
University of Swaziland



Re-sit Examination – July 2017

BSc I, BEd I, BEng I, BASS I

Title of Paper : Introduction to Calculus

Course Number : MAT112

Time Allowed — : Three (3) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY THREE (3) questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN
BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. $\lim_{x \rightarrow 2} \left(\frac{8 - x^3}{x^2 - 4} \right)$ [4 marks]

b. $\lim_{\theta \rightarrow 0} \left(\frac{\tan 4\theta}{\theta} \right)$ [3 marks]

A.2 Find $\frac{dy}{dx}$ if

a. $y = (1 - 4x^3)^{15}$ [3 marks]

b. $y = \frac{2 + 3x^2}{3 - 2x^2}$ [4 marks]

c. $y = \ln(\sec x + \tan x)$ [4 marks]

A.3 Use the *limit definition* to find $\frac{df}{dx}$ given

$$f(x) = \frac{1}{x}. \quad [7 \text{ marks}]$$

A.4 Integrate

a. $\int_0^1 \frac{x \, dx}{1 + x^2}$ [3 marks]

b. $\int x^2 \sin\left(\frac{1}{2}x\right) \, dx$ [4 marks]

c. $\int_0^{\frac{\pi}{4}} 8 \tan^2 \theta \, d\theta$ [4 marks]

d. $\int \frac{dx}{x - x^2}$ [4 marks]

Section B

Answer ANY THREE (3) Questions in this section

B.1 a. Evaluate

i. $\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - \sqrt{4-x}}{x}$ [7 marks]

ii. $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \tan \theta}$ [3 marks]

b. Use the *limit definition* to find y' given

$$y = \sqrt{4x - 1}. \quad [10 \text{ marks}]$$

B.2 a. Find $\frac{dy}{dx}$ and simplify given

$$y = \frac{e^x - e^{-x}}{e^x + e^{-x}}. \quad [6 \text{ marks}]$$

b. Consider the function

$$y = 10 + 18x^2 - x^4.$$

i. Find the stationary points of y . [6 marks]

ii. Determine the nature of each stationary point. [4 marks]

iii. Make a sketch of the graph of y . [4 marks]

B.3 a. Differentiate and simplify

$$y = \ln(1 + x^2) - \tan^{-1} x - \frac{x}{1 + x^2}. \quad [8 \text{ marks}]$$

b. A closed cylindrical can is to hold 54π litres of liquid. Find the dimensions of such a can which requires the minimum material to construct. [12 marks]

B.4 a. Integrate

$$\int e^{-2x} \sin 3x \, dx. \quad [10 \text{ marks}]$$

b. Derive the formula

$$A = \pi r^2$$

for the area of a circle with radius r . [10 marks]

B.5 Integrate

a. $\int \frac{dx}{e^{2x} - 2e^x}$ [13 marks]

b. $\int_0^3 \frac{dx}{\sqrt{9 - x^2}}$ [7 marks]

END OF EXAMINATION
