UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE & ENGINEERING

DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

SIGNALS AND SYSTEMS I

COURSE CODE – EEE331

MAIN EXAMINATION 2019

DURATION OF THE EXAMINATION - 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. There are **FIVE** questions in this paper. Answer any **FOUR** questions.

2. Each question carries 25 marks.

3. Start each new question on a fresh page.

4. Useful tables are attached at the end of the question paper.

5. Make sure that this exam contains 7 pages including this one.

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QUESTION ONE (*25 marks*)

(a) (6 pts.) Sketch and label carefully the following signals.

(i) $x(t) = e^t\,(u(t+2) + u(t-3))$

(ii) $x(n) = 2u(-n+2) - 3u(n-2)$

(b) (i) (6 pts.) Calculate the energy of the signal $x(t)$ given in Fig. 1.

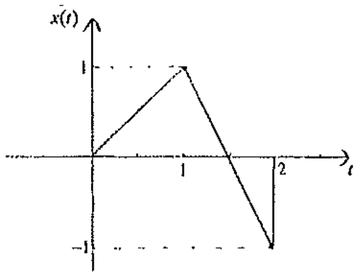

Fig. 1

(ii) (4 pts.) Sketch the odd component of the signal in Fig. 1 (Show your steps!)

(c) (9 pts) Fill in the following table (for each column, give a "yes/no" answer and briefly justify it):

System	Linear	Time-invariant	Causal
$y(n) = 3x(n+1)u(n) + 1$			
$y(t) = 2x(t)\cos(t)$			
$y(t) = tx(2t)$			

QUESTION TWO (*25 marks*)

(a) (15 pts) Let $x(t) = \begin{cases} t, & 0 < t < 1 \\ 2-t, & 1 \le t \le 2 \end{cases}$ be a periodic signal with fundamental period $T = 2$ and Fourier coefficients a_k.

(i) Determine the value of a_0.

(ii) Determine the Fourier series representation of $\frac{dx(t)}{dt}$.

(b) (10 pts) Consider the following circuit (fig. 2):

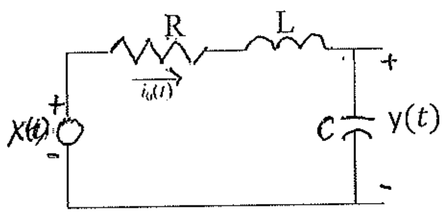

Fig. 2

Write down the input-output differential equation for this circuit in terms of the input voltage $x(t)$ and the output voltage $y(t)$. (Note that $i_c(t) = C\frac{dV_c(t)}{dt}$ and $v_L(t) = L\frac{di_L(t)}{dt}$.)

QUESTION THREE (*25 marks*)

(a) (8 pts.) Given a first-order system

$$y(n) + \frac{1}{2}y(n-1) = 2x(n)$$

Find the system response with initial condition of rest for $x(n) = \left(\frac{1}{2}\right)^n u(n)$.

Express your result in closed form.

(b) (12 pts) An LTI system generates the output $y(t) = (e^{-2t} - e^{-3t})u(t)$ in response to the input $x(t) = e^{-2t}u(t)$. Determine the unit impulse response $h(t)$ of the system. Sketch the magnitude $|H(\omega)|$.

(c) (5 pts) Consider a discrete-time system which has input of signal $x(n)$ and output of $y(n) = \cos\left(\frac{\pi}{4}x(n)\right)$. Evaluate and draw the impulse response of the above system.

QUESTION FOUR (*25 marks*)

(a) (12 pts) Determine the Laplace transform and the ROC for each of the following signals:

(i) $x(t) = e^{at}u(t-2)$
Use the definition of Laplace transform. Sketch the pole – zero for plot for a < 0.

(ii) $x(t) = te^{at}u(t) + e^{at}u(t-4) + u(t-5) + 2\delta(t)$

(iii) $x(t) = \sin(3t)\cos(3t)$

(b) (6+7 pts) Find the inverse Laplace transform of

(i) $F(s) = \frac{s^2 - s + 1}{s^2(s-1)}$ $\quad 0 < \text{Re}\{s\} < 1$

(ii) $F(s) = \frac{s^2 + 6s + 7}{s^2 + 3s + 2}$ $\quad \text{Re}\{s\} > -1$

QUESTION FIVE (*25 marks*)

(a) (10 pts) Let $x(t)$ be a signal, and let $X(\omega)$ be its Fourier transform. Find the Fourier transform of the signal $y(t) = x(\beta(t-\alpha))$. Where $\beta \neq 0$ and $\alpha \in \mathcal{R}$.

(b) (15 pts.) Using the definition of the Fourier transform, compute the Fourier transform of the following:

(i) $u(t) - u(t+2)$

(ii) $e^{3t}\sin(2t)\,u(-t)$

Table of Laplace Transforms

delta function	$\delta(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	1
shifted delta function	$\delta(t-a)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	e^{-as}
unit step	$u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}$
ramp	$tu(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2}{s^3}$
n-th power	t^n	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\lvert t\rvert}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$-F'(s)$
frequency n-th differentiation	$t^nf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$(-1)^nF^{(n)}(s)$
time differentiation	$f'(t)=\frac{d}{dt}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$sF(s)-f(0)$
time 2nd differentiation	$f''(t)=\frac{d^2}{dt^2}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^2F(s)-sf(0)-f'(0)$
time n-th differentiation	$f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau=(u*f)(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$

Properties of Laplace Transforms

i) Time-shift (delay): $f(t-t_0) \overset{L}{\longleftrightarrow} F(s)e^{-st_0}, \quad t_0 > 0$

ii) Time differentiation: $\dfrac{df(t)}{dt} \overset{L}{\longleftrightarrow} sF(s) - f(0)$

iii) Time integration: $\displaystyle\int_0^t f(t)dt \overset{L}{\longleftrightarrow} \frac{F(s)}{s}$

iv) Linearity: $af(t) + bg(t) \overset{L}{\longleftrightarrow} aF(s) + bF(s)$

v) Convolution Integral: $x(t) * h(t) \overset{L}{\longleftrightarrow} X(s)H(s)$

vi) Frequency-shift: $e^{\alpha t} f(t) \overset{L}{\longleftrightarrow} F(s-\alpha)$

vii) Multiplying by t: $tf(t) \overset{L}{\longleftrightarrow} -\dfrac{dF(s)}{ds}$

viii) Scaling: $f(at) \overset{L}{\longleftrightarrow} \dfrac{1}{a} F\left(\dfrac{s}{a}\right), \quad a > 0$

ix) Initial Value Theorem: $\lim_{s\to\infty}\{sF(s)\} = f(0)$

x) Final Value Theorem: $\lim_{s\to 0}\{sF(s)\} = f(\infty)$

TABLE 1 Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$a > 0$
2	$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$a > 0$
3	$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$a > 0$
4	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	$a > 0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$a > 0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a > 0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a > 0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table 2 Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\lvert a \rvert} X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\, du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

Useful Formulae:-

Trigonometric Identity:- $sin^2(\theta) = \frac{1}{2}[1 - \cos(2\theta)]$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

Euler's relation:- $e^{j\theta} = \cos(\theta) + jsin(\theta)$

$$Cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right), \quad sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$