UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2008/09

BSc.II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE (5) QUESTIONS 3. ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY TEH INVIGILATOR.

QUESTION 1

(a) Show that $f(x) = 3x + 5$ is continuous at $x_0 = 2$. [3]

(b) (i) Let the points P, Q, R, S, have Cartesian coordinates $(p_1, p_2), (q_1, q_2), (r_1, r_2)$ and (s_1, s_2), respectively. State and prove the necessary and sufficient conditions for $\overline{PQ}$ and $\overline{RS}$ to represent the same vector.

(ii) Apply this theorem to find the coordinates of point T such that $\overline{PQ} \sim \overline{QT}$, if $P = (4,1), \quad Q = (2,-2)$. [5,2]

(c) Use the quadratic approximation formula to compute $\sqrt{1+x}$ for small $|x|$, and estimate the error. In particular, compute $\sqrt{1.02}$. [6]

(d) Find the fourth Taylor polynomial, at $x_0 = 0$, for $f(x) = \sqrt{1+x}$. [4]

QUESTION 2

(a) Let the points P, Q, R have the Cartesian coordinates $(1,2,3), (4,-1,7)$ and $(6,-1,4)$, respectively. Compute $\theta = (P\hat{Q}R)$. [3]

(b) Find the volume of a parallelopiped spaned by $\overline{a} = \overline{OA}, \quad b = \overline{OB}$ and $\overline{c} = \overline{OC}$. Thus prove the mixed triple product formula

$\overline{a} \cdot (\overline{b} \times \overline{c}) = (\overline{a} \times \overline{b}) \cdot \overline{c}$. [5]

(c) Find the volume obtained by rotating the region bounded by the arc of the parabola $y = x^2$ from $x = 0$ to $x = 2$, the y-axis, and the line $y = 4$ about the y-axis. [5]

(d) What is the equation, in polar coordinates, of

(i) A straight line,

(ii) A vertical line,

(iii) A horizontal line? [3,2,2]

QUESTION 3

(a) Let A, B and C be three 2×2 matrices. Determine which of the following is true. Give examples

(i) $AB = 0 \Rightarrow A = 0$ or $B = 0$.

(ii) $AB = AC \Rightarrow A = 0$ or $B = C$. [2,2]

(b) Consider the matrix

$$A = \begin{bmatrix} -1 & 0 & 2 \\ -4 & 1 & -3 \\ 7 & 0 & -2 \end{bmatrix}.$$

(i) Find the minors and the cofactors for each entry.

(ii) Expand, simplify and evaluate the determinant.

(iii) Check your solution by applying a direct formular for the determinant of a 3×3 matrix. [4,2,3]

(c) Find the maximum and minimum values of $f(x, y) = x^2 + y$ subject to the constraint

$x^2 + y^2 = 4$. [7]

QUESTION 4

(a) Find the inverse and check the result, or state that the inverse does not exist, giving a reason

(i) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(ii) $\begin{bmatrix} 0.5 & 0 & -0.5 \\ -0.1 & 0.2 & 0.3 \\ 0.5 & 0 & -1.5 \end{bmatrix}$. [2,4]

(b) Solve the following system

$$\begin{aligned} -x_1 + x_2 + 2x_3 &= 2 \\ 3x_1 - x_2 + x_3 &= 6 \\ -x_1 + 3x_2 + 4x_3 &= 4, \end{aligned}$$

applying

(i) Gauss elimination method,

(ii) Cramer's rule. [4,4]

(c) Let D be the region defined by the inequalities

$x^2 + y^2 < 1, \quad 0 < z < x^2 + y^2.$

Pass to the cylindrical coordinates to find

$\iiint_D x^2 y^2 dx\, dy\, dz.$ [6]

QUESTION 5

(a) Find the equation of the line through $(1,2)$ which is (i) parallel, (ii) perpendicular to the line $2x - 4y + 5 = 0$. [3,3]

(b) State L'Hospital rule. [3]

(c) Apply the above rule to evaluate the following limits

(i) $\lim\limits_{x \to 0} \dfrac{\sin x - e^x + 1}{x^2}$,

(ii) $\lim\limits_{x \to 0^+} \dfrac{\log \sin x}{\log \tan x}$. [3,3]

(d) Water is poured into a cylindrical container of radius 6cm at the rate of $36cm^3$/sec. How fast is the level of the water rising? [5]

QUESTION 6

(a) State and prove Rolle's theorem. [5]

(b) If $f(x) = x^{\frac{2}{3}}$, find all numbers in the open interval $(-1, 1)$ for which the mean value theorem is satisfied. [3]

(c) Find the area of the region enclosed between the curve

$y = x^3$ and the line $y = x$. [5]

(d) Find the area of the surface generated by rotating the curve c

$c : y = \frac{1}{3}x^3, \quad 0 < x < 2$, about the x-axis, [7]

QUESTION 7

(a) Find the partial derivatives (at $x = 2, \quad y = 3$) of

$$f(x, y) = 3x^3y + 4xy^2 - 2x + 4y - 5.$$

[5]

(b) Find the partial derivatives with respect to u and v of $z = e^{xy}$ where $x = u^2$ and $y = uv$. [7]

(c) Compute the volume under the surface

$$z = f(x, y) = xy + 1$$

over the region D, where $D : 0 < x < 2, \quad 0 < y < 4.$ [8]

END OF EXAMINATION