UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE & ENGINEERING

DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

SIGNALS AND SYSTEMS I

COURSE CODE – EEE331/EE331

MAIN EXAMINATION

DECEMBER 2018

DURATION OF THE EXAMINATION - 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. There are **FIVE** questions in this paper. Answer any **FOUR** questions.

3. Each question carries 25 marks.

4. Show all your steps clearly in any calculations/work

5. Start each new question on a fresh page.

6. Useful Fourier and Laplace transform properties are **attached**.

7. Make sure that this exam contains 7 pages including this one.

DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION ONE (*25 marks*)

(a) (6 pts) Let $x(n) = \cos(\omega(n+p)+\theta)$.

Determine the period of $x(n)$ for each of the following cases.

(i) $\omega = \frac{3\pi}{4}, \quad p = 2 \quad , \quad \theta = \frac{\pi}{4}$

(ii) $\omega = \frac{3}{4}, \quad p = 1 \quad , \quad \theta = \frac{1}{4}$

(b) (9 pts) Calculate the total energy (E_∞) and average power (P_∞) over infinite duration for each of the following signals:

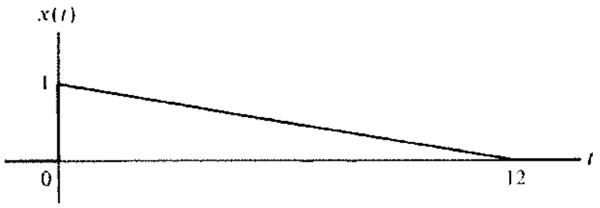
(i) $x(t) = 2e^{-8|t|}$

(ii) $x(t) = u(t)$

(c) (10 pts) For $x(t)$ indicated below, sketch the following:

(i) $x(1-t)[u(t+1) - u(t-2)]$

(ii) $x(t)[u(t+1) - u(2-t)]$



QUESTION TWO (*25 marks*)

(a) (15 pts) A continuous-time LTI system is represented by the first-order differential equation for the relationship of input $x(t)$ and output $y(t)$,

$$\frac{dy(t)}{dt} + 5y(t) = 2x(t)$$

Find the output $y(t)$ for $t > 0$, if the auxiliary condition $y(0) = 3$ and the input signal is $x(t) = 9e^{-2t}u(t)$.

(b) (10 pts) Given a first-order system: $y(n) + \frac{1}{2}y(n-1) = 2x(n)$

Find the system response with initial condition of rest for $x(n) = \left(\frac{1}{2}\right)^n u(n)$.

Express your result in closed form.

QUESTION THREE (*25 marks*)

(a) (5 pts) Given a continuous-time system with the input-output relation described by,

$$y(t) = x(t+3) - x(1-t)$$

Determine whether the above system is Causal, Linear and/or Time-invariant.

(b) (20 pts) Determine the fundamental frequency and the Fourier series coefficients for the following signals:

(i) $x(t) = sin^2(t)$

(ii) $x(n) = \sin(2\pi n/3) \cdot \cos(\pi n/2)$

QUESTION FOUR (*25 marks*)

(a) (10 pts) The output $y(t)$ of a continuous-time LTI system is $2e^{-3t}\,u(t)$ when the input $x(t)$ is $u(t)$.

(i) Find the impulse response $h(t)$ of the system using Laplace Transform.

(ii) Find the output $y(t)$ of the system when the input $x(t)$ is $e^{-t}u(t)$ using Laplace Transform.

(b) (7 pts) Determine the Laplace transform, pole and zero locations, and associated ROC for $e^{-a|t|}$, $a > 0$.

(c) (8 pts) Find the inverse transform of the following signals:

(i) $X(s) = \frac{s^2+6s+7}{s^2+3s+2}$ Re{s} > -1

QUESTION FIVE (*25 marks*)

(a) (10 pts) Using the Fourier transform analysis equation, determine the Fourier transforms of:

(i) $x(t) = e^{-2|t-1|}$

(ii) $x(t) = e^{-3t}\sin(2t)u(t)$

(b) (5 pts) Find the inverse Fourier transform of $X(\omega) = \frac{1}{(3+j\omega)^2}$.

(c) (10 pts) Consider the signal $x(t)$ with Fourier transform

$$X(\omega) = \begin{cases} 1, & \text{for } |\omega| \le 20\pi \\ 0, & otherwise \end{cases}.$$

For the following signals, find a mathematical expression for the Fourier transform and plot its magnitude.

(i) $y(t) = 2x(t)cos(15\pi t)$

(ii) $q(t) = x(-\frac{t}{5})$

Table of Laplace Transforms

delta function	$\delta(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	1
shifted delta function	$\delta(t-a)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	e^{-as}
unit step	$u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}$
ramp	$tu(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2}{s^3}$
n-th power	t^n	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\|t\|}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$-F'(s)$
frequency n-th differentiation	$t^nf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$(-1)^nF^{(n)}(s)$
time differentiation	$f'(t)=\frac{d}{dt}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$sF(s)-f(0)$
time 2nd differentiation	$f''(t)=\frac{d^2}{dt^2}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^2F(s)-sf(0)-f'(0)$
time n-th differentiation	$f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau=(u*f)(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$

Properties of Laplace Transforms

i) Time-shift (delay): $f(t-t_0) \overset{L}{\longleftrightarrow} F(s)e^{-st_0}, \quad t_0 > 0$

ii) Time differentiation: $\frac{df(t)}{dt} \overset{L}{\longleftrightarrow} sF(s) - f(0)$

iii) Time integration: $\int_0^t f(t)dt \overset{L}{\longleftrightarrow} \frac{F(s)}{s}$

iv) Linearity: $af(t) + bg(t) \overset{L}{\longleftrightarrow} aF(s) + bF(s)$

v) Convolution Integral: $x(t) * h(t) \overset{L}{\longleftrightarrow} X(s)H(s)$

vi) Frequency-shift: $e^{\alpha t} f(t) \overset{L}{\longleftrightarrow} F(s-\alpha)$

vii) Multiplying by t: $tf(t) \overset{L}{\longleftrightarrow} -\frac{dF(s)}{ds}$

viii) Scaling: $f(at) \overset{L}{\longleftrightarrow} \frac{1}{a} F\left(\frac{s}{a}\right), \quad a > 0$

ix) Initial Value Theorem: $\lim_{s\to\infty}\{sF(s)\} = f(0)$

x) Final Value Theorem: $\lim_{s\to 0}\{sF(s)\} = f(\infty)$

TABLE 1 Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\dfrac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\dfrac{\omega\tau}{2}\right)$	
18	$\dfrac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\dfrac{\omega}{2W}\right)$	
19	$\Delta\left(\dfrac{t}{\tau}\right)$	$\dfrac{\tau}{2}\operatorname{sinc}^2\left(\dfrac{\omega\tau}{4}\right)$	
20	$\dfrac{W}{2\pi}\operatorname{sinc}^2\left(\dfrac{Wt}{2}\right)$	$\Delta\left(\dfrac{\omega}{2W}\right)$	
21	$\displaystyle\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\displaystyle\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\dfrac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table 2 Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\|a\|} X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\, du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

Useful Formulae:-

Trigonometric Identity:- $sin^2(\theta) = \frac{1}{2}[1 - \cos(2\theta)]$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

Euler's relation:- $e^{j\theta} = \cos(\theta) + jsin(\theta)$

$$Cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right), \quad sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$