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University of Swaziland

Supplementary Examination, July 2012

BSc I, EEng I, BEd I

Title of Paper : Algebra, Trig. and Analytic Geometry

Course Number : M111

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

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Question 1

(a) The third term of an AP is 9, while the thirtieth is 117. Find the

i. first term and the common difference. [4]

ii. sum of all the terms between the third and thirtieth, inclusive. [6]

(b) Expand

$$\left(a^2 - \frac{2b^2}{a}\right)^5$$

and simplify term by term. [10]

Question 2

(a) Find the coordinates of the centre and the radius of the circle

$$x^2 + y^2 - 4y + 20x + 4 = 0. \qquad \text{[6 marks]}$$

(b) Evaluate

$$\begin{vmatrix} 1 & 2 & 0 & -2 \\ -2 & 0 & 1 & 5 \\ 2 & 3 & -1 & 0 \\ 0 & -1 & 0 & 2 \end{vmatrix}$$

[6]

(c) Prove

$$\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A} = 1 - \cos A \sin A.$$

[6]

Question 3

(a) Find the 12th term of the binomial expansion of

$$\left(\frac{x^2}{y} - \frac{y^2}{x}\right)^{15}. \qquad [6]$$

(b) A ball falls from a height of 4 metres. If it rebounds to 88% of the height of the previous fall each time, find the total distance it travels as it bounces repeatedly to a rest. [6 marks]

(c) Factorise

$$P(x) = 2x^3 + x^2 - 13x + 6,$$

and hence find all its roots. [8 marks]

Question 4

(a) Solve for x :

i. $3^{2x-1} = 5$ [4]

ii. $\log_3(17 - 4x) = 2 + \log_3(2x - 3)$ [6]

(b) Use mathematical induction to prove

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \tfrac{1}{6}n(n+1)(2n+1),$$

where $n \in \mathbb{Z}$ and $n \geqslant 1$. [10]

Question 5

(a) Use synthetic division to divide

$$\frac{x^4 - 16}{x + 2} \qquad [5]$$

(b) Use Cramer's rule to solve

$$\begin{aligned} x + 5y + z &= 4, \\ 2x - y - z &= 6, \\ x + 2y + 3z &= -5. \end{aligned}$$

[15]

Question 6

(a) Divide

$$\frac{x^4 - 16}{x^2 - 2}. \qquad [7]$$

(b) Find the the first 4 terms of the binomial expansion of

$$(1 + 2x)^{-\frac{1}{2}}. \qquad [7]$$

(c) Find all the cube roots of $-27i$. [6]

Question 7

(a) Evaluate

$$\frac{20i}{4-3i}+\frac{20i}{4i+3},$$

and express your answer in the form $a+ib$. [5]

(b) Solve for x (in the range $0 \leqslant x < 2\pi$)

$$2\cos^2 x + \cos x - 1 = 0.$$ [8]

(c) Use mathematical induction to prove that

$$P(n) = 1 + 3^{2n-1}, \quad n \in \mathbb{Z}, \quad n \geqslant 1,$$

is always divisible by 4. [7]