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University of Swaziland

Final Examination, 2011/2012

BSc II, Bass II, BEd II

Title of Paper : Calculus I

Course Number : M211

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION 1

1.1 Find the absolute maximum and absolute minimum values of the function

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

on the interval $[-2, 3]$. [5]

1.2 Consider the function $f(x) = 3x^5 - 5x^3 + 3$.

1.2.1 Find the intervals on which f is increasing or decreasing. [4]

1.2.2 Find the local maximum and local minimum values of f. [2]

1.2.3 Find the intervals on which f is concave up and concave down. [4]

1.2.4 Find the inflection points of f. [2]

1.2.5 Sketch a graph of f using the information in 1.2.1 – 1.2.4. [3]

QUESTION 2

Evaluate the following limits.

2.1 $\lim\limits_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$ [4]

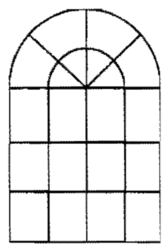
2.2 $\lim\limits_{x\to\infty} x^3 e^{-x}$ [5]

2.3 $\lim\limits_{x\to 0} (\csc x - \cot x)$ [5]

2.4 $\lim\limits_{x\to 0} (1 - 2x)^{1/x}$ [6]

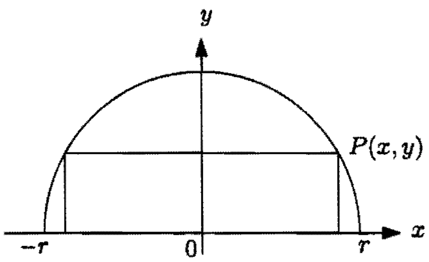
QUESTION 3

3.1 A Norman window has the shape of a rectangle surmounted by a semicircle as shown below.



If the perimeter of the window is 10 m, find the dimensions of the window, so that the greatest possible amount of light is permitted. [10]

3.2 Show that the area of the largest rectangle that can be inscribed in a semicircle of radius r is r^2. (**Hint:** use the picture below). [10]



QUESTION 4

4.1 Find the volume of a solid whose base is the triangular region with vertices $(0,0)$, $(1,0)$ and $(0,1)$ and whose cross-sections perpendicular to the y-axis are squares. [1(

4.2 Use cylindrical shells to find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and the x-axis about the line $x = 2$. [1(

QUESTION 5

5.1 Find the length of the curve with parametric equations $x = \cos t$, $y = t + \sin t$, $0 \leq t \leq \pi$. [***Hint:*** $1 + \cos t = 2\cos^2\left(\dfrac{t}{2}\right)$.] [1(

5.2 Find the length of the curve $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $1 \leq x \leq 2$. [1(

QUESTION 6

Test each of the following series for convergence or divergence. State the test used.

6.1 $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$ [4]

6.2 $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$ [5]

6.3 $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$ [5]

6.4 $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$ [5]

QUESTION 7

7.1 Show that the p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$. [1

7.2 Find the radius of convergence and the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}.$$

[1

END OF EXAMINATION PAPER