47.18	37.12	33.20	31.05	29.75	28.83	28.16	27.65	27.24	
0.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	
0.050	5.59	5.14	4.76	4.75	4.53	4.39	4.26	4.21	4.15	
6	0.025	8.81	7.26	6.60	5.99	5.59	5.20	5.00	4.52	
0.010	19.75	16.52	9.78	9.15	8.75	8.47	8.26	8.10	7.98	
0.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	8.39	
0.100	3.59	3.26	3.07	2.95	2.88	2.83	2.78	2.75	2.72	
0.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	
7	0.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	
0.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.09	5.91	
0.001	22.41	18.46	15.83	14.39	13.48	12.45	12.05	11.77		



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TABLE 8 continued

Degrees of freedom numerator, d.f.N										Degrees of freedom denominator, d.f.D			
Right-tail area		10	12	15	18	20	25	30	40	50	60	120	1000
0.100	60.19	60.71	61.22	61.74	62.26	62.53	62.69	62.79	63.06	63.30	63.50	63.70	63.90
0.050	241.88	243.91	245.95	248.01	249.26	250.19	251.14	251.77	252.20	253.25	254.19	255.14	256.10
1	986.53	976.71	984.87	983.10	988.08	981.14	1005.6	1008.1	1009.8	1011.0	1011.7	1012.4	1013.1
0.010	6055.8	6105.3	6157.3	6208.7	6233.8	6260.6	6286.8	6313.0	6339.4	6359.7	6369.7	6379.2	6389.1
0.001	605521	610558	615754	620898	624037	626094	628712	631337	633972	635901	636901	637927	638927
0.100	5.39	5.41	5.42	5.44	5.45	5.46	5.47	5.48	5.49	5.50	5.51	5.52	5.53
0.050	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.48	19.49	19.49	19.49	19.49
2	0.025	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.48	39.49	39.50	39.50
0.010	593.40	598.42	599.43	599.45	599.46	599.47	599.47	599.48	599.48	599.49	599.49	599.49	599.49
0.001	599.40	599.42	599.43	599.45	599.46	599.47	599.47	599.48	599.48	599.49	599.49	599.49	599.49
0.100	5.10	5.23	5.30	5.38	5.46	5.54	5.62	5.70	5.78	5.86	5.94	5.98	5.98
0.050	8.79	8.74	8.70	8.66	8.62	8.59	8.56	8.53	8.50	8.47	8.55	8.53	8.53
3	0.025	14.42	14.34	14.25	14.17	14.11	14.06	14.01	13.99	13.95	13.91	13.87	13.87
0.010	27.23	27.05	26.87	26.69	26.51	26.35	26.21	26.04	25.87	25.70	25.55	25.40	25.40
0.001	129.25	128.32	127.37	126.42	125.45	124.56	123.57	122.57	121.57	120.57	120.47	120.37	120.37
0.100	5.39	5.50	5.67	5.84	5.93	5.98	6.02	6.06	6.09	6.12	6.15	6.18	6.18
0.050	5.86	5.91	5.96	6.02	6.06	6.10	6.14	6.18	6.22	6.26	6.30	6.34	6.34
4	0.025	8.84	8.76	8.68	8.56	8.45	8.36	8.26	8.16	8.06	8.00	8.06	8.06
0.010	14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.65	13.56	13.47	13.47	13.47
0.001	48.05	47.41	46.75	46.10	45.70	45.43	45.09	44.88	44.75	44.60	44.49	44.49	44.49
0.100	5.30	5.37	5.42	5.49	5.56	5.63	5.70	5.77	5.84	5.91	5.98	6.05	6.05
0.050	6.74	6.82	6.86	6.92	6.96	7.02	7.08	7.14	7.20	7.26	7.32	7.38	7.38
5	0.025	6.62	6.52	6.43	6.33	6.27	6.23	6.18	6.14	6.12	6.07	6.02	6.02
0.010	10.05	9.89	9.72	9.59	9.38	9.24	9.10	9.00	8.90	8.80	8.70	8.60	8.60
0.001	26.92	26.62	26.42	25.91	25.58	25.08	24.87	24.60	24.44	24.33	24.05	23.82	23.82
0.100	5.24	5.30	5.37	5.44	5.51	5.58	5.65	5.72	5.78	5.84	5.90	5.96	5.96
0.050	4.06	4.00	3.94	3.87	3.81	3.75	3.70	3.67	3.64	3.61	3.58	3.55	3.55
6	0.025	5.46	5.37	5.27	5.17	5.07	5.01	4.98	4.95	4.92	4.89	4.86	4.86
0.010	7.87	7.72	7.56	7.40	7.23	7.04	6.87	6.69	6.49	6.29	6.09	5.89	5.89
0.001	18.41	17.99	17.56	17.12	16.85	16.67	16.44	16.21	16.01	15.77	15.56	15.36	15.36
0.100	2.70	2.67	2.63	2.59	2.57	2.55	2.52	2.51	2.49	2.47	2.45	2.43	2.43
0.050	3.64	3.57	3.51	3.44	3.36	3.28	3.22	3.15	3.08	3.02	3.01	2.97	2.97
7	0.025	4.76	4.67	4.57	4.47	4.37	4.28	4.20	4.12	4.04	3.96	3.84	3.84
0.010	6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.86	5.82	5.77	5.72	5.66	5.66
0.001	14.08	13.71	13.32	12.93	12.53	12.19	12.02	11.82	11.62	11.41	11.21	11.01	11.01
0.100	2.54	2.50	2.46	2.42	2.38	2.35	2.32	2.29	2.26	2.23	2.20	2.17	2.17
0.050	3.95	3.82	3.78	3.72	3.68	3.64	3.60	3.54	3.49	3.44	3.39	3.33	3.33
8	0.025	4.30	4.20	4.10	4.00	3.94	3.89	3.84	3.78	3.73	3.68	3.63	3.63
0.010	5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.07	5.03	4.95	4.87	4.81	4.81
0.001	11.54	11.19	10.84	10.48	10.11	9.82	9.50	9.23	9.00	8.73	9.53	9.35	9.35

TABLE 8 (continued)

		Degrees of freedom numerator, $d_{f,1}$								
		1	2	3	4	5	6	7	8	9
Right-tail area										
0.000	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	
0.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	
0.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	
0.010	10.55	8.02	6.99	6.42	5.96	5.80	5.67	5.47	5.35	
0.001	22.86	16.38	13.50	12.56	11.71	11.13	10.70	10.37	10.11	
0.000	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	
0.050	4.66	4.10	3.71	3.49	3.33	3.22	3.14	3.07	3.02	
0.025	6.94	5.45	4.83	4.47	4.24	4.07	3.95	3.85	3.78	
0.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	
0.001	21.04	14.81	12.55	11.28	10.48	9.93	9.52	9.20	8.96	
0.000	3.22	2.86	2.65	2.54	2.45	2.39	2.34	2.30	2.27	
0.050	4.84	3.98	3.59	3.38	3.20	3.09	3.01	2.95	2.90	
0.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	
0.010	9.55	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	
0.001	19.63	13.31	11.56	10.35	9.58	9.05	8.66	8.35	8.12	
0.000	3.18	2.85	2.61	2.48	2.39	2.33	2.28	2.24	2.21	
0.050	4.75	3.89	3.48	3.26	3.11	3.00	2.91	2.85	2.80	
0.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	
0.010	9.53	6.93	5.95	5.41	5.05	4.82	4.64	4.59	4.53	
0.001	18.64	12.37	10.80	9.63	8.89	8.38	8.09	7.71	7.48	
0.000	3.14	2.76	2.55	2.43	2.35	2.28	2.23	2.20	2.16	
0.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	
0.025	6.41	4.87	4.35	4.00	3.77	3.60	3.48	3.39	3.31	
0.010	9.20	6.52	5.74	5.21	4.86	4.52	4.46	4.30	4.18	
0.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98	
0.000	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	
0.050	4.60	3.74	3.31	3.01	2.76	2.65	2.78	2.70	2.65	
0.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	
0.010	8.86	6.55	5.55	5.04	4.69	4.46	4.29	4.14	4.03	
0.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58	
0.000	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	
0.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	
0.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	
0.010	8.68	6.56	5.42	4.89	4.56	4.32	4.14	4.00	3.89	
0.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26	
0.000	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	
0.050	4.49	3.63	3.24	3.01	2.85	2.74	2.65	2.59	2.54	
0.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	
0.010	8.53	6.33	5.23	4.77	4.44	4.20	4.03	3.89	3.78	
0.001	16.12	10.47	9.01	7.94	7.27	6.80	6.45	6.19	5.98	