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# University of Swaziland



## Final Examination, May 2012

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### BSc I, EEng I, BEd I, BASS I

**Title of Paper** : Introduction to Calculus

**Course Number** : M115

**Time Allowed** : Three (3) hours

**Instructions** :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

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**Question 1**

(a) Evaluate

(i)  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - x - 2}$  [3 marks]

(ii)  $\lim_{x \rightarrow \infty} \frac{x^2 - 4}{x^2 - x - 2}$  [3 marks]

(b) Differentiate and simplify

(i)  $y = (x - 1)\sqrt{3 + 2x}$  [4 marks]

(ii)  $y = \tan^{-1}\left(\frac{2}{x}\right)$  [3 marks]

(c) Show that  $y = e^{-t} \sin 2t$  satisfies the differential equation

$$\ddot{y} + 2\dot{y} + 5y = 0, \quad [7 \text{ marks}]$$

where  $\dot{y} = \frac{dy}{dt}$ .

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**Question 2**

(a) Differentiate

(i)  $y = x^{3-2x}$  [5 marks]

(ii)  $F(x) = \ln\left(\frac{x}{1-x}\right)$  [5 marks]

(b) Integrate

$$\int \frac{4x + 5}{x^3 + 2x + x} dx. \quad [10 \text{ marks}]$$

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**Question 3**

(a) Find  $\frac{df}{dx}$  using the *limit definition*, for

$$f(x) = \frac{x}{1-2x}. \quad [8 \text{ marks}]$$

(b) Integrate

i.  $\int \frac{3x-9}{\sqrt{4+6x-x^2}} dx$  [6 marks]

ii.  $\int_0^{\frac{1}{2}\pi} \sin^2 \theta \cos^3 \theta d\theta$  [6 marks]

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**Question 4**

(a) Evaluate

i.  $\lim_{\theta \rightarrow 0} \frac{\theta^2}{1-\cos \theta}$  [4 marks]

ii.  $\lim_{\theta \rightarrow 0} \frac{\ln(e^2 + \theta)}{e^\theta + e^{-\theta}}$  [3 marks]

(b) Use Leibnitz rule to find  $\frac{d^4 f}{dx^4}$ , given

$$f(x) = x^5 e^{-2x}. \quad [7 \text{ marks}]$$

(b) Find the exact value of the area of the region enclosed between the curves  $y = x^2$  and  $y = 16 - x^2$ . [6 marks]

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**Question 5**(a) Find  $y'$ .

i.  $y = e^x + \ln(2x) - \sin 2x - \frac{1}{e^{2x}} + \frac{3}{2x^{\frac{4}{3}}}$  [3 marks]

ii.  $y = \frac{1 - 2x^2}{2 + 3x^2}$  [6 marks]

(b) Find the equation of the normal to the curve  $x^2y - 2xy^2 - 3 = 0$  at the point  $(-1, 1)$ . [4 marks]

(c) Integrate

$$\int x^4 \sin 2x \, dx.$$
 [7 marks]

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**Question 6**

(a) Consider the statement:

*Every inflexion point is a stationary point.*

Is the statement true or false? Explain. [5 marks]

(b) Given

$$y = \frac{x}{\sqrt{2x+1}},$$

find  $y''$ . [8 marks]

(c) Integrate

$$\int \frac{dx}{x^2 + 4x + 13}.$$
 [7 marks]

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**Question 7**

(a) Differentiate

$$\Gamma(\theta) = \sin^4 \theta + 2 \cos^2 \theta - \cos^4 \theta$$

and show that  $\frac{d\Gamma}{d\theta} = 0$ . [6 marks]

(b) Integrate

i.  $\int_0^2 \sqrt{4 - x^2} dx$  [7 marks]

ii.  $\int_0^4 \frac{dx}{2 + \sqrt{x}}$  [7 marks]

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