

**UNIVERSITY OF SWAZILAND**

**MAIN EXAMINATION PAPER 2016**

**TITLE OF PAPER : INFERENCE STATISTICS**

**COURSE CODE : ST 220**

**TIME ALLOWED : THREE (3) HOURS**

**REQUIREMENTS : CALCULATOR AND STATISTICAL TABLES**

**INSTRUCTIONS : THIS PAPER HAS SIX (6) QUESTIONS AND TWO SECTIONS. ANSWER ALL QUESTIONS IN SECTION ONE, ANY THREE (3) QUESTIONS IN SECTION TWO**

## SECTION ONE

### (ANSWER ALL QUESTIONS)

#### Question 1

**[10 marks, 1 mark each]**

Choose the correct answer from the alternatives provided.

1. Which of the following is not a property of the normal distribution?
  - (a) It is symmetric about its mean
  - (b) It is bell-shaped
  - (c) It is common
  - (d) It is unimodal
2. The government claims that students earn an average of SZL4500 during their summer break from studies. A random sample of students gave a sample average of SZL3975 and a 95% confidence interval was found to be  $(SZL3525 < \mu < SZL4425)$ . This interval is interpreted to mean that:
  - (a) if the study were to be repeated many times, there is a 95% probability that the true average summer earnings is not SZL4500 as the government claims.
  - (b) because our specific confidence interval does not contain the value SZL4500 there is a 95% probability that the true average summer earnings is not SZL4500.
  - (c) if we were to repeat our survey many times, then about 95% of all the confidence intervals will contain the value SZL4500.
  - (d) if we repeat our survey many times, then about 95% of our confidence intervals will contain the true value of the average earnings of students.
  - (e) there is a 95% probability that the true average earnings are between SZL3525 and SZL4425 for all students.
3. Which of the following statements about confidence intervals is *incorrect*?
  - (a) If we keep the sample size fixed, the confidence interval gets wider as we increase the confidence coefficient.
  - (b) A confidence interval for a mean always contains the sample mean.
  - (c) If we keep the confidence coefficient fixed, the confidence interval gets narrower as we increase the sample size.
  - (d) If the population standard deviation increases, the confidence interval decreases in width.
  - (e) If the confidence intervals for two means do not overlap very much, there is evidence that the two population means are different.
4. A 95 percent confidence interval for the mean time taken to process new insurance policies is (11, 12) days. This interval can be interpreted to mean that:
  - (a) only 5 percent of all policies take less than 11 or more than 12 days to process
  - (b) only 5 percent of all policies take between 11 and 12 days to process

- (c) about 95 out of every 100 such intervals constructed from random samples of the same size will contain the population mean processing time
  - (d) the probability is .95 that all policies take between 11 and 12 days to process
  - (e) none of the above
5. A turkey producer knows from previous experience that profits are maximized by selling turkeys when their average weight is 12 kilograms. Before determining whether to put all their full grown turkeys on the market this month, the producer wishes to estimate their mean weight. Prior knowledge indicates that turkey weights have a standard deviation of around 1.5 kilograms. The number of turkeys that must be sampled in order to estimate their true mean weight to within 0.5 kilograms with 95% confidence is:
- (a) 35
  - (b) 5
  - (c) 65
  - (d) 10
  - (e) 150
6. A confidence statement includes what two things?
- (a) margin of error and bias
  - (b) bias and variability
  - (c) bias and confidence level
  - (d) confidence level and margin of error
7. I read an advertisement recently in which a credit card company promised that I could reduce my debt by 150 percent. Which of the following statements is (are) true?
- (a) This is possible if my debt is more than 150 dollars.
  - (b) This is possible if my debt has recently increased by at least 150 percent.
  - (c) The company's claim makes no sense.
  - (d) Both (a) and (b).
8. The alternative hypothesis for the Chi-square test of independence is that the variables are
- (a) dependent
  - (b) related
  - (c) independent
  - (d) always zero
9. The diameter of ball bearings are known to be normally distributed with unknown mean and variance. A random sample of size 25 gave a mean 2.5 cm. The 95% confidence interval had length 4 cm. Then
- (a) The sample variance is 4.86.
  - (b) The sample variance is 26.03.

- (c) The population variance is 4.84.
  - (d) The population variance is 23.47.
  - (e) The sample variance is 23.47.
10. The 0.01 level of significance is used in an experiment and a two-tailed hypothesis test applied. Computed  $z$  is found to be -2.0. This indicates:
- (a)  $H_0$  should be accepted
  - (b) We should reject  $H_0$  and accept  $H_1$
  - (c) We should have used the 0.05 level of significance.
  - (d) None of these is correct.

## Question 2

[15 marks, 1 mark each]

State whether each of these statements is true or false, giving brief reasons why this is so (*Note that no marks will be awarded for a simple true/false reply*)

1. When using a large random sample, we cannot assume that its mean forms part of a normal distribution.
2. The least squares regression line minimizes the sum of absolute deviations.
3. The power of a test is the probability of a type 2 error.
4. If two variables are correlated then they must have a linear relationship.
5. The sampling distribution of the mean is distributed the same way as the original observations.
6. A chi-squared value can be positive.
7. The significance level of a test is greater than the probability of a Type 1 error.
8. The mean of a dataset is always smaller than the mode.
9. Two 95% confidence intervals for the same parameter should have the same length.
10. If  $\mathcal{P}(A|B) > \mathcal{P}(B|A)$  then  $\mathcal{P}(A) > \mathcal{P}(B)$ .
11. If  $H_0$  is false, a high level of power increases the probability we will reject it.
12. Other things being equal, larger confidence levels provide a smaller margin of error.
13. The critical region for rejection of  $H_0$  is the area under the curve which contains all the values of the statistic which fail to allow rejection of  $H_0$ .
14. Confidence statements are statements applicable only to the sample of individuals measured.
15. Nonsampling errors can not occur in a census.

## SECTION TWO

### (ANSWER ANY THREE QUESTIONS)

#### Question 3

**[25 marks, 8+6+8+3]**

It is assumed that there is a linear relationship between yield of apple trees and the amount of fertiliser supplied to them. In order to test this assumption, nine apple trees of the same type were randomly selected and supplied weekly with a fixed quantity ( $x$  grams) of fertiliser. The yield of each apple tree ( $y$  kilograms) was recorded.

$x$	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$y$	3.9	4.3	5.5	6.4	6.9	7.1	7.3	7.7	8.0

- Calculate the least squares line of  $y$  on  $x$  ( $\sum x^2 = 96$ ,  $\sum xy = 186.75$  and  $\sum y^2 = 379.51$ ).
- Compute and interpret the coefficient of determination.
- Is the *relationship* between the *amount of fertiliser* supplied and *yield* meaningful (or significant)? Use  $\alpha = 0.05$ .
- What prediction will you give for a tree that is treated weekly with 0.0032 kg of fertiliser?

#### Question 4

**[20 marks, 10+15]**

- It is suspected that the number of faulty products produced by a manufacturing industry varies depending on which day of the week they were made. In order to test this a random sample of 200 products is taken from the warehouse, and after inspection yields the following results.

Product	Day of manufacture		
	Monday	Tuesday-Thursday	Friday
Perfect	32		94
Faulty	3		21

Use the  $\chi^2$  test to decide if these results indicate that there is a connection between faulty products and when they were made.

- The production manager of Raylite batteries, a car battery manufacturer, wants to know whether the three machines used for this process (labelled A, B and C) produce equal amount of rejects. A random sample of shifts for each machine was selected and the number of rejects produced per shift was recorded. The number of shifts selected were 6, 4 and 5 for machines A, B and C respectively. The average number of rejects for machines A and C were 10.5 and 14.0 respectively. The following information is provided as well.

	Degrees of freedom	Sum of squares	Mean sum of squares	F value
Battery				4.165
Residuals		76.5		
Total				

By completing the table using the information provided, can the production manager of Raylite batteries concluded that the three machines used to manufacture car batteries produce rejects at the *same average rate* per shift. Use  $\alpha = 0.05$  and show the ANOVA table. Also state the necessary assumptions.

## Question 5

[25 marks, 10+10+5]

- (a) A consumer report examined potential differences between two brands of tyres. The mean life of the tyres is of primary concern. The available data, measured in thousands of miles, are provided below:

	Sample size $n$	Sample mean $m$	Sample standard deviation $s$
Brand A	34	21.4	1.5
Brand B	38	22.3	1.8

Assuming that the population variances for the two *brands* are equal, use an appropriate hypothesis test to determine whether the mean lives of the two brands are different.

- (b) The student union of a large university gathered a random sample of 525 students to determine whether they are in favour of a new grading system. The results are summarized in the table below.

	Sample size	Number in favour or new grading system
Humanities	325	221
Science	200	120

- (i) Do the results indicate a difference between humanities and science in the population proportions in favour of the new grading system? Conduct an appropriate test and comment of your results.
- (ii) Give a 97% confidence interval for the difference between the two proportions in the population.

## Question 6

[25 marks, 10+4+6+5]

- (a) Ten randomly selected oil wells in a large field of oil wells produced 21, 19, 20, 22, 24, 21, 19, 22, 22, and 20 barrels of crude oil per day. Is this evidence at the 0.01 level of significance that the oil wells are not producing an average of 22.5 barrels of crude oil per day?
- (b) Suppose the probability is 0.30 that any given student in a large class can provide the answer to an assigned problem. What is the probability that the fourth student randomly selected by the instructor will be the first one who can provide the answer to the problem?
- (c) A test is taken by some students, their marks are recorded and we are interested in the properties of the sample mean. Under the assumption that the marks follow a Normal distribution with exact mean 60 and variance 81, calculate the probability that the mark of a randomly selected student?

(i) is greater than 59.5 exactly; and

(ii) lies between 59 and 60.5 exactly.

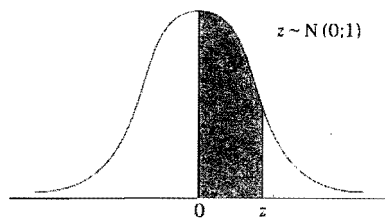
- (d) A blended wine is intended to comprise two parts of Sauvignon to one part of Merlot. The amounts dispensed to make up a nominal 75cl bottle of this wine are  $X$  cl of Sauvignon and  $Y$  cl of Merlot, where  $X$  and  $Y$  are assumed to be independent Normally distributed random variables with respective means 52 and 26 cl and respective variances 1 and 0.5625. Find the probability that the actual volume of wine dispensed into a bottle is less than the nominal volume.

# APPENDIX 1: LIST OF STATISTICAL TABLES

## TABLE 1

### The standard normal distribution (z)

This table gives the area under the standard normal curve between 0 and z  
i.e.  $P[0 < Z < z]$

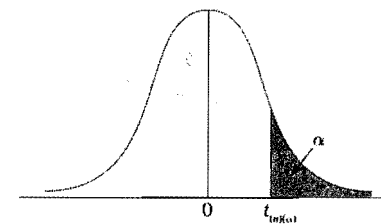


Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0358
0.1	0.0438	0.0478	0.0518	0.0558	0.0598	0.0638	0.0677	0.0716	0.0756	0.0795
0.2	0.0834	0.0873	0.0911	0.0949	0.0987	0.1025	0.1063	0.1101	0.1139	0.1176
0.3	0.1213	0.1251	0.1288	0.1325	0.1361	0.1398	0.1434	0.1470	0.1506	0.1541
0.4	0.1577	0.1611	0.1645	0.1679	0.1712	0.1746	0.1780	0.1814	0.1847	0.1881
0.5	0.1915	0.1948	0.1981	0.2014	0.2047	0.2079	0.2112	0.2144	0.2177	0.2209
0.6	0.2241	0.2273	0.2304	0.2335	0.2366	0.2396	0.2427	0.2457	0.2487	0.2517
0.7	0.2547	0.2576	0.2605	0.2633	0.2661	0.2688	0.2715	0.2743	0.2769	0.2796
0.8	0.2823	0.2849	0.2875	0.2899	0.2924	0.2948	0.2972	0.2995	0.3019	0.3042
0.9	0.3065	0.3088	0.3109	0.3129	0.3149	0.3169	0.3188	0.3206	0.3224	0.3241
1.0	0.3258	0.3275	0.3291	0.3307	0.3322	0.3338	0.3353	0.3368	0.3382	0.3396
1.1	0.3410	0.3425	0.3438	0.3451	0.3465	0.3478	0.3491	0.3504	0.3517	0.3529
1.2	0.3541	0.3554	0.3566	0.3578	0.3589	0.3601	0.3612	0.3623	0.3634	0.3645
1.3	0.3655	0.3665	0.3675	0.3685	0.3695	0.3705	0.3715	0.3724	0.3733	0.3743
1.4	0.3752	0.3761	0.3770	0.3779	0.3788	0.3796	0.3805	0.3813	0.3821	0.3829
1.5	0.3838	0.3846	0.3854	0.3861	0.3869	0.3876	0.3883	0.3890	0.3897	0.3904
1.6	0.3910	0.3917	0.3924	0.3930	0.3936	0.3941	0.3946	0.3951	0.3956	0.3961
1.7	0.3966	0.3970	0.3975	0.3979	0.3983	0.3987	0.3990	0.3994	0.3998	0.4001
1.8	0.4005	0.4008	0.4011	0.4014	0.4017	0.4019	0.4022	0.4025	0.4027	0.4029
1.9	0.4031	0.4033	0.4035	0.4037	0.4039	0.4041	0.4043	0.4045	0.4046	0.4048
2.0	0.4049	0.4050	0.4051	0.4052	0.4053	0.4054	0.4055	0.4056	0.4056	0.4057
2.1	0.4057	0.4058	0.4058	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
2.2	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
2.3	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
2.4	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
2.5	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
2.6	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
2.7	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
2.8	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
2.9	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
3.0	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
3.1	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
3.2	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
3.3	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
3.4	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
3.5	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
3.6	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
3.7	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
3.8	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
3.9	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059
4.0	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059	0.4059

## TABLE 2

### The t distribution

This table gives the value of  $t_{(n)(\alpha)}$   
where  $n$  is the degrees of freedom  
i.e.  $\alpha = P[t \geq t_{(n)(\alpha)}]$



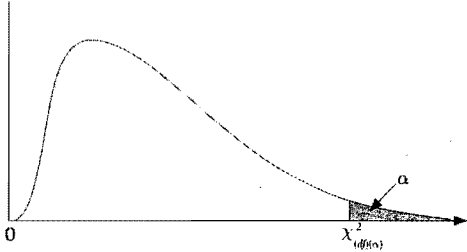
	0.100	0.050	0.025	0.010	0.005	0.0025
1	0.699	0.999	1.279	1.879	2.259	2.759
2	0.699	0.999	1.279	1.879	2.259	2.759
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87	0.699	0.999	1.279	1.879	2.259	2.759
88	0.699	0.999	1.279	1.879	2.259	2.759
89	0.699	0.999	1.279	1.879	2.259	2.759
90	0.699	0.999	1.279	1.879	2.259	2.759
91	0.699	0.999	1.279	1.879	2.259	2.759



TABLE 3

The Chi-Squared distribution ( $\chi^2$ )

This table gives the value of  $\chi^2_{(df)(\alpha)}$  where  $df$  is the degrees of freedom i.e.  $\chi^2_{(df)(\alpha)} = P[\chi^2 > \chi^2_{(df)(\alpha)}]$

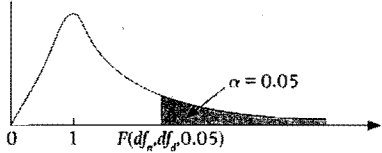


	0.100	0.050	0.025	0.01	0.005	0.0025
1	3.841	5.024	6.635	16.013	19.023	21.009
2	3.000	3.841	5.991	10.597	12.592	13.824
3	2.366	3.000	5.408	9.348	11.345	12.838
4	1.936	2.366	4.838	8.445	10.297	11.998
5	1.676	1.936	4.351	7.779	9.488	11.154
6	1.501	1.676	3.940	7.231	8.845	10.591
7	1.390	1.501	3.619	6.841	8.343	10.142
8	1.312	1.390	3.357	6.501	7.963	9.778
9	1.259	1.312	3.178	6.256	7.633	9.488
10	1.219	1.259	3.000	6.032	7.378	9.236
11	1.183	1.219	2.878	5.858	7.163	9.025
12	1.155	1.183	2.764	5.691	6.958	8.837
13	1.131	1.155	2.658	5.538	6.779	8.661
14	1.109	1.131	2.561	5.393	6.615	8.501
15	1.089	1.109	2.476	5.259	6.467	8.357
16	1.071	1.089	2.392	5.136	6.327	8.224
17	1.055	1.071	2.319	5.021	6.193	8.099
18	1.040	1.055	2.246	4.913	6.064	7.981
19	1.026	1.040	2.173	4.812	5.941	7.871
20	1.013	1.026	2.101	4.718	5.823	7.764
21	1.001	1.013	2.030	4.630	5.710	7.661
22	0.990	1.001	1.960	4.547	5.601	7.561
23	0.980	0.990	1.891	4.469	5.496	7.464
24	0.971	0.980	1.823	4.396	5.394	7.371
25	0.963	0.971	1.757	4.327	5.296	7.281
26	0.956	0.963	1.692	4.262	5.201	7.193
27	0.949	0.956	1.628	4.200	5.110	7.108
28	0.943	0.949	1.565	4.141	5.022	7.025
29	0.937	0.943	1.503	4.085	4.937	6.944
30	0.932	0.937	1.442	4.032	4.854	6.865
40	0.909	0.909	1.344	3.757	4.551	6.446
50	0.891	0.891	1.276	3.599	4.353	6.259
60	0.879	0.879	1.224	3.456	4.190	6.094
70	0.869	0.869	1.183	3.328	4.055	5.964
80	0.861	0.861	1.149	3.215	3.935	5.858
90	0.854	0.854	1.120	3.115	3.830	5.764
100	0.850	0.850	1.093	3.029	3.745	5.689

TABLE 4 (a)

F distribution ( $\alpha = 0.05$ )

The entries in this table are critical values of  $F$  for which the area under the curve to the right is equal to 0.05.



	Degrees of freedom for numerator									
	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	234.02	237.08	239.49	241.25	242.59
2	18.513	18.008	17.592	17.259	16.977	16.733	16.517	16.324	16.150	16.000
3	10.128	9.552	9.161	8.845	8.581	8.357	8.162	7.993	7.840	7.700
4	7.709	7.115	6.711	6.393	6.128	5.893	5.696	5.525	5.370	5.230
5	6.591	5.994	5.588	5.269	4.993	4.758	4.560	4.388	4.232	4.092
6	5.965	5.367	4.960	4.641	4.365	4.130	3.932	3.760	3.604	3.464
7	5.591	4.992	4.584	4.265	3.989	3.754	3.556	3.384	3.228	3.088
8	5.318	4.719	4.311	3.992	3.716	3.481	3.283	3.111	2.955	2.815
9	5.101	4.502	4.094	3.775	3.499	3.264	3.066	2.894	2.738	2.598
10	4.923	4.324	3.916	3.597	3.321	3.086	2.888	2.716	2.560	2.420
11	4.770	4.171	3.763	3.444	3.168	2.933	2.735	2.563	2.407	2.267
12	4.634	4.035	3.627	3.308	3.032	2.797	2.599	2.427	2.271	2.131
13	4.510	3.911	3.503	3.184	2.908	2.673	2.475	2.303	2.147	2.007
14	4.396	3.797	3.389	3.070	2.794	2.559	2.361	2.189	2.033	1.893
15	4.292	3.693	3.285	2.966	2.690	2.455	2.257	2.085	1.929	1.789
16	4.198	3.599	3.191	2.872	2.596	2.361	2.163	1.991	1.835	1.695
17	4.113	3.514	3.106	2.787	2.511	2.276	2.078	1.906	1.750	1.610
18	4.037	3.438	3.030	2.711	2.435	2.200	1.992	1.820	1.664	1.524
19	3.969	3.370	2.962	2.643	2.367	2.132	1.924	1.752	1.596	1.456
20	3.908	3.309	2.901	2.582	2.306	2.071	1.863	1.691	1.535	1.395
21	3.853	3.254	2.846	2.527	2.251	2.016	1.808	1.636	1.480	1.340
22	3.802	3.203	2.795	2.476	2.199	1.964	1.756	1.584	1.428	1.288
23	3.755	3.156	2.748	2.429	2.153	1.918	1.710	1.538	1.382	1.242
24	3.711	3.112	2.703	2.384	2.107	1.872	1.664	1.492	1.336	1.196
25	3.669	3.070	2.660	2.341	2.064	1.829	1.621	1.449	1.293	1.153
26	3.629	3.030	2.619	2.299	2.022	1.787	1.579	1.407	1.251	1.111
27	3.590	2.991	2.579	2.259	1.982	1.747	1.539	1.367	1.211	1.071
28	3.553	2.953	2.540	2.220	1.943	1.708	1.490	1.318	1.162	1.022
29	3.517	2.916	2.502	2.182	1.905	1.670	1.452	1.280	1.124	0.984
30	3.483	2.880	2.465	2.145	1.868	1.633	1.415	1.243	1.087	0.947
40	3.347	2.744	2.330	2.011	1.734	1.499	1.281	1.109	0.953	0.813
50	3.284	2.681	2.267	1.948	1.671	1.436	1.218	1.046	0.890	0.750
60	3.234	2.631	2.217	1.898	1.621	1.386	1.168	0.996	0.840	0.700
70	3.192	2.589	2.175	1.856	1.579	1.344	1.126	0.954	0.798	0.658
80	3.157	2.554	2.140	1.821	1.544	1.309	1.091	0.919	0.763	0.623
90	3.127	2.524	2.110	1.791	1.514	1.279	1.061	0.889	0.733	0.593
100	3.101	2.498	2.084	1.765	1.488	1.253	1.035	0.863	0.707	0.567

TABLE 4 (a) continued

*F* distribution ( $\alpha = 0.05$ )

Degrees of freedom for denominator	Degrees of freedom for numerator	
	1	2
1	161.277	199.509
2	18.5128	18.5128
3	10.1286	10.1286
4	7.70861	7.70861
5	6.59166	6.59166
6	5.98737	5.98737
7	5.59142	5.59142
8	5.31800	5.31800
9	5.10099	5.10099
10	4.96464	4.96464
12	4.75346	4.75346
14	4.59923	4.59923
16	4.47713	4.47713
18	4.37454	4.37454
20	4.28757	4.28757
25	4.13284	4.13284
30	4.04541	4.04541
40	3.91942	3.91942
50	3.83715	3.83715
60	3.77453	3.77453
80	3.69957	3.69957
100	3.64515	3.64515
120	3.60322	3.60322
140	3.57189	3.57189
160	3.54797	3.54797
180	3.52949	3.52949
200	3.51443	3.51443
250	3.47535	3.47535
300	3.44438	3.44438
400	3.39732	3.39732
500	3.36176	3.36176
600	3.33741	3.33741
800	3.30327	3.30327
1000	3.28325	3.28325
1200	3.26823	3.26823
1400	3.25621	3.25621
1600	3.24619	3.24619
1800	3.23717	3.23717
2000	3.22915	3.22915
2500	3.20413	3.20413
3000	3.18411	3.18411
4000	3.15409	3.15409
5000	3.13407	3.13407
6000	3.11905	3.11905
8000	3.08903	3.08903
10000	3.07401	3.07401
12000	3.06400	3.06400
14000	3.05600	3.05600
16000	3.04900	3.04900
18000	3.04300	3.04300
20000	3.03800	3.03800
25000	3.02800	3.02800
30000	3.02000	3.02000
40000	3.00500	3.00500
50000	2.99500	2.99500
60000	2.98700	2.98700
80000	2.97200	2.97200
100000	2.96200	2.96200
120000	2.95700	2.95700
140000	2.95300	2.95300
160000	2.95000	2.95000
180000	2.94800	2.94800
200000	2.94600	2.94600
250000	2.94100	2.94100
300000	2.93700	2.93700
400000	2.93000	2.93000
500000	2.92500	2.92500
600000	2.92100	2.92100
800000	2.91200	2.91200
1000000	2.90700	2.90700
1200000	2.90300	2.90300
1400000	2.90000	2.90000
1600000	2.89800	2.89800
1800000	2.89600	2.89600
2000000	2.89500	2.89500
2500000	2.89000	2.89000
3000000	2.88600	2.88600
4000000	2.87900	2.87900
5000000	2.87400	2.87400
6000000	2.87000	2.87000
8000000	2.86100	2.86100
10000000	2.85600	2.85600
12000000	2.85200	2.85200
14000000	2.84900	2.84900
16000000	2.84700	2.84700
18000000	2.84500	2.84500
20000000	2.84400	2.84400
25000000	2.83900	2.83900
30000000	2.83500	2.83500
40000000	2.82800	2.82800
50000000	2.82300	2.82300
60000000	2.81900	2.81900
80000000	2.81000	2.81000
100000000	2.80500	2.80500
120000000	2.80100	2.80100
140000000	2.79900	2.79900
160000000	2.79700	2.79700
180000000	2.79600	2.79600
200000000	2.79500	2.79500
250000000	2.79000	2.79000
300000000	2.78600	2.78600
400000000	2.77900	2.77900
500000000	2.77400	2.77400
600000000	2.77000	2.77000
800000000	2.76100	2.76100
1000000000	2.75600	2.75600
1200000000	2.75200	2.75200
1400000000	2.75000	2.75000
1600000000	2.74800	2.74800
1800000000	2.74600	2.74600
2000000000	2.74500	2.74500
2500000000	2.74000	2.74000
3000000000	2.73600	2.73600
4000000000	2.72900	2.72900
5000000000	2.72400	2.72400
6000000000	2.72000	2.72000
8000000000	2.71100	2.71100
10000000000	2.70600	2.70600
12000000000	2.70200	2.70200
14000000000	2.70000	2.70000
16000000000	2.69800	2.69800
18000000000	2.69600	2.69600
20000000000	2.69500	2.69500
25000000000	2.69000	2.69000
30000000000	2.68600	2.68600
40000000000	2.67900	2.67900
50000000000	2.67400	2.67400
60000000000	2.67000	2.67000
80000000000	2.66100	2.66100
100000000000	2.65600	2.65600
120000000000	2.65200	2.65200
140000000000	2.65000	2.65000
160000000000	2.64800	2.64800
180000000000	2.64600	2.64600
200000000000	2.64500	2.64500
250000000000	2.64000	2.64000
300000000000	2.63600	2.63600
400000000000	2.62900	2.62900
500000000000	2.62400	2.62400
600000000000	2.62000	2.62000
800000000000	2.61100	2.61100
1000000000000	2.60600	2.60600
1200000000000	2.60200	2.60200
1400000000000	2.60000	2.60000
1600000000000	2.59800	2.59800
1800000000000	2.59600	2.59600
2000000000000	2.59500	2.59500
2500000000000	2.59000	2.59000
3000000000000	2.58600	2.58600
4000000000000	2.57900	2.57900
5000000000000	2.57400	2.57400
6000000000000	2.57000	2.57000
8000000000000	2.56100	2.56100
10000000000000	2.55600	2.55600
12000000000000	2.55200	2.55200
14000000000000	2.55000	2.55000
16000000000000	2.54800	2.54800
18000000000000	2.54600	2.54600
20000000000000	2.54500	2.54500
25000000000000	2.54000	2.54000
30000000000000	2.53600	2.53600
40000000000000	2.52900	2.52900
50000000000000	2.52400	2.52400
60000000000000	2.52000	2.52000
80000000000000	2.51100	2.51100
100000000000000	2.50600	2.50600
120000000000000	2.50200	2.50200
140000000000000	2.50000	2.50000
160000000000000	2.49800	2.49800
180000000000000	2.49600	2.49600
200000000000000	2.49500	2.49500
250000000000000	2.49000	2.49000
300000000000000	2.48600	2.48600
400000000000000	2.47900	2.47900
500000000000000	2.47400	2.47400
600000000000000	2.47000	2.47000
800000000000000	2.46100	2.46100
1000000000000000	2.45600	2.45600
1200000000000000	2.45200	2.45200
1400000000000000	2.45000	2.45000
1600000000000000	2.44800	2.44800
1800000000000000	2.44600	2.44600
2000000000000000	2.44500	2.44500
2500000000000000	2.44000	2.44000
3000000000000000	2.43600	2.43600
4000000000000000	2.42900	2.42900
5000000000000000	2.42400	2.42400
6000000000000000	2.42000	2.42000
8000000000000000	2.41100	2.41100
10000000000000000	2.40600	2.40600
12000000000000000	2.40200	2.40200
14000000000000000	2.40000	2.40000
16000000000000000	2.39800	2.39800
18000000000000000	2.39600	2.39600
20000000000000000	2.39500	2.39500
25000000000000000	2.39000	2.39000
30000000000000000	2.38600	2.38600
40000000000000000	2.37900	2.37900
50000000000000000	2.37400	2.37400
60000000000000000	2.37000	2.37000
80000000000000000	2.36100	2.36100
100000000000000000	2.35600	2.35600
120000000000000000	2.35200	2.35200
140000000000000000	2.35000	2.35000
160000000000000000	2.34800	2.34800
180000000000000000	2.34600	2.34600
200000000000000000	2.34500	2.34500
250000000000000000	2.34000	2.34000
300000000000000000	2.33600	2.33600
400000000000000000	2.32900	2.32900
500000000000000000	2.32400	2.32400
600000000000000000	2.32000	2.32000
800000000000000000	2.31100	2.31100
1000000000000000000	2.30600	2.30600
1200000000000000000	2.30200	2.30200
1400000000000000000	2.30000	2.30000
1600000000000000000	2.29800	2.29800
1800000000000000000	2.29600	2.29600
2000000000000000000	2.29500	2.29500
2500000000000000000	2.29000	2.29000
3000000000000000000	2.28600	2.28600
4000000000000000000	2.27900	2.27900
5000000000000000000	2.27400	2.27400
6000000000000000000	2.27000	2.27000
8000000000000000000	2.26100	2.26100
10000000000000000000	2.25600	2.25600
12000000000000000000	2.25200	2.25200
14000000000000000000	2.25000	2.25000
16000000000000000000	2.24800	2.24800
18000000000000000000	2.24600	2.24600
20000000000000000000	2.24500	2.24500
25000000000000000000	2.24000	2.24000
30000000000000000000	2.23600	2.23600
40000000000000000000	2.22900	2.22900
50000000000000000000	2.22400	2.22400
60000000000000000000	2.22000	2.22000
80000000000000000000	2.21100	2.21100
100000000000000000000	2.20600	2.20600
120000000000000000000	2.20200	2.20200
140000000000000000000	2.20000	2.20000
160000000000000000000	2.19800	2.19800
180000000000000000000	2.19600	2.19600
200000000000000000000	2.19500	2.19500
250000000000000000000	2.19000	2.19000
300000000000000000000	2.18600	2.18600
400000000000000000000	2.17900	2.17900
500000000000000000000	2.17400	2.17400
600000000000000000000	2.17000	2.17000
800000000000000000000	2.16100	2.16100
1000000000000000000000	2.15600	2.15600
1200000000000000000000	2.15200	2.15200
1400000000000000000000	2.15000	2.15000
1600000000000000000000	2.14800	2.14800
1800000000000000000000	2.14600	2.14600
2000000000000000000000	2.14500	2.14500
2500000000000000000000	2.14000	2.14000
3000000000000000000000	2.13600	2.13600
4000000000000000000000	2.12900	2.12900
5000000000000000000000	2.12400	2.12400
6000000000000000000000	2.12000	2.12000
8000000000000000000000	2.11100	2.11100
10000000000000000000000	2.10600	2.10600
12000000000000000000000	2.10200	2.10200
14000000000000000000000	2.10000	2.10000
16000000000000000000000	2.09800	2.09800
18000000000000000000000	2.09600	2.09600

## MEASURES OF DISPERSION AND SKEWNESS

**Range** Range = Maximum value – Minimum value + 1  

$$= x_{\max} - x_{\min} + 1$$

**Variance** Mathematical – ungrouped data

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{(n-1)}$$

Computational – ungrouped data

$$s^2 = \frac{\sum x_i^2 - n\bar{x}^2}{(n-1)}$$

**Standard deviation**  $s = \sqrt{s^2}$

**Coefficient of variation**  $CV = \frac{s}{\bar{x}} \times 100\%$

**Pearson's coefficient of skewness**

$$sk_p = \frac{n\sum (x_i - \bar{x})^3}{(n-1)(n-2)s^3}$$

$$sk_p = \frac{3(\text{Mean} - \text{Median})}{\text{Standard deviation}} \quad (\text{approximation})$$

## PROBABILITY CONCEPTS

**Conditional probability**  $P(A/B) = \frac{P(A \cap B)}{P(B)}$

**Addition rule** Non-mutually exclusive events  
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Mutually exclusive events  
 $P(A \cup B) = P(A) + P(B)$

**Multiplication rule** Statistically dependent events  
 $P(A \cap B) = P(A/B) \times P(B)$

Statistically independent events  
 $P(A \cap B) = P(A) \times P(B)$

**n! = n factorial**  $n \times (n-1) \times (n-2) \times (n-3) \times \dots \times 3 \times 2 \times 1$

**Permutations**  ${}_nP_r = \frac{n!}{(n-r)!}$

**Combinations**  ${}_nC_r = \frac{n!}{r!(n-r)!}$

## PROBABILITY DISTRIBUTIONS

**Binomial distribution**  $P(x) = {}_nC_x p^x (1-p)^{(n-x)}$  for  $x = 0, 1, 2, 3, \dots, n$

$P(x \text{ successes}) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$  for  $x = 0, 1, 2, 3, \dots, n$

**Binomial descriptive measures**

Mean  $\mu = np$

Standard deviation  $\sigma = \sqrt{np(1-p)}$

**Poisson distribution**  $P(x) = \frac{e^{-a} a^x}{x!}$  for  $x = 0, 1, 2, 3 \dots$

**Poisson descriptive measures**

Mean  $\mu = a$

Standard deviation  $\sigma = \sqrt{a}$

**Standard normal probability**  $z = \frac{x - \mu}{\sigma}$

## CONFIDENCE INTERVALS

**Single mean**  $n$  large; variance known

$$\bar{x} - z \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z \frac{\sigma}{\sqrt{n}}$$

(lower limit) (upper limit)

$n$  small; variance unknown

$$\bar{x} - t_{(n-1), \frac{\alpha}{2}} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{(n-1), \frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

(lower limit) (upper limit)

**Single proportion**  $p - z \sqrt{\frac{p(1-p)}{n}} \leq \pi \leq p + z \sqrt{\frac{p(1-p)}{n}}$

(lower limit) (upper limit)

## HYPOTHESES TESTS

**Single mean** Variance known

$$z\text{-stat} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Variance unknown;  $n$  small

$$t\text{-stat} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

9.2

**Single proportion**  $t\text{-stat} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$

**Difference between two means** Variances known

$$z\text{-stat} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

9.1

Variances unknown;  $n_1$  and  $n_2$  small

$$t\text{-stat} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} \text{ where } s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

2

**Paired t-test**  $t\text{-stat} = \frac{\bar{x}_d - \mu_d}{\frac{s_d}{\sqrt{n}}}$

where  $\mu_d = (\mu_1 - \mu_2)$

and  $s_d = \sqrt{\frac{\sum (x_d - \bar{x}_d)^2}{n - 1}}$

**Differences between two proportions**

$$z\text{-stat} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\hat{\pi}(1 - \hat{\pi})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \text{ where } \hat{\pi} = \frac{x_1 + x_2}{n_1 + n_2}; p_1 = \frac{x_1}{n_1}; p_2 = \frac{x_2}{n_2} \quad 9.8$$

**Chi-Squared**  $\chi^2\text{-stat} = \sum \frac{(f_o - f_e)^2}{f_e}$

**Overall mean**  $\bar{x} = \frac{\sum \sum x_{ij}}{N}$

**Total sum of squares (SSTotal)**

$$= \sum_i \sum_j (x_{ij} - \bar{x})^2$$

$$\text{SST} = \sum_j n_j (\bar{x}_j - \bar{x})^2$$

$$\text{SSE} = \sum_i \sum_j (x_{ij} - \bar{x}_j)^2$$

$$\text{SSTotal} = \text{SST} + \text{SSE}$$

$$\text{MSTotal} = \frac{\text{SSTotal}}{N - 1}$$

$$\text{MST} = \frac{\text{SST}}{k - 1}$$

$$\text{MSE} = \frac{\text{SSE}}{N - k}$$

$$F\text{-stat} = \frac{\text{MST}}{\text{MSE}}$$