

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2012/2013

IDE-Dip. Comm

TITLE OF PAPER : QUANTITATIVE TECHNIQUES

COURSE NUMBER : MS 202

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS.
3. NON PROGRAMMABLE
CALCULATORS MAY BE USED.

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) A firm produces two types of calculators, x thousand of type A and y thousand of type B per year. If the revenue and cost equations for the year are (in millions of dollars)

$$R(x, y) = 2x + 3y$$

$$C(x, y) = x^2 - 2xy + 2y^2 + 6x - 9y + 5$$

- i. Determine how many of each type of calculator should be produced per year to maximize profit? [5 marks]
 - ii. What is the maximum profit? [2 marks]
 - iii. Verify that this is indeed the maximum profit. [3 marks]
- (b) i. Find and classify all critical points of the function
 $f(x, y) = 4x^3 - 36x^2 + 3y^2 + 12xy + 156x - 12y + 24$. [10 marks]

QUESTION 2

2. (a) i. Reduce the determinant

$$\begin{vmatrix} 1 & 2 & 1 \\ 1 & -1 & -1 \\ 2 & 1 & 1 \end{vmatrix} \quad (1)$$

to triangular form and hence evaluate it. [6 marks]

- ii. Solution of the linear system

$$\begin{array}{rcrcrcrcrcl} x & + & 2y & + & z & = & 3 \\ x & - & y & - & z & = & 0 \\ 2x & + & y & + & z & = & -1 \end{array}$$

depends on the determinant in (1). Without solving this linear system, can a solution be found? Justify your answer. [2 marks]

- (b) An economy is based on 3 industries: agriculture, health and services.

Each $E1$ in agriculture requires 25c in health, 15c in services, and 10c in services. Each $E1$ in health takes 24c in agriculture, 5c in health and 18c in services, while each $E1$ in services uses 8c in agriculture, 8c in health and 4c in health.

Find the production schedule for the economy if demand is for $E50$ million in agriculture, $E79.9$ million in health, and $E85.4$ million in services.

[12 marks]

QUESTION 3

3. A company manufactures stools and tables. Each stool requires 1 hour of carpentry, 1 hour of painting and 2 hours of finishing. Similarly, a table needs 2 hours of carpentry, 1 hour of painting and 1 hour of finishing. During each production period, the carpentry, painting and finishing departments can only work for up to 10 hours, 7 hours and 12 hours respectively. The company makes E40 profit per stool and E30 profit per table.
- (a) The problem is to determine the number of stools and tables that should be made in order to maximize profits. Formulate this as a linear programming problem. [8 marks]
- (b) Solve linear programming problem by the simplex method. [12 marks]

QUESTION 4

4. Two dietary drinks are used to supply protein and carbohydrates. The first drink provides 1 unit of protein and 3 units of carbohydrates in each litre. The second drink supplies 2 units of protein and 2 units of carbohydrates in each litre. An athlete requires 3 units of protein and 5 units of carbohydrates. The first drink costs E4 per litre and the second costs E2 per litre.
- (a) The problem is to find the amount of each drink the athlete should consume to minimize the cost and still meet the minimum dietary requirements. Formulate this as a linear programming problem. [8 marks]
- (b) Solve linear programming problem by graphical method. [12 marks]

QUESTION 5

5. An clothing company ships cotton from 3 farms, X , Y and Z , to its 3 factories, A , B and C . Table (1) shows the demand, availabilities and unit costs of transportation.

	A	B	C	Availability
X	5	4	3	10
Y	8	4	3	30
Z	9	7	5	30
Demand	30	20	20	

Table 1: Demand, supply and unit cost values

Starting with the north-west corner solution and using the stepping-stone method, determine the transportation pattern that minimises the total cost.

What is the minimum total transportation cost? [20 marks]

QUESTION 6

6. (a) A company wishes to assign its employees 1, 2, 3, 4 to 4 different projects. The assignment costs are given as follows:

Cost	A	B	C	D
1	10	14	16	13
2	12	13	15	12
3	9	12	12	11
4	14	16	18	16

Determine the optimal assignment schedule that minimizes the total cost. [10 marks]

- (b) A company has 4 employees 1, 2, 3, 4 to assign to 4 projects A, B, C, D based on the following scores:

Score	A	B	C	D
1	80	55	45	45
2	58	35	70	50
3	70	50	80	65
4	90	70	40	80

Determine the optimal assignment schedule that maximizes the total score. [10 marks]

QUESTION 7

7. (a) A loan of $E500$ is due in 6 months with interest charged at 5.3% per annum. The debtor makes a first payment of $E300$ in 2 months, followed by a payment of $E100$ in 4 months. Find the balance payable on due date under the Merchant's rule. [6 marks]
- (b) A farmer wishes to purchase a new tractor in 6 years time. He figures that he will need $E130000$ then. What sum must he invest the end of each month in a fund paying 9.5% compounded monthly in order to accumulate the price of the tractor? [8 marks]
- (c) What sum of money should be set aside to provide an income of $E720$ every 6 months for the next 8 years if the money earns interest at rate 6% compounded semi-annually? [6 marks]