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University of Swaziland

Supplementary Examination, 2011/12

BSc II, Bass II, BEd II, BEng

Title of Paper : Calculus II

Course Number : M212

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Evaluate the double integral of $f(x,y) = xy^2 + x^2$ over the region bounded by the curves $y = x^2$ and $x = y^2$. [10]

(b) Evaluate

$$\int\int_R (4 - x^2 - y)\mathrm{d}x\mathrm{d}y$$

where R is the region bounded by $x = \sqrt{4-y},\ x = 0,\ y = 0.$ [10]

Question 2

(a)

(i) Sketch the graph of the curve

$$r = 1 + \sin\theta.$$ [3]

(ii) Find the area enclosed by the curve in (i). [5]

(b) Find the volume under the surface

$$z = f(x,y) = x^4y^4$$

and above the circle $x^2 + y^2 = 1$. [12]

Question 3

(a) Consider Laplace's equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0,$$

where $z = f(x, y)$. Show that under the transformation $x = r\cos\theta,\ y = r\sin\theta$, Laplace's equation takes the form

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{\partial^2 f}{\partial \theta^2} = 0. \qquad [10]$$

(b) Find the directional derivative of

$$f(x, y) = x^3 e^y + xy$$

in the direction of the vector from $P_0(4, 0, 16)$ to $P_1(-3, 1, 4)$. [6]

(c) Show that $f(x, y) = \cos(x + y)$ is a solution to

$$\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y} = 0. \qquad [4]$$

Question 4

Consider the cardioid

$$\left(x^2 + y^2 - x\right)^2 = x^2 + y^2.$$

(a) Transform the equation of the cardioid from cartesian to polar coordinates.

(ii) Sketch the cardioid.

(c) Find the area enclosed by the cardioid.

(d) Find the length of the cardioid.

Question 5

(a) Find and classify the critical points of

$$f(x, y) = x^3 + y^3 - 3xy. \qquad [8]$$

(b) Use the method of Lagrange multipliers to find the extreme values of

$$f(x, y) = xy$$

subject to

$$4x^2 + 8y^2 = 16.$$

[12]

Question 6

(a) Find $\mathrm{d}f$ when $f(x, y) = x^2 e^y \cos(xy)$. [5]

(b) Find $g(x, y)$ such that

$$\mathrm{d}g = \left[2y^2(\sin x + x\cos x) - ye^{xy}\right]\mathrm{d}x + \left[4xy\sin x - xe^{xy} + 2y\right]\mathrm{d}y.$$

[5]

(c) Show that the functions

(i) $f(x, y, z) = \sqrt{x^2 + y^2 + z}$

(ii) $f(x, y) = e^x \cos y$

are harmonic. [10]

Question 7

(a) Find the maximum and minimum values of the function

$$f(x, y, z) = xyz$$

subject to $x^2 + y^2 + z^2 = 1$. [12]

(b) Find f_{xx}, f_{xy} and f_{yy} for

$$f(x, y) = x^2 + xy + y^2 \sin\left(\frac{x}{y}\right).$$

[8]